ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)
Homework #12

Due on 12/4/2019, 9 a.m., in class

1. Show that for sufficiently smooth vector fields f and g and scalar function h, we have (15 points)

$$\mathcal{L}_{[f,g]}h(x) = \mathcal{L}_f \mathcal{L}_g h(x) - \mathcal{L}_g \mathcal{L}_f h(x)$$

2. The dynamics of a one-link robot arm with elastically coupled actuator can be written as

$$J_1 \ddot{q}_1 + F_1 \dot{q}_1 + K/N(q_2 - q_1/N) = T$$
$$J_2 \ddot{q}_2 + F_2 \dot{q}_2 + K(q_2 - q_1/N) + mg\ell \cos q_2 = 0$$

Let $K = N = J_1 = J_2 = F_1 = F_2 = m = g = \ell = 1$. Our goal is to design the torque input T to globally stabilize the system towards the equilibrium point

$$x^* = \begin{bmatrix} q_1^* \\ \dot{q}_1^* \\ q_2^* \\ \dot{q}_2^* \end{bmatrix} = \begin{bmatrix} \pi/2 \\ 0 \\ \pi/2 \\ 0 \end{bmatrix}$$

2.1 Put the system dynamics in state-space form with $u = T$ as input (2 points)

2.2 Show that the system is feedback linearizable *without finding any output functions* h. (10 points)

2.3 Now find an output function $y = h(x)$ that gives relative degree $\rho = 3$. (10 points)
Hint: Make sure that your function satisfies $h(x^*) = 0$, because it has to map our desired equilibrium point x^* to the origin of the new coordinates.

2.4 Using the output function you found in 2.3, find a change of coordinates $\xi = T(x)$ that puts the system into the normal form (5 points)

$$\dot{\xi}_1 = \xi_2$$
$$\dot{\xi}_2 = \xi_3$$
$$\dot{\xi}_3 = \xi_4$$
$$\dot{\xi}_4 = \gamma(x)\big[u - \alpha(x)\big]$$

2.5 Using this change of coordinates, design a feedback control law $u = u(x)$ which globally stabilizes the origin (10 points)
Hint: For 2.5, use pole placement in ξ coordinates after cancelling out $\alpha(x)$ and $\gamma(x)$, but remember to put everything back together in the x coordinates.
You can use MATLAB for symbolic computations in all parts.