ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)
Extra Homework

Due on 12/2/2019, 9 a.m., in class

1. **Sensitivity Equation:** Consider the scalar system

$$\dot{x} = -\lambda x^3$$

   1.1 Write down the sensitivity equation for the parameter $\lambda > 0$. (3 points)
   1.2 Compute the sensitivity function in closed form. You may use MATLAB to check your answer but make sure to include hand derivation of the solution. (5 points)
   1.3 What is the first-order approximation

$$x(t;\lambda) = x(t;\lambda_0) + S(t)(\lambda - \lambda_0)$$

   at $\lambda_0 = 2$? (2 points)

2. **Lyapunov Stability:** Prove that the system

$$\dot{x}_1 = -x_2^3$$
$$\dot{x}_2 = x_1 - x_2$$

   is globally asymptotically stable by (both of)

   2.1 the La Salle's invariance principle; (5 points)
   2.2 finding a Lyapunov function with negative definite $\dot{V}$. (5 points)
   Hint: For 2.2, you may append your V from 2.1 by a general quadratic term and find the appropriate coefficients.

3. **Chetaev's Theorem:** Prove that the following system

$$\dot{x}_1 = x_2 + x_1^3 + 3x_1x_2^2$$
$$\dot{x}_2 = -x_1 + x_2^3 + 3x_2x_1^2$$

   is unstable (3 points). Include an illustration of the set U from which the solutions can escape the origin (2 points).

4. **La Salle's Invariance Principle:** Consider the system

$$\dot{x}_1 = x_2 + x_1x_3$$
$$\dot{x}_2 = -x_1 - x_2 + x_2x_3$$
$$\dot{x}_3 = -x_1^2 - x_2^2$$

   Which of the state globally variables converge to zero and which may not? Why? (5 points)

5. **Averaging Theory:** Consider the system

$$\dot{x}_1 = \sin\omega t\big[x_1 + \sin\omega t\big]\big[x_2 + \cos\omega t - x_1 - \sin\omega t\big]$$
$$\dot{x}_2 = \cos\omega t\big[x_2 + \cos\omega t\big]\big[x_1 + \sin\omega t - x_2 - \cos\omega t\big]$$

   Show that for sufficiently large $\omega > 0$, the system has an exponentially stable limit cycle in an $O(1/\omega)$ neighborhood of the origin. (10 points)