# ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)

## Homework #5

**Due on 10/16/2019, 9 a.m., in class**

1. **Scalar LTI system (1 points).** Consider the scalar system

$$\dot{x}(t) = (a + b\sin(t))x(t), \qquad t \geq t_0 = 0 \tag{1}$$

Let $a = -1, b = 0$. Using

$$V(t, x) = \frac{1}{2}p(t)x(t)^2, \qquad q(t) = q = -1, \tag{2}$$

prove that this system is exponentially stable using Theorem 4.12 in the book. Compare your result with the solution of the corresponding (time-invariant) Lyapunov equation.

*Hint:* choose $p(0)$ carefully!

2. **Scalar LTV system (2 points).** Now consider the same system (1) but with $a = 1, b = 1$. Still use the same $V$ and $q$ as in (2).

   2.1 (1 point) Write down the appropriate differential equation for $p(t)$.

   2.2 (1 point) Since this equation does not have an analytical solution, solving it numerically using `ode45` (time interval $[0, T], T = 100$, arbitrary $p(0)$). Plot the result. Is the resulting $p(t)$ positive definite? (which, as you know, is different from being merely positive, right?)

3. **Scalar LTV system (3 points).** Again consider (1)-(2), but this time with $a = -1, b = 1$.

   3.1 (1 points) Repeat 2.1 and 2.2 for this system.

   3.2 (1 points) Can you find an appropriate $p(0)$ for which the resulting $p(t)$ is positive definite? Compare this with the systems of Problems 1 and 2 in terms of the importance of $p(0)$. Do you see a pattern?

   3.3 (1 points) Try solving the differential equation for $p(t)$ *backwards*, starting from an arbitrary $p(T)$ and solving backwards until $t = 0$ (still use a large $T$ such as $T = 100$). Plot the result.

   *Hint:* the command `ode45(odefun, T:-0.01:0, p_T)` does this automatically!

   Is the resulting $p(t)$ positive definite? Is the system exponentially stable?

4. **Vector LTV system (5 points).** Now consider the system

$$\dot{x}(t) = \begin{bmatrix} -2 + \cos(t) & e^{-t}\sin(t) \\ e^{-2t} & -3 + 2e^{-t}\cos(t) \end{bmatrix} x(t)$$

Using what you have learned so far, show that this system is exponentially stable.

*Hint:* A complete proof requires solving for $P(t)$ analytically, but it is OK (and our only option) here to find an appropriate $P(t)$ numerically.