# AIDED NAVIGATION

## GPS with High Rate Sensors

## About the Author

**Jay A. Farrell** received B.S. degrees (1986) in physics and electrical engineering from Iowa State University, and M.S. (1988) and Ph.D. (1989) degrees in electrical engineering from the University of Notre Dame. At Charles Stark Draper Lab (1989–1994), he was principal investigator on projects involving intelligent and learning control systems for autonomous vehicles. Dr. Farrell received the Engineering Vice President's Best Technical Publication Award in 1990, and Recognition Awards for Outstanding Performance and Achievement in 1991 and 1993. He is a Professor and former Chair of the Department of Electrical Engineering at the University of California, Riverside. He has served as Vice President of Finance and Vice President of Technical Activities for the IEEE Control Systems Society. He is a Fellow of the IEEE (2008) and author of over 150 technical publications. He is author of the book *Aided Navigation: GPS with High Rate Sensors* (McGraw-Hill, 2008). He is also co-author of the books *The Global Positioning System and Inertial Navigation* (McGraw-Hill, 1998) and *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches* (John Wiley, 2006).

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*To my students and colleagues, without whom this book would not have been possible.*

# Contents

# Preface

Technological innovations over recent decades have enabled an ever increasing array of inexpensive, high accuracy, aided navigation systems suitable for student projects, research test-bed usage, and commercial and military applications. These innovations include: small, low cost, low power sensors; GPS technology; and, high performance, low cost, computational equipment for embedded processing.

Aided navigation involves two categories of sensors. The output signals from sensors in the first category are integrated using a kinematic model of the system. The result of this integration provides a reference trajectory. Example kinematic input sensors include inertial measurement units, Doppler radar or sonar, and wheel encoders. Elements of the second category of sensors are used to estimate the error between this reference trajectory and trajectory of the vehicle. In the estimation process, these aiding sensors may also determine various calibration parameters to improve the future performance of the system. The advent of Global Navigation Satellite Systems (GNSS) has provided an accurate and reliable aiding measurement source for navigation systems in certain outdoor applications. A prototypical example of a GNSS sensor suitable for aided operations is the Global Positioning System (GPS).

Aided navigation is motivated when an application, such as an autonomous vehicle, requires accurate high bandwidth information about the vehicle state reliably at a high sample rate. The aiding sensors may satisfy the accuracy requirement, but not the reliability, bandwidth, and high sample rate specifications. The reference trajectory computed from the kinematic input sensors may satisfy the reliability, bandwidth, and high sample rate specifications; however, due to its integrative nature, errors that accumulate over time can cause the unaided system to eventually fail to meet the accuracy specification.

The aiding sensors and the reference trajectory computed from the kinematic input sensors have complementary characteristics; therefore, it is natural to consider their implementation within an integrated approach.

## Objective

My main objective in writing this book is to provide a self-contained reference on aided navigation system design that is appropriate for use in a classroom setting. The methodology presented herein is an industry standard approach. It has been used by many persons in various applications with a high-level of success.

The text is written with the expectation that the reader has the standard background of a senior in a bachelor-of-science (BS) engineering program.

I have taken special care in presenting the material at the BS level. For example, the topic of optimal filtering is derived both in the least squares context and in the standard stochastic processes context. Examples are included throughout the text to relate theoretical concepts back to applications, to motivate the importance of specific concepts, and to illustrate the tradeoffs between alternative techniques. In addition, detailed derivations are included. The intended audience for the book includes engineers, students, researchers, scientists, and project managers who may be interested in either designing or using an aided navigation system in a given application.

## Outline

The text presents a systematic method for designing and analyzing aided navigation systems along with the essential theory to support that methodology. The book is divided into two parts.

Part I contains seven chapters. Chapter 1 motivates the design and analysis methodology. This chapter introduces simple examples and comparisons to illustrate the method and to stimulate interest in the theoretical information presented in Chapters 2–6. Chapter 2 defines frames-of-reference, transformations between frames-of-reference, and methods for maintaining the transformations between rotating frames-of-reference. Chapters 3 and 4 focus on model development. An accurate mathematical model is a necessary precondition to being able to make accurate quantitative statements about navigation system performance during the design process. Chapter 3 presents various concepts from (deterministic) systems theory. Chapter 4 introduces certain necessary concepts from the theory of stochastic processes. Chapter 5 introduces and derives optimal state estimation methods (i.e., Kalman filtering). Knowledge of optimal and sub-optimal state estimation are essential to the implementation and performance analysis of navigation systems. Chapter 6 discusses methods for performance analysis. Chapter 7 returns to the methodology suggested in Chapter 1; however, now with the tools provided in Chapters 2-6 the same examples can be rigorously and quantitatively analyzed. Examples are used extensively throughout Chapters 1-7.

Part II of the text provides in-depth discussion of several specific navigation applications. Chapter 8 provides a self-contained description of the basic GPS solution, differential GPS, Doppler processing, and various carrier phase processing techniques. This presentation of the various GPS techniques using unified notation greatly facilitates the understanding of the techniques and their relative tradeoffs. For clarity of the presentation and comparison of methods within Chapter 8, all techniques presented in that chapter consider only point-wise data processing (i.e., no Kalman fil-

tering). GPS is discussed herein as a prototype of the various GNSS which are now or soon will be available. Chapters 9–12 each focus on a specific aided navigation application. Each chapter derives the kinematic model and presents the navigation mechanization equations based on the kinematic model; derives and analyzes the dynamic model for the navigation error state; and, presents equations for predicting the aiding measurements and for modeling the residual aiding measurements. Different forms of performance analysis are included in each of the chapters including covariance analysis, observability analysis, and data analysis from application or simulation. Chapter 9 discusses aided encoder-based dead-reckoning. Chapter 10 discusses an attitude and heading reference system that uses gyros as inputs to the kinematic model and accelerometers as gravity sensors for error correction. Chapter 11 discusses aided inertial navigation. Chapter 12 discusses a specific application of an aided inertial system. Together these chapters provide examples of methods that the reader can modify to fit their particular application needs.

Four appendices are included. Appendix A defines the notation and constants used in the text. Appendix B reviews various linear algebra concepts that are used in the main body of the text. Appendix C presents material from the GPS interface specification that is necessary to process GPS pseudorange, phase, and Doppler measurements. Example calculations are also included. Appendix D presents a short tutorial on quaternions. This appendix discusses their definition, operations, and kinematic model. Example calculations are included in each appendix.

## Motivation

This book is motivated in part by the reader response to my prior book [48]. That response requested two major changes. First, that the material be restructured to support the book's use in a classroom setting. Second, that the subject matter be widened to the broader topic of aided navigation. Therefore, while the present book draws a significant amount of its source material from [48], the objectives of this book are distinct enough and the amount of new material is significant enough to necessitate the new title.

The present book discusses the general topic of aided navigation system design. Chapters 1 and 7 present the aided navigation system design methodology. Chapters 2–6 present the theoretical material required to understand and implement that method. Chapter 8 provides a detailed discussion of GPS. Chapters 9–12 provide detailed discussion of the aided navigation system design and analysis methodology.

This book is appropriate for use as a textbook in a senior or first year graduate level course. As such, the book includes extensive use of examples and end-of-chapter problems throughout Part I. As a textbook, the

lectures could cover Part I, with a student end-of-course project to design and implement an aided navigation system. The systems in Chapters 9, 10, and 11 are particularly appropriate for senior design projects.

Many engineering programs require senior-level students to complete a substantial design project. A challenge for faculty involved in directing such projects is leading the student team through the quantitative definition of the specification and analysis of system performance relative to the specifications. A main reason that the aided navigation system design and analysis methodology presented herein is widely used in commercial and military applications is that it is easily amenable to such quantitative analysis of performance relative to specifications. Performance analysis is discussed in Chapter 6.

## Book Website

Associated with this book is the publisher hosted website. The website contains various resources related to this book:

- The source code used to create examples using MATLAB;
- Data sets to support examples;
- Data sets that are requested by readers and that are reasonable for the author to provide;
- An errata list;
- Clarifications as requested by readers.

The official web site for this text can be found at

www.mhprofessional.com

then searching by isbn, title, or author. The book web site that results will contain a download section containing the above material.

## Errata

While I have worked to ensure that this book is free from errors, previous experience has taught me that this is unlikely to occur. An errata list will be made available through the publisher website and through my university homepage, currently

www.ee.ucr.edu/∼farrell

Readers who detect errors not already on the errata list are encouraged to report them electronically to the author. My e-mail address is currently

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Identification of errors, or suggestions of either additional material (e.g., examples, exercises, or topics), is greatly appreciated.

## Acknowledgements

First, I appreciate the many readers of [48] who communicated to me interesting ideas, corrections, and useful suggestions. Those communications helped to motivate and define this book. Second, I am grateful to acknowledge the help and collaboration of my students and colleagues in the development and proofreading of this book. The first set of reviewers included Anning Chen, Licheng Luo, Angello Pozo, and Arvind Ramanandan. The second set of reviewers included Jinrong Cheng, Wenjie Dong, Yu Lu, Paul Miller, Rolf Rysdyk, and Peng Xie. In addition, Wenjie Dong collaborated in the writing of Section 4.9.1; Arvind Ramanandan collaborated in the writing of Section 4.9.2; Yunchun Yang provided information essential to the writing of Chapter 10; Yu Lu collaborated in the writing of Section C.4 and provided source material helpful in the writing of Chapter 9; and, collaboration with Paul Miller was essential to the writing of Chapter 12. Of course, I take responsibility for any errors or omissions in the final presentation of all the material contained in the book. Third, I appreciate the help and support of the production team, mainly Wendy Rinaldi and Jean Bodeaux. Finally, I appreciate the patience of my family throughout this project.

This book was typeset using LaTeX. The examples and graphs for figures were implemented in MATLAB and SIMULINK. Drawings for figures were created using Macromedia FreeHand MX.

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