ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)

Homework #10

Due on 11/20/2019, 9 a.m., in class

1. **Perturbation Theory:** Consider the perturbed system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2 + \epsilon x_1^3$$

   1.1 Find a dynamical system whose trajectories are $O(\epsilon)$ approximations of the above system (5 points)

   1.2 Find a dynamical system whose trajectories are $O(\epsilon^2)$ approximations of the above system (5 points)

   1.3 Determine whether the above approximations hold on the infinite interval (5 points).

2. **Averaging:** Consider the system

$$\dot{x}_1 = \big(x_2 \sin \omega t - 2\big)x_1 - x_3 \tag{1a}$$
$$\dot{x}_2 = -x_2 + \big(x_2^2 \sin \omega t - 2x_3 \cos \omega t\big) \cos \omega t \tag{1b}$$
$$\dot{x}_3 = 2x_2 - \sin(x_3) + \big(4x_2 \sin \omega t + x_3\big) \sin \omega t \tag{1c}$$

   2.1 Put the system in the standard form of averaging theory and find the average system (5 points)

   2.2 Show that for sufficiently large $\omega > 0$, the original system (1) has an exponentially stable equilibrium point at the origin (5 points).

3. **Singular Perturbations:** Consider a control system given by

$$\dot{x} = A_{11}x + A_{12}z + B_1 u$$
$$\epsilon \dot{z} = A_{21}x + A_{22}z$$

where A_{22} is Hurwitz. Assume that there exists K of appropriate dimension such that

$$A_{11} - A_{12}A_{22}^{-1}A_{21} + B_1 K$$

is also Hurwitz. Show that for sufficiently small ϵ, the partial state feedback controller

$$u = Kx$$

exponentially stabilizes the origin $(x, z) = (0, 0)$. (10 points)