ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)
Homework #3

Due on 10/2/2019, 9 a.m., in class

1. **Lyapunov Stability.** Consider the system

$$\dot{x} = y - (2x^2 + y^2)x$$
$$\dot{y} = -x - 2(2x^2 + y^2)y$$

   Prove that the origin is globally asymptotically stable (5 points).

2. **Chetaev's Theorem.** Consider the system

$$\dot{x} = |x|x + (1 + |x|)xy$$
$$\dot{y} = -\frac{1}{8}(1 + |x|)x^2$$

   Prove that the origin is unstable. Plot (by hand or MATLAB) the set U in the Chetaev's theorem (5 points).

3. **La Salle's Invariance Principle.** Consider the system

$$\dot{x} = y$$
$$\dot{y} = -x^3 - y^3 - z^3$$
$$\dot{z} = -z + y$$

   Using $V(x, y, z) = \frac{1}{4}x^4 + \frac{1}{2}y^2 + \frac{1}{4}z^4$, determine whether the origin is stable, unstable, or asymptotically stable (5 points).