ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)

Homework #4

Due on 10/9/2019, 9 a.m., in class

1. **Lyapunov Stability.** Consider the system

$$\dot{x} = y$$
$$\dot{y} = -\frac{x}{1+y^2}$$

   Prove that the origin is stable (5 points).

   Hint: Seek a Lyapunov function of the form $V(x, y) = \phi(x) + \psi(y)$.

2. **Chetaev's Theorem.** Consider the system

$$\dot{x} = |x|x + xy\sqrt{|y|}$$
$$\dot{y} = -y + |x|\sqrt{|y|}$$

   Show that the origin is unstable. Plot the set U in Chetaev's theorem (by hand or MATLAB) (5 points).

3. **La Salle's Invariance Principle.** Consider the system

$$\dot{x} = -x + yx + z\cos(x)$$
$$\dot{y} = -x^2$$
$$\dot{z} = -x\cos(x)$$

   a) Find all the equilibria and determine the stability property of the origin (3 points).

   b) Prove that $\lim_{t\to\infty} x(t) = 0$ (2 points).

   c) Prove that $\lim_{t\to\infty} z(t) = 0$ (2 points).