ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)
Homework #11

Due on 11/27/2019, 9 a.m., in class

1. **Input-Output Feedback Linearization:** Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 - x_3\\ \dot{x}_2 &= -x_1 x_3 - x_2 + u\\ \dot{x}_3 &= -x_1 + u\\ y &= x_3\end{aligned}$$

   1.1 What is the system's relative degree? (3 points)

   1.2 Is the system input-output feedback linearizable? Why? (2 points)

   1.3 Find the feedback control input $u(x)$ that makes the input-output relationship linear. (5 points)

   1.4 What is the system's zero dynamics? Is it minimum phase? (5 points)

   1.5 Find the change of coordinates that puts the system in normal form. (5 points)
   Hint: For the last part, apart from ξ whose choice is trivial, you need to find η_1 and η_2 both of which have to satisfy $\frac{\partial \eta_i}{\partial x} g(x) = 0, i = 1, 2$. Don't look for complicated functions. There are very simple η_i that satisfy this, just make sure that ξ, η_1, and η_2 are linearly independent (otherwise they won't make a "change of variables", right?!)

2. **Functional Distributions:** Consider the distribution

$$\Delta = \text{span}\{f_1, f_2\}$$

   where

$$f_1(x) = \begin{bmatrix} x_1 \\ 1 \\ 0 \\ x_3 \end{bmatrix}, \qquad f_2(x) = \begin{bmatrix} e^{-x_2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

   2.1 Is Δ non-singular? Why? (3 points)

   2.2 Is Δ involutive? Why? (5 points)