ESE 617/MEAM 613: Nonlinear Systems & Control (Fall 2019)
Homework #7

Due on 11/4/2019, 9 a.m., in class
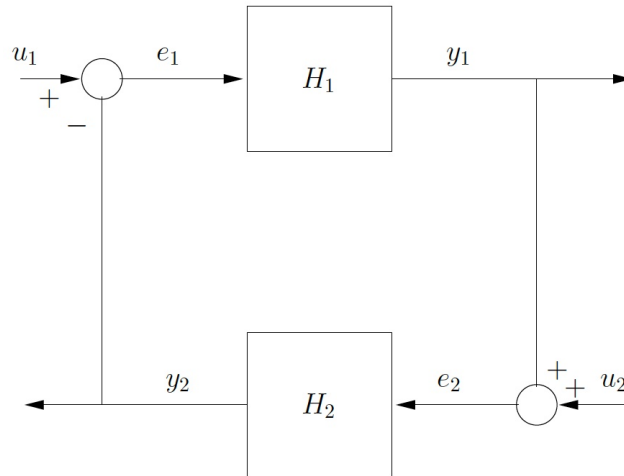
1. (5 points) Show that the following system is $\mathcal{L}_\infty$ stable:

$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = -x_1 - \sigma(x_1) - x_2 + u$$
$$y = x_2$$

where σ is locally Lipschitz, bounded, and satisfies $z\sigma(z) \geq 0$ for all $z \in \mathbb{R}$.

Hint: Use a standard Lyapunov function with an additional term that takes care of $\sigma(x_1)$.

2. (5 points) Show that the following system is $\mathcal{L}_2$ stable:

$$H_1 : \begin{cases} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = -x_1^3 - x_2 + e_1 \\ y_1 = x_2 \end{cases} \qquad H_2 : \begin{cases} \dot{x}_3 = -x_3^3 + e_2 \\ y_2 = \frac{1}{2}x_3^3 \end{cases}$$

where $u_2 = 0$, $u = u_1$, and $y = y_1$.