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An improved diameter scaling correlation for turbulent flow of drag-reducing polymer solutions

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Abstract

The friction coefficient was measured for developed flow of drag-reducing polymer solutions in tubes of 2, 5, 10, 20 and 52 mm i.d. Our results were processed along with other authors' data in terms of different parameters in order to investigate the possibility of developing a simple empirical method for the prediction of the pipe diameter effect on friction. We found that the drag-reduction coefficient (DR), if expressed as a function of the fluid bulk velocity (V), becomes independent of the tube diameter in the subcritical region (i.e. without fluid degradation), with the deviations being smaller than about 5% for all the diameters and velocities covered. This correlation proved to be not only better than similar procedures based on friction velocity, but also more convenient and physically more meaningful. It was also found that the logarithmic layer shift in the 3-layers velocity profile is also better correlated with the bulk velocity than with the friction velocity. Finally, existing models for drag-reduction involving non-dimensional correlations between the integral flow parameters and the fluid properties were also reevaluated in light of these findings. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Drag-reduction; Diameter effect; Turbulent flow; Polymer solutions; Scaling

1. Introduction

For turbulent flow of Newtonian fluids through smooth circular tubes, the friction coefficient or wall shear stress is a function of the Reynolds number only. For drag-reducing fluids in the non-asymptotic region this is not the case, and another parameter – which is function of the fluid and pipe diameter – is also necessary. In other words, for a given Reynolds number and a given fluid, the tube diameter will still appear as another parameter. This is the so-called 'diameter effect' for drag-reducing fluids. This effect is a vexing issue from the practical point of view, as test data obtained with one tube diameter cannot be readily extrapolated to other diameters, and data from different sources usually cannot be compared directly.

It is not very difficult to measure the drag-reduction level ('DR') for a given fluid in the laboratory in tubes with diameters around 10 or 20 mm, but much bigger pipes become problematic. For instance, we are conducting a field test in the hydronic cooling system of a building (Engineering II, UCSB campus),

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where the pipe diameters vary from 1.5″ to 6″, but we cannot easily measure drag-reduction in the laboratory using pipes of the largest size, so the only way to predict DR in such a building system is to rely on some scaling procedure.

Up to this point it does not appear that a complete understanding of the drag-reduction phenomenon has been achieved, and consequently no theoretical models exist that can predict appropriately this diameter effect. There exist some semi-empirical models based on fluid and flow parameters such as the Deborah or Weissenberg numbers (e.g. those by Katsibas et al. [1], Darby et al. [2,3], and Mironov and Shishov [4] that are intended to be predictive of general drag-reduction issues and could therefore also predict in principle the diameter effect. This type of models, however, – even though providing some physical insight into the fundamentals of the phenomenon – are still generally not providing fully conclusive answers. They may differ widely in nature, the diameter effect was not used as a major criterion for evaluation of their applicability, and they were generally not evaluated with experimental data covering a wide range of diameters.

Since the diameter effect is important for practical applications, and is also an interesting theoretical problem, some attempts were made to solve this problem separately from the overall drag-reduction issue. Those scaling techniques were more or less empirical, as were the global models of drag-reduction, but were focused on the diameter effect alone, the identification and measurement of the relevant fluid properties becoming therefore less important since diameter effect is generally defined as the variation of friction with pipe diameter for a given fluid of constant properties. Diameter effect also assumes that one considers the region of non-asymptotic flow since this effect is no longer visible under asymptotic conditions. This type of approach is the one we followed in the present work where we focused strictly on the diameter effect and did not attempt to generate models of wider drag-reduction applicability.

Although these scaling procedures are limited in scope compared to the general models for drag-reduction, they are typically subjected to more rigorous testing with experimental data covering wide range of diameters. If in addition to satisfying the scaling needs, such a procedure succeeds in revealing some physical reality, we may then also obtain some insight or even a partial solution to the global drag-reduction problem. Of course, successful scaling techniques – even fully empirical – would also provide a useful practical tool for scaling experimental data from small laboratory pipes to the large ones used in practical applications such as hydronic heating and cooling systems, for example, which are very attractive potential targets for implementation of drag-reducing additives [5]. (The practical application side of the problem also implies that simplicity of use of a scaling technique may be as desirable as high accuracy and that a few percent uncertainty for DR is readily acceptable for practical purposes). This is the type of scaling technique we were looking for in our work.

Several procedures have been proposed for the scaling-up of the friction coefficient in drag-reducing flows from small to large diameters – although much less work has been done on the heat transfer issue [6]. Most of the proposed methods like those presented by Granville [7,8] and Matthys and Sabersky [9], are based on considerations of the velocity profile in the developed pipe flow. They make use of the experimentally-confirmed concept of buffer layer, which is often modeled by the introduction of a linear displacement (ΔB^+) of the Newtonian profile in the law-of-the-wall universal velocity profile [10–12]. When this buffer layer extends to the pipe centerline, asymptotic drag-reduction results, and the diameter effect is no longer visible. Typically, all these scaling procedures use a velocity similarity law, assuming that the ΔB^+ term is independent of the diameter for a given fluid and friction velocity. All these techniques are reasonably successful over moderate range of diameters but being graphical or numerical are somewhat cumbersome to use. Hoyt [13], starting from the same 3-layers velocity profile, used the idea of negative roughness for his simpler analytical scaling procedure. Most of the 3-layers methods are equivalent fundamentally and give similar levels of correlation spread.

An early approach to the diameter scaling deserves special mention. Whitsitt et al. [14] proposed a procedure that correlates DR with the solution friction velocity (u_p^*), based on the assumption that wall shear stress is the mechanism that controls DR. This procedure has the drawback that both variables (DR and u_p^*) contain the unknown parameter (τ_w) and an iterative procedure is therefore necessary to predict the DR of the solution. A number of other authors e.g. Astarita et al. [15], Lee et al. [16], Savins and Seyer [17], then simplified Whitsitt's procedure by using the solvent friction velocity (u_w^*) instead of the solution friction velocity (u_p^*). The physical connection between the solvent friction velocity and the actual solution flow is then limited as the wall shear stress in an associated water flow has no direct relationship to the actual wall shear stress in the flow of the drag-reducing solution. Surprisingly, though, this procedure nevertheless showed better correlation from small to larger diameters than that based on solution friction velocity.

All the scaling procedures that assume velocity similarity law also have problems scaling from very small to large diameters. Hoyt and Sellin [18] suggested that the reason for this limited success is the relatively stronger effect of the laminar sublayer in small tubes than in larger tubes. They suggested that, in general, only tubes with diameters larger than 10 mm can be scaled adequately by almost all proposed methods. Interestingly, the Savins and Seyer method [17], which is more empirical (scaling with friction velocity), was singled out in the Hoyt and Sellin analysis as more satisfactory than more complex procedures when scaling between tubes with diameters smaller than 10 mm and the larger ones.

All the methods discussed above were developed for, and tested with, polymer solutions data. Recently, however, drag-reducing surfactant solutions have become of particular interest due to their greater potential for industrial applications, and in the midst of surfactant studies, the diameter effect problem has been revisited somewhat. Schmitt et al. [19] proposed two different empirical correlations for surfactants depending upon the shear stress level: DR vs. τ_w for high stresses and τ_w vs. V for low stresses. The former is the same as the procedure proposed by Whitsitt et al. for polymer solutions, but the τ_w vs. V approach is not and would lead to large errors in friction prediction for most polymer solutions. We have conducted many experimental studies ourselves with surfactant solutions, including scaling studies [20], and some of these will be presented in follow-up articles.

2. Experimental setup

It is important to realize that the availability of an appropriate data base for drag-reduction in tubes of various diameters is crucial for the development of a successful scaling procedure. To address this issue, we have built a setup allowing us to obtain accurate measurements over a wide range of parameters (e.g. velocities up to 20 m/s and total pressure drops up to 1000 psi). Four custom stainless steel test tubes with nominal i.d. of 2, 5, 10 and 20 mm were installed and instrumented [21]. It should be mentioned that the accuracy of the tube diameter measurements in particular, is very critical because the friction coefficient is a function of the diameter almost to the fifth power at a given Reynolds number for water. Accordingly, the accuracy of these and all other necessary measurements were addressed with particular attention, as we believe that only reliable and accurate measurements can provide a clue to the empirical solution of the diameter scaling problem. High-quality pressure tap holes are also

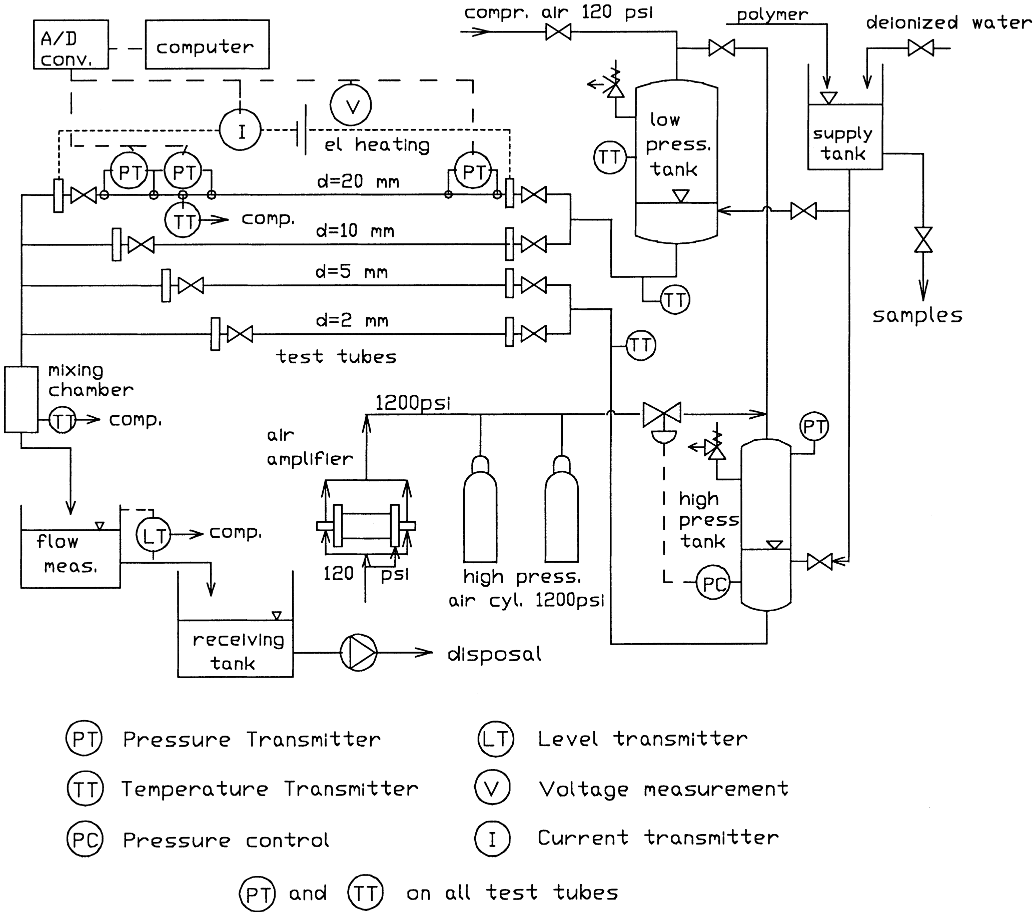


Fig. 1. Setup for diameter effect measurements: pipes of 2, 5, 10 and 20 mm i.d.

essential to avoid viscoelastic hole pressure errors, and these holes were therefore made by electric discharge machining (EDM). Fig. 1 shows a schematic of part of the setup we built for this purpose.

In addition to those four test tubes, a fifth test tube with an i.d. of 51.7 mm was also installed, with a circulation loop including a 7.5 HP centrifugal pump and a large calibrated tank for volumetric flow rate measurement. The main section of the test tube was made of 2″ diameter, schedule 40, PVC pipe. The PVC pipe provided easy handling, a smooth surface, and corrosion resistance. The measurements were taken over a 6 m-long PVC pipe section only, since a length of about 11 m had to be provided for the entry length, and about 2 m had to be left downstream from the second pressure tap section. The viscoelastic pressure hole errors due to the imperfect holes that can be achieved in a plastic pipe could then be very large relative to the small pressure drop measured, however, and could reduce the accuracy of the measurements to an unacceptably low level. On the other hand, pressure drop measurements in stainless steel tubes of the same diameter showed at least 10 times smaller pressure hole error, due to our ability to make much smoother and more uniform pressure holes in stainless steel than in PVC. Accordingly, in order to eliminate the problem with the 51.7 mm test tube, two sections of stainless steel tube (of lengths 2 and 1.5 m approximately) were inserted in the plastic line as shown in Fig. 2.

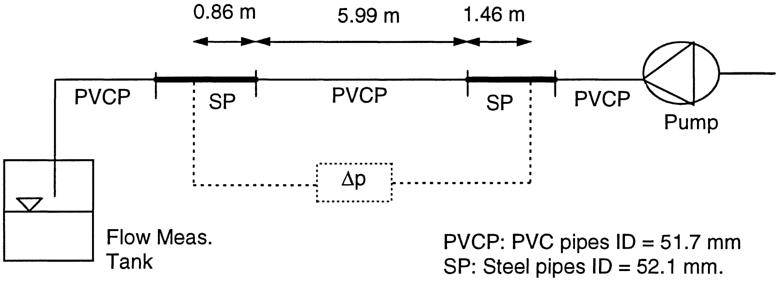


Fig. 2. Test pipe layout for the 52 mm i.d. tube. The pipe is assembled from sections of PVC pipe with stainless steel pipe inserts where the taps for pressure drop measurements are located.

Each of these stainless steel sections provide one location for the pressure drop measurement. At each pressure tap location, four radial holes were drilled and reamed, and the inner tube surface was honed. This procedure is believed to reduce the pressure drop uncertainty due to the viscoelastic hole errors to less than about 2%. The maximal pressure transducer uncertainty guaranteed by the manufacturer's calibration is 0.25% of full scale. There is a slight difference in diameter between the PVC and steel pipe (51.7 mm for the PVC and 52.1 mm for the stainless steel or about 1%). As in the case of the small tubes, the largest uncertainty in the results is assumed to be due to tube diameter uncertainty. Although a large diameter can be measured very accurately directly at the end of tube sections (we found less than 0.2% deviation for all sections in one case), it is more realistic to use an uncertainty level twice as large (0.4%), because the plastic tubes are subjected to possible deformations. Particular care was taken, however, to ensure that the transition from one section to another was as smooth as possible, and our experience shows that the 1% difference in diameter does not introduce any disturbance to the flow which would affect the pressure drop over a 6 m long pipe more than the estimated overall uncertainty margin of 1–2%. (The difference in diameter is of course taken into account when calculating the velocity). All together, we expect our uncertainty margin for the calculated drag-reduction coefficient for the 51.7 mm tube to be about 2% or 3% [21]. This is also the level of uncertainty we estimated for the other four pipes. (This relatively high accuracy is due to the fact that the error in pipe diameter cancels out in the calculation of the drag-reduction coefficient). The level of uncertainty on the velocity ranges from about 5% in the smallest pipe to 2% or 3% for the biggest ones (again, most of the error being due to the pipe diameter uncertainty), and about the same for the Reynolds number. The actual uncertainty level for a given run depends of course also on the specific testing conditions for that run.

This installation was designed and built for heat transfer measurements as well, but we limit our discussion here to friction results with the heat transfer data to be presented elsewhere.

3. Discussion of experimental results

Some data obtained with a solution of polyacrylamide (Separan AP-273 by Dow Chemicals) in deionized water are shown in Figs. 3 and 4 for two concentrations (20 and 12 wppm), in the form of friction coefficient as a function of solvent Reynolds number. The solutions were prepared by the introduction of alcohol-wetted powder in a water jet and then let to stabilize for a period of a few days.

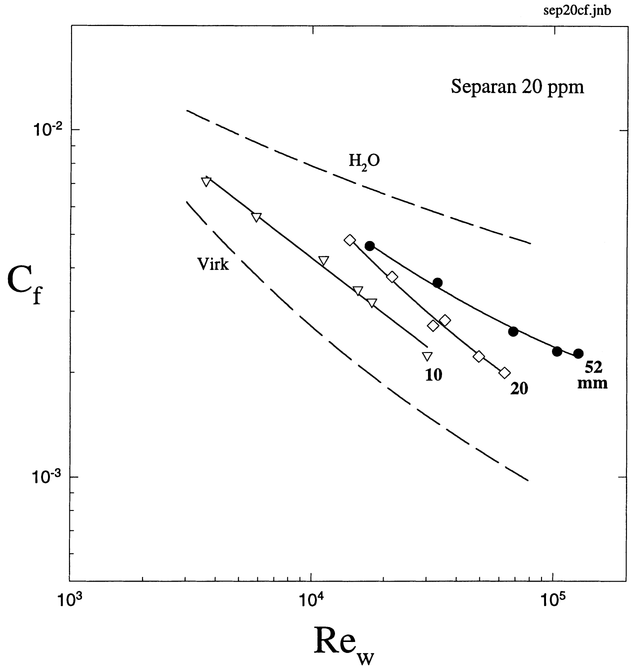


Fig. 3. Friction coefficients for three different pipe diameters (52, 20 and 10 mm) as a function of solvent Reynolds number for a 20 ppm polyacrylamide solution (Separan AP-273).

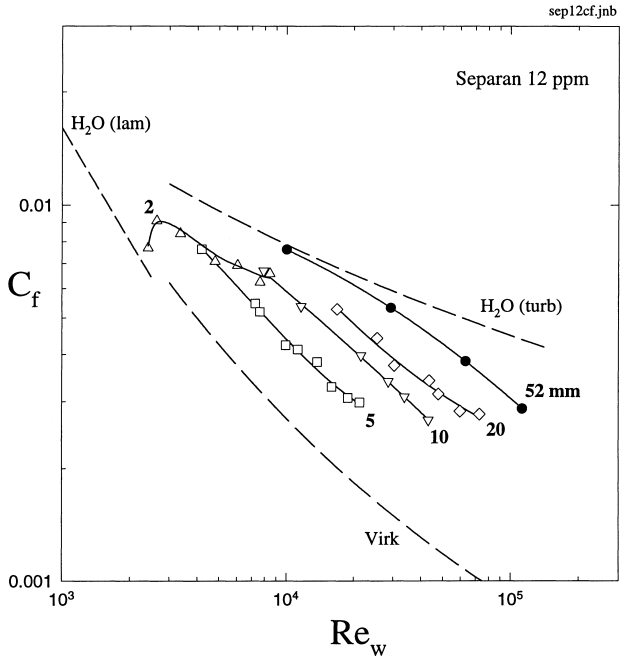


Fig. 4. Friction coefficients for five different pipe diameters (52, 20, 10, 5 and 2 mm) as a function of solvent Reynolds number for a 12 ppm polyacrylamide solution (Separan AP-273). (Note the degradation experienced in the 2 mm tube).

The water and solution conductivities were continuously monitored showing some increase with time due to contaminants, with an effective water quality for most experiments somewhere between deionized and tap water. The 20 ppm solution was subsequently mechanically degraded slightly prior to the tests in order to avoid it being asymptotic in the smaller pipes. Both solutions showed viscosities very close to that of water, suggesting a true dilute molecular regime.

As seen in those figures, and as expected, the friction coefficient for the polymer solution is a strong function of the tube diameter in the non-asymptotic regime. Reynolds numbers based on the solvent viscosity (water) are used here because the viscosity of polymer solutions at such low concentrations does not show any noticeable deviation from the viscosity of pure water; and a small difference in viscosity is acceptable for our diameter scaling procedure purposes.

The drag-reduction level – DR – is used as the dependent variable in all the scaling procedures we discuss hereafter, and it is calculated as

$$\mathrm{DR} = \frac{C_{\mathrm{fw}} - C_{\mathrm{fp}}}{C_{\mathrm{fw}}},$$

where C_f stands for friction coefficient and the subscripts w and p refer to water and the polymer solution, respectively. The use of this parameter DR is typical for most empirical scaling procedures, and provides a direct comparison of friction for water and drag-reducing solution as a function of the different independent variables used. A word of caution is due here in that a change in C_f at high levels of DR is not reflected as a proportional change in the value of DR, since the latter is limited to the value corresponding to asymptotic conditions. However, for practical purposes the DR presentation is satisfactory since we are primarily interested in the comparison between the given drag-reducing solution and water. Except for the 12 ppm solution in the 2 mm pipe, the data do not show obvious evidence of mechanical degradation.

For the same experimental results shown in Fig. 3. Figure 5 shows the parameter DR plotted as a function of the solution (polymer) friction velocity (u_p^*), the solvent (water) friction velocity (u_w^*) and the bulk velocity (V), where the friction velocity is defined as $u^* = \sqrt{\tau_\mathrm{w}/\rho}$, with τ_w and u^* the wall shear stress and the friction velocity, respectively. (The solvent friction velocity is calculated with the shear stress for water at the same Reynolds number). The first two representations (u_p^* and u_w^*) are those used earlier by several authors. The third one (V) is the one we have proposed previously for surfactant solutions [20] and believe to be preferable as discussed below.

Although these representations show reasonably good invariance with pipe diameter, it is apparent that the spread of the experimental data is somewhat reduced when the solvent friction velocity (u_w^*) is used instead of the solution friction velocity (u_p^*), as mentioned in earlier work. The fact that u_w^* as the invariant provides better correlation than u_p^* appears somewhat puzzling at first, given the common belief that the shear stress is the governing phenomenon in the subcritical region [17], and also since the actual solution properties should be more relevant than those of an hypothetical solvent fluid.

Our third representation shows DR as a function of the bulk velocity (V) of the fluid, which shows an even better correlation than the two procedures based on friction velocities over this entire range of diameters and velocities. (We did not have enough fluid from this particular batch to run experiments in the 5 and 2 mm pipes).

Fig. 6 shows the same data as Fig. 4 (12 wppm Separan AP-273) plotted in the same three representations. In this case, the measurements were taken in tubes with diameters 2, 5, 10, 20 and 52 mm, and the same improvement is seen as the dependent parameter changes from u_p^* to u_w^* to V.

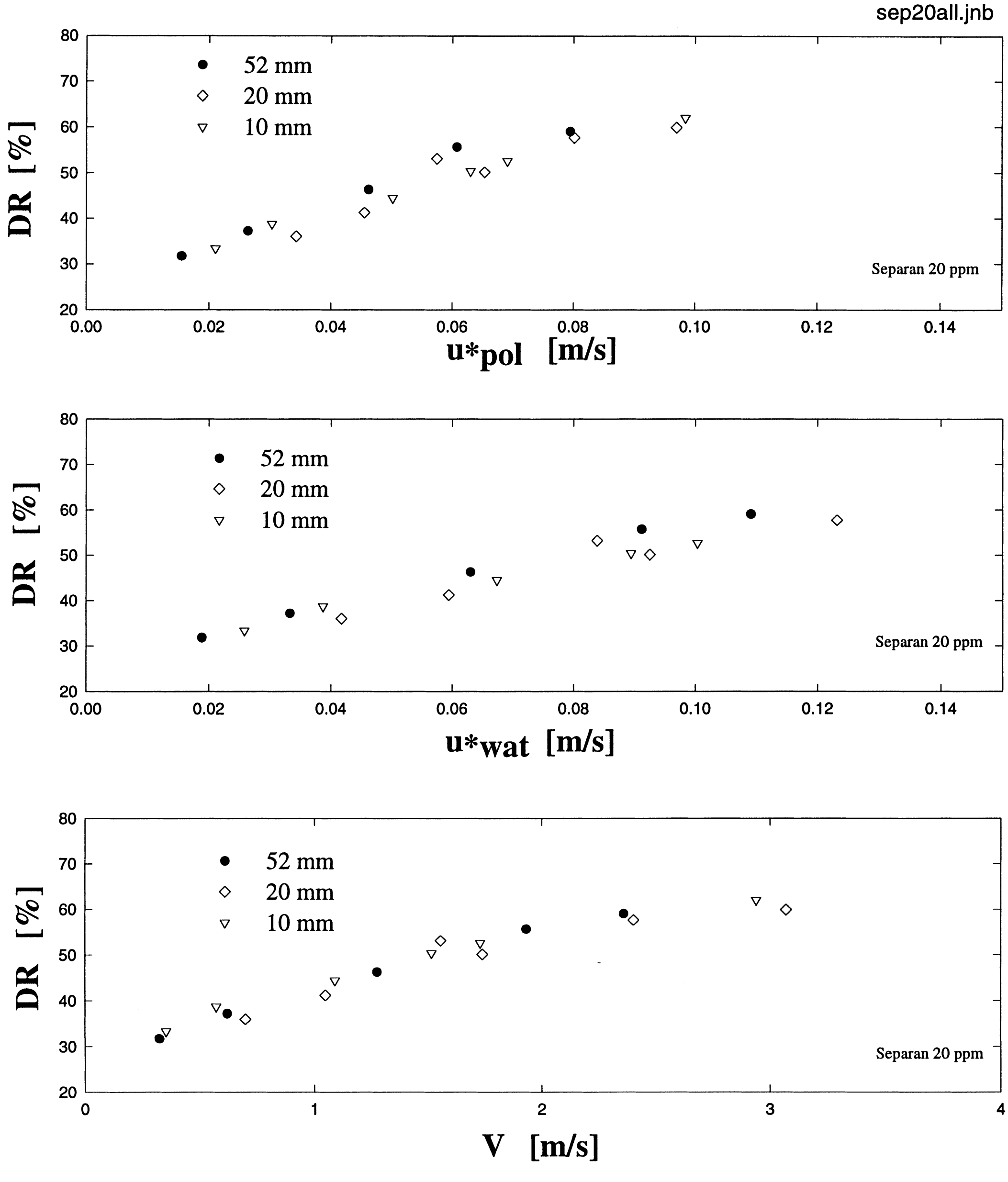


Fig. 5. Drag-reduction level as a function of the friction velocity of the solution (polymer), friction velocity of the solvent (water), and bulk velocity for the 20 ppm polyacrylamide solution in pipes of 10, 20, and 52 mm i.d.

Disregarding the smallest pipe (2 mm) for which degradation occurred at velocities above 1 m/s, the spread of the measured DR data as a function of *V* is less than about 5% for both solutions shown in Figs. 5 and 6, which can be considered to be within the limits of experimental uncertainty. For velocities below 1 m/s even the 2 mm tube appears well correlated, although there are few data in that region.

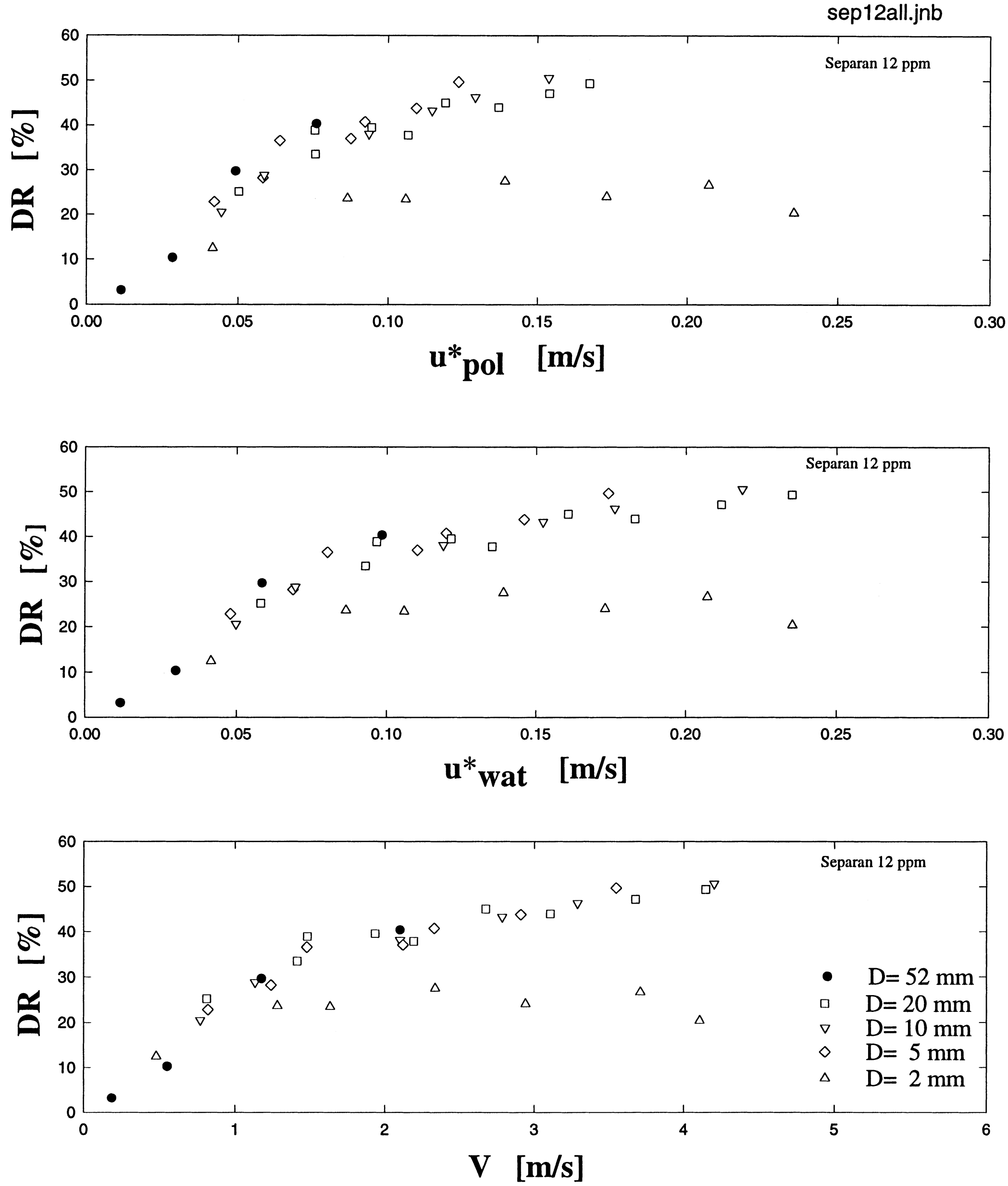


Fig. 6. Drag-reduction level as a function of friction velocity of the solution, friction velocity of the solvent and bulk velocity for the 12 ppm solution in pipes of 2, 5, 10, 20, and 52 mm i.d. The smallest pipe (2 mm) led to mechanical degradation of the fluid.

To test this scaling procedure further, we applied it to two other sets of data published in the literature. Fig. 7 shows data reported in Sellin and Ollis [22] and Ollis [23] with diameters ranging from 1 to 50 mm. The polymer is Alcomer (Polyethylene oxide) with a concentration of 10 ppm and we used only the data in the region without degradation. The success of this scaling procedure is evident, all data

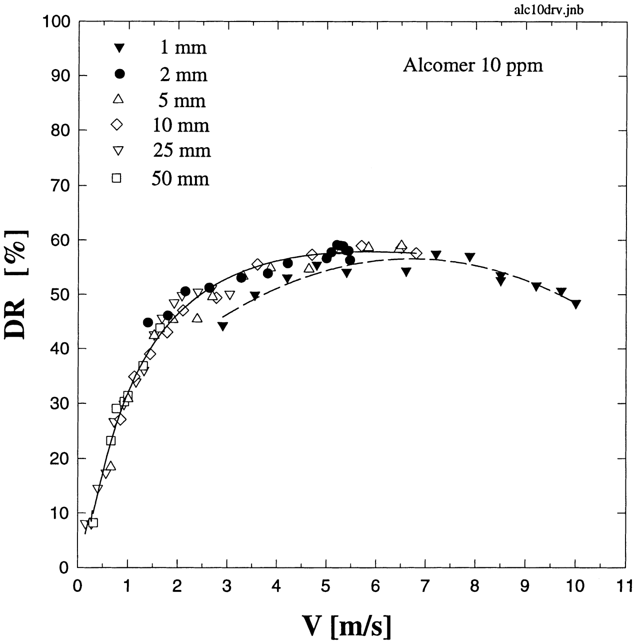


Fig. 7. Drag-reduction as a function of bulk velocity for a 10 ppm Alcomer (Polyox) solution in 1, 2, 5, 10, 25, and 50 mm i.d. Data from Ollis [23].

falling within about 5% of the correlation curve. (Note that the 1 mm pipe data depart slightly from the rest. These data do not correlate with those for the other pipes as in that range of velocities the 1 mm pipe shows already asymptotic drag-reduction). Especially striking is the good scaling even for small diameters, as was the case with our own data and unlike what was observed with other scaling techniques, as discussed above. Fig. 8 shows the application of our scaling procedure for data reported by DeLoof et al. [24] for tubes with large diameters (from 52.5 to 208 mm). Again there is no systematic difference, with all deviations possibly due to experimental error.

It is interesting to examine how this scaling procedure relates to those based on friction velocity. Although the underlying principle for our correlation parameter is physically very different from that in the two previously considered methods (i.e. using bulk velocity instead of wall shear stress as the invariant), the difference in the spread of the results is not extremely large, which may look strange at first sight. The reason for this is that the wall shear stress is a quadratic function of the velocity ($\tau_w = C_f \rho V^2/2$) but only a weak function of other factors as the friction coefficient is only a $-1/4$ power of the Reynolds number for turbulent flow of water. The solvent friction velocity (an artificial concept for an actual solution flow) calculated at the same bulk velocity is therefore, approximately proportional to the actual solution bulk velocity, whereas the solution friction velocity – which shows a stronger dependence of the friction coefficient on the Reynolds number – will not be related as simply to the bulk velocity. The reason why the solvent friction velocity – even though not physically very relevant – works better as an invariant than the solution friction velocity may then be understood as resulting from the fact that the former is really a ‘hidden’ form of the bulk velocity itself.

The fact that DR can be scaled better with bulk velocity than with friction velocity suggests the possibility that bulk velocity is a more important parameter for DR phenomena than the shear stress

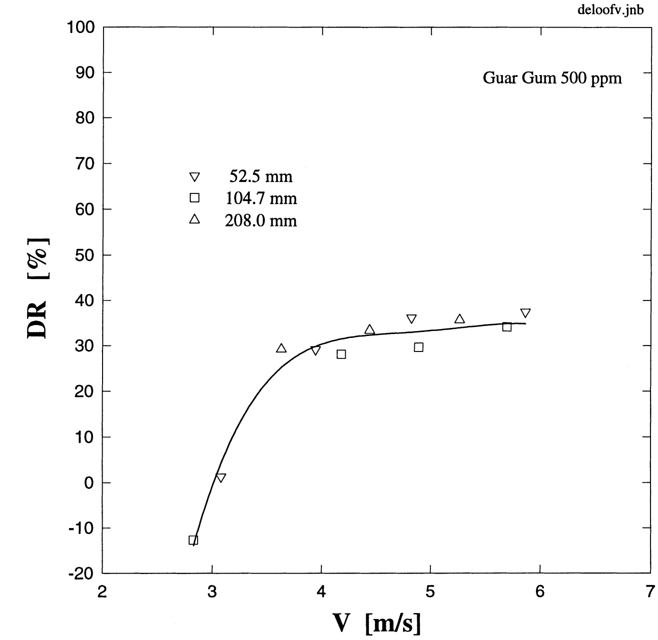


Fig. 8. Drag-reduction as a function of bulk velocity for a 500 ppm Guar Gum solution in pipes of 52, 104, and 208 mm i.d. Data from DeLoof et al. [24].

(friction velocity), which is usually assumed to be the critical one. It may be then interesting to apply this hypothesis to different aspects of our knowledge of DR phenomena. Let us consider the concept that DR in pipes of different diameters might be a function of the bulk velocity only for a given fluid in the intermediate regime. As we saw, experimental data from three sources indicate that this applies within the limits of experimental error, the spread in DR for pipes ranging from 5 to 50 mm and from 50 to 200 mm being of the order of 5% or less for three separate databases. The problem of the diameter effect can be then approached from two directions: (1) an analysis of the velocity profile (local flow parameters), and (2) the relationship between global flow parameters (bulk velocity, friction velocity, etc). With this experimental law for scaling, we can now go back and see if any new information or understanding of the general drag-reduction phenomenon can be generated. Some preliminary thoughts to this effect are mentioned below.

Let us note first that scaling technique based on bulk velocity would not be satisfactory near the onset of drag-reduction since this onset is believed to take place at a given shear stress independent of the pipe diameter. Scaling with friction velocity would therefore, work better in that region. From a practical scaling point of view, the onset issue is less important, however, since applications would typically be aimed at high levels of reduction, and also since drag-reduction increases very fast with velocity near onset with the result that the difference between scaling with shear and bulk velocity would not be large, especially at those low levels of drag-reduction. On the fundamental side, however, the onset problem is certainly very interesting, of course. It may well be that drag-reduction involves two different types of processes: one where the fluid properties are affected through molecular conformational change perhaps primarily due to shear stress, and the other where 'organized' (e.g. stretched or aligned) molecules affect the flow–fluid interactions, perhaps best characterized by bulk velocity as an overall flow characteristic. If this is indeed the case, a scaling technique using only one of the two parameters would not be sufficient to cover adequately the whole range of velocities from onset to asymptote, since the first process (shear stress) might be dominant near onset and the second (bulk velocity) in the intermediate regime. Be that as it may, we are indeed interested more in the intermediate regime than in the onset region since this is the most interesting one for practical applications, and have indeed aimed at a scaling technique in that region, the focus of this paper.

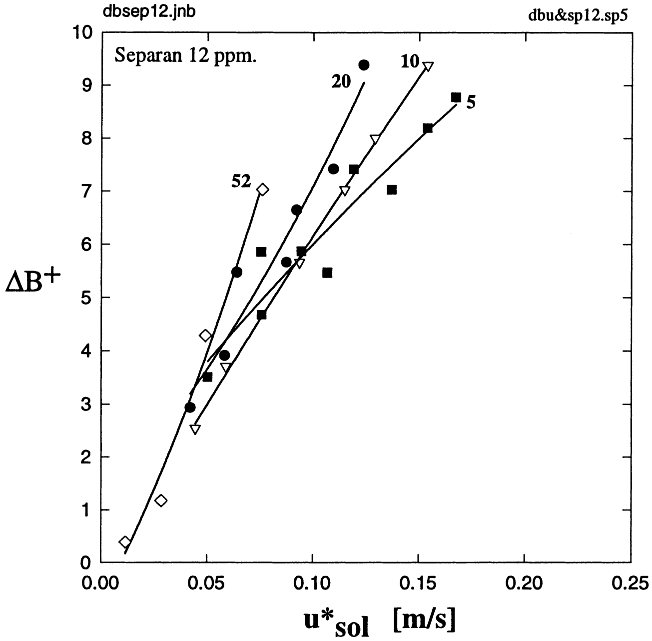
4. Implications of the results

4.1. Velocity profile

The friction coefficient is an integral expression of the velocity profile in that the friction coefficient can be calculated by integration if the velocity profile is known. Our scaling law is a relationship of global flow parameters (friction coefficient and velocity), and cannot provide enough information a priori to deduce the velocity field (local flow parameters). Fortunately, there is ample experimental information about velocity profiles, and we can use our (experimentally-proven) scaling procedure to test or modify the velocity profile models proposed previously. Several experiments have confirmed that most polymeric solutions (at least those which belong to the 'Type A' [25]), follow Virk's 3-layers model [26] very closely, with an elastic layer added between the viscous and the turbulent layers typical of the velocity profile for Newtonian fluids. Virk postulated that the thickness of the elastic layer (in the usual wall coordinates) is independent of pipe diameter, and for a given fluid is a unique function of wall shear stress or friction velocity. This is based on the assumption that most drag-reduction phenomena are dominated by the wall shear stress (the onset of drag-reduction, for example). That same 3-layers model was the basis for the scaling procedures based on the integration of the velocity profile [7,9,13,22] (which differ only in the calculation procedures, but should show the same results) with the logarithmic layer shift assumed to be a function of friction velocity alone. These approaches appear, however, to give less satisfactory invariance than our scaling with bulk velocity, as suggested by the results shown in Figs. 5 and 6. Since our scaling procedure results imply that bulk velocity may be a more relevant global flow parameter for drag-reducing flow than friction velocity, it is worth checking if the thickness of the elastic layer may not be a function of bulk velocity instead of the friction velocity. As the resolution of the velocity profile measurements is limited, this idea can in fact perhaps be better tested by global measurements of friction coefficients in pipes of different diameters than by direct velocity profile measurements.

For this purpose, we used our experimental data for the 12 ppm Separan AP273 solution in pipes of different diameters to calculate the thickness of the elastic layer (ΔB^+), through integration of the 3-layers profile. The calculated ΔB^+ can then be presented as a function of either friction velocity or bulk velocity as the independent variable. Some results are presented in Fig. 9 for the various diameters used (except for 2 mm pipe in which degradation took place).

The calculated ΔB^+ is seen to be indeed closer to being independent of pipe diameter at a given bulk velocity than at a given friction velocity. (Importantly, the apparently large relative spread of the data does not translate into large variations in predicted DR, however, as seen in Fig. 5). The 3-layers profile together with the concept of ΔB^+ being a function of the bulk velocity only – will therefore – allow better scaling, as did the empirical correlation DR $= f(V)$ that we discussed above. (The former is of course much less convenient, however, given the need for numerical processing).



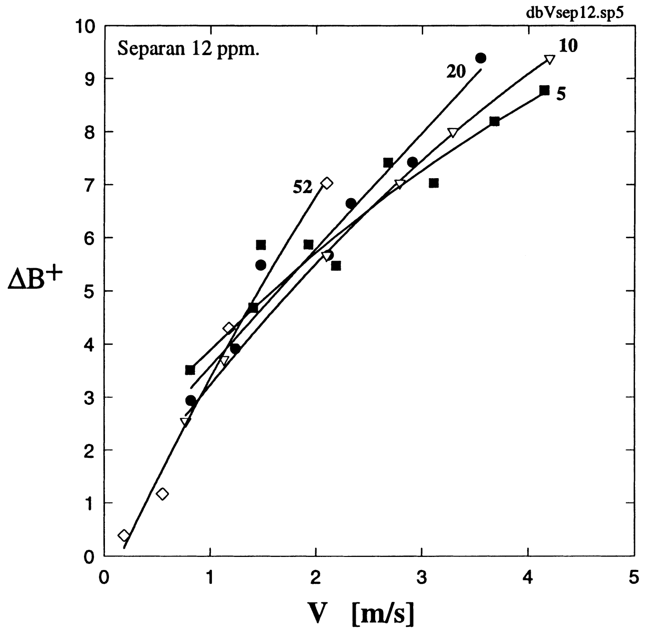


Fig. 9. ΔB^+ as a function of solution friction velocity and bulk velocity for the 12 ppm polyacrylamide in pipes of 5, 10, 20, and 52 mm i.d.

Equivalently, this can be shown by predicting DR from ΔB^+, as shown in Fig. 10. This figure shows the predicted friction coefficient for a 52 mm pipe based on ΔB^+ values obtained from the friction data generated in the 10 mm pipe, with ΔB^+ assumed to be either a function of V or of u_p^*, but independent

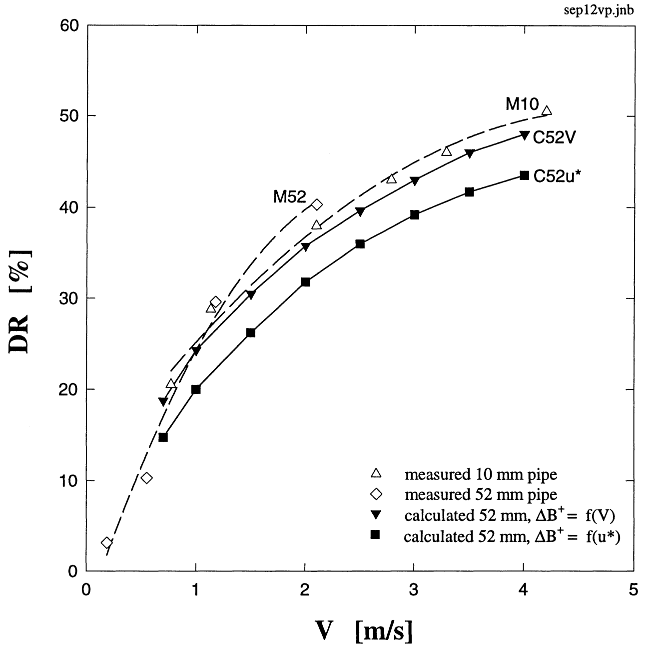


Fig. 10. Drag-reduction as a function of bulk velocity. The open symbols are the measured 10 and 52 mm data. The solid symbols are predictions of DR in the 52 mm pipe based on the 10 mm pipe data, assuming a 3-layers profile with ΔB^+ either a function of V or u_p^*.

of diameter. We see that using ΔB^+ as a function of V only as a fluid characteristic results in predictions of DR for the 52 mm pipe that are closer to the measured values than if ΔB^+ as a function of u_p^* is used, confirming that the 3-layers profile gives better results with ΔB^+ as a function of bulk velocity rather than as a function of friction velocity.

Furthermore, the DR calculated for the 52 mm pipe at a given velocity by the use of the 3-layers model with $\Delta B^+ = f(V)$ from the 10 mm data is very close (within 2%) to DR measured for the 10 mm pipe at the same velocity. In other words, a fluid which is well represented by this improved 3-layers model (3-layers profile plus $\Delta B^+ = f(V)$) should also satisfy the DR $= f(V)$ invariant correlation.

In this regard, measurements of velocity profiles for most polymer solutions showed very good agreement with the 3-layers profile [27], and the solutions used in the present work are indeed among those that were shown to conform to the 3-layers profile. One might then conclude that most polymer solutions should scale well with DR $= f(V)$. There is evidence, however, that some fluids show different types of velocity profiles – especially some surfactant solutions [28] –, and the diameter effect for those fluids may not be readily correlated by DR $= f(V)$.

4.2. Dimensional analysis

Our scaling procedure is an empirical solution (*i.e.* experimental and phenomenological only) to the diameter effect problem. It is concerned with the effect of diameter on drag-reduction for a given fluid. As such, any correlation of the type DR $= f(V)$ is dimensionally incorrect since it relates non-dimensional quantity with a dimensional one. This is because the fluid properties are not considered explicitly, since the procedure is valid only for a given fluid. We could then attempt to extend the scope of our procedure to provide a more general solution for drag-reduction based on fluid properties if these were known. We can also, on the other hand, see what new information our scaling procedure can add to the existing general 'theories of drag-reduction'. By 'theory' we mean the possible relations of global flow parameters which define the friction coefficient in a drag-reducing flow. In principle these are phenomenological concepts derived from dimensional analysis and intuitive understanding of the phenomenon. Two prominent and competing theories are those involving characteristic time or characteristic length as relevant parameter. Dimensional analysis alone can not determine which is better, of course, since both approaches can be dimensionally correct. Generally, though, the concept of a fluid characteristic time has seemed to be more compatible with experimental evidence. Let us now see how these theories look like in the light of our scaling with bulk velocity results, and let us analyze some of the most obvious types of general 'theories' of drag-reduction.

4.2.1. Fluid characteristic length

This theory postulates that the dominant fluid property responsible for drag-reduction is the characteristic length of the additive molecules or structure. For polymer solutions this could be the radius of gyration of the macromolecules, for example. A dimensionally correct relationship defining the friction coefficient should contain the Reynolds number (as for a Newtonian fluid), plus another non-dimensional number based on the fluid characteristic length ('l') and some other fluid and flow parameters, most likely the kinematic viscosity, geometry and velocity

$$C_\mathrm{f} = f(\mathrm{Re}, D, V, l, \nu).$$

If we assume as a working hypothesis that the effect of the Reynolds number can be separated from that of the other parameters, we can use DR instead of the friction coefficient

$$\mathrm{DR} = f(D, V, l, \nu).$$

We can now use our experimental finding that the effect of diameter for a given fluid is eliminated by the use of velocity as dependent parameter. This implies that the only flow parameter is the bulk velocity:

$$\mathrm{DR} = f(V, l, \nu),$$

assuming that l is not a function of the pipe diameter. Using those parameters we can create a non-dimensional number

$$\mathrm{DR} = f\left(\frac{Vl}{\nu}\right).$$

This relationship, besides being dimensionally correct, seems also convincing as far as polymer solutions are concerned, and was in fact recommended by researchers in the early stages of drag-reduction research, although with friction velocity instead of bulk velocity as discussed above.

4.2.2. Fluid characteristic time

Following a reasoning similar to that above, but using a fluid characteristic time instead of a characteristic length, we can write for example

$$\mathrm{DR} = f\left(\frac{V^2\lambda}{\nu}\right),$$

assuming again that λ is not a function of pipe diameter. This relationship satisfies our experimental evidence of scaling with bulk velocity, i.e. the drag-reduction for a given fluid is a function of bulk velocity only and the diameter is not a parameter. As mentioned above, the fluid relaxation time has been, in general, viewed more favorably than the fluid characteristic length as an additional parameter. The non-dimensional group based on the characteristic time may be therefore, a good candidate for use as additional parameter for these fluids.

Other parameters including a characteristic time are the Weissenberg and Deborah numbers. The former has been proposed as a non-dimensional parameter for drag-reduction (e.g. Kwack et al. [29]). It involves a fluid characteristic time, and compares this fluid characteristic time to the flow characteristic time, which is often taken to be *V*/*D*. If $\mathrm{We} = \lambda V/D$, we could have

$$\mathrm{DR} = f\left(\frac{\lambda V}{D}\right).$$

In this case, since the non-dimensional parameter is already defined, our experimental evidence of scaling with bulk velocity is used to test the 'theory' rather than to choose the parameter. As can be readily seen, the use of such a parameter is in contradiction with our experimental finding, as we observed that for a given fluid (i.e. relaxation time), the drag-reduction is a function of *V* only, not of *V*/*D* as would be the case if the Weissenberg number (with constant λ) were an appropriate choice. Another form of the Weissenberg number is one where the flow time is taken as the reciprocal of the wall shear rate. Then $\mathrm{We} = \lambda u^{*2}/\nu$ and

$$\mathrm{DR} = f\left(\frac{\lambda u^{*2}}{\nu}\right),$$

which even though it does not include a diameter would still lead to scaling with friction velocity.

This was shown above to be not as good as scaling with bulk velocity, and again this type of Weissenberg number may not be the best for scaling. If a Deborah number is generated by introducing a Reynolds number-based factor into the Weissenberg number as is sometimes done, the same type of conclusions would be obtained since we are considering here scaling at a given Reynolds number. Overall, it appears therefore, that the use of a characteristic time might be suitable, but that simple Weissenberg or Deborah numbers may not be the best parameters to scale and characterize drag-reducing flows.

4.2.3. Characteristic time and characteristic length

Finally, we can consider the possibility that both characteristic time and characteristic length act as independent parameters. Although less likely, this concept would satisfy the requirement of dimensional analysis, as well as our experimental observation of scaling with bulk velocity, if expressed in the following way, for example:

$$\mathrm{DR} = f\left(\frac{V\lambda}{l}\right).$$

5. Summary and conclusions

In this article we address the issue of the diameter scaling for friction in turbulent pipe flow of drag-reducing polymer solutions. Rather than using the more complicated scaling techniques based on velocity profile modeling, we focused on simpler empirical correlations, more suitable for application-oriented purposes. Two main empirical representations have been proposed in the past: DR vs. solution friction velocity, and DR vs. solvent friction velocity. The data obtained in our setup confirmed a reasonable invariance with the pipe diameter when plotted in both of these formats, with the latter showing better invariance, although less physical meaning.

We found, however, an even simpler scaling procedure which shows still more invariance with pipe diameter. In this approach we plot the drag-reduction level as a function of the actual bulk velocity of the solution. Besides providing a better correlation of the experimental data than the other two, the physical meaning of this approach is also restored, since the bulk velocity is of course a more relevant parameter for drag-reducing flow than a hypothetical solvent friction velocity. As we saw, the data for all the pipes in the subcritical (no degradation) non-asymptotic regime are very well correlated by a single curve in the DR vs. V representation, with the maximum absolute deviation in DR for tubes with diameters ranging from 2 to 52 mm – a ratio of 25 – being about 5%. The scheme works well with larger pipes too, and is therefore, very attractive, especially considering its simplicity, as a useful tool for application-oriented studies. Although the data are few, it appears that if there is no degradation even very small diameters (2 mm) correlate well with larger ones, which was previously considered unlikely. This approach works remarkably well for the fluids tested here, but we will continue to test other fluids to determine the limits of its generality.

Not only did the bulk velocity proved to be a better global scaling parameter than the friction velocity, it was also found to be a better parameter with respect to velocity profile modeling. We showed indeed that the 3-layers model fits better the measured friction coefficient data in pipes of different diameters if ΔB^+ is considered to be a function of the bulk velocity, rather than the friction velocity. The fluids which show a velocity profile conforming with the 3-layers model (with $\Delta B^+ = f(V)$) should also be well correlated by $\mathrm{DR} = f(V)$. Altogether, it appears that the bulk velocity might be a more important parameter in the intermediate regime than the wall shear stress for these drag-reducing flows. Fluids exhibiting other types of velocity profiles may or may not scale readily with bulk velocity.

If scaling with bulk velocity (an experimental observation) is used to test the proposed general 'theories' of drag-reduction, it can be concluded that simple Weissenberg or Deborah numbers as the additional non-dimensional number for drag-reducing viscoelastic flows may not be the best choice. Other characteristic length – and characteristic time-based parameters, however, are more compatible with our observations.

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