ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)

Homework #2

Due on 9/25/2019, 9 a.m., in class

1. **Poincare-Bendixson theorem and the supercritical Hopf bifurcation** (3 points) Prove, using the Poincare-Bendixson Criterion (Lemma 2.1), that the normal form of the supercritical Hopf bifurcation,

$$\dot{x}_1 = x_1(\mu - x_1^2 - x_2^2) - x_2$$
$$\dot{x}_2 = x_2(\mu - x_1^2 - x_2^2) + x_1$$

has a periodic orbit when $\mu > 0$.

2. **Comparison principle** (5 points). Consider the scalar differential inequality

$$\dot{v}(t) \leq -cv(t) + \ell_1(t)v(t) + \ell_2(t), \qquad v(0) \geq 0, \tag{1}$$

where ℓ_1 and ℓ_2 are continuous functions with bounded L_1 norm defined as

$$\|f\|_1 = \int_0^\infty |(t)|dt,$$

for any function $f(t)$, and $c > 0$ is a parameter. Using the comparison principle, show that

$$v(t) \leq \big(v(0)e^{-ct} + \|\ell_2\|_1\big)e^{\|\ell_1\|_1}.$$

Hint: solve the ODE version of 1 by multiplying both sides by $e^{\int_0^t [c-\ell_1(\tau)]d\tau}$.

3. **Sensitivity equations** (3 points) Consider the system

$$\dot{x}_1 = \tan^{-1}(\alpha x_1) - x_1 x_2$$
$$\dot{x}_2 = \beta x_1^2 - \gamma x_2,$$

where α, β, γ are parameters.

   (i) Derive the sensitivity equations around the nominal values $\alpha_0 = 1$, $\beta_0 = 0$, and $\gamma_0 = 1$. (2 points)

   (ii) Simulate the resulting 8-dimensional ODE using *a single call* to MATLAB `ode45`. Plot the results and attach the code. (1 point)