ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)

Homework #6

Due on 10/28/2019, 9 a.m., in class

1. Show that the following systems are ISS (2 points each):

   (i)
   $$\dot{x} = -x^3 + xu$$

   (ii)
   $$\dot{x} = -x + u^3$$

   (iii)
   $$\begin{aligned} \dot{x} &= -x^3 + xy \\ \dot{y} &= -y + u^3 \end{aligned}$$

2. (4 points) Show that the following system is ISS

   $$\begin{aligned} \dot{x} &= -x + y^3 \\ \dot{y} &= -y - \frac{x}{\sqrt{1+x^2}} + z^2 \\ \dot{z} &= -z + u \end{aligned}$$

   using the Lyapunov function

   $$V(x, y, z) = \sqrt{1+x^2} - 1 + \frac{1}{4}y^4 + \frac{1}{2}z^8$$

   Hint: After simplifying $\dot{V}$, use the infinity norm $\|(x, y, z)\|_\infty = \max\{|x|, |y|, |z|\}$ of the state to collect the terms in x and y and z into a single term in $\|(x, y, z)\|_\infty$. Also, beware of the fact that for any class $\mathcal{K}$ function α and $a, b, c > 0$,

   $$\alpha(a + b + c) \leq \alpha(3a) + \alpha(3b) + \alpha(3c)$$

   which you can easily show (right?).