Memory Hierarchy (IV)

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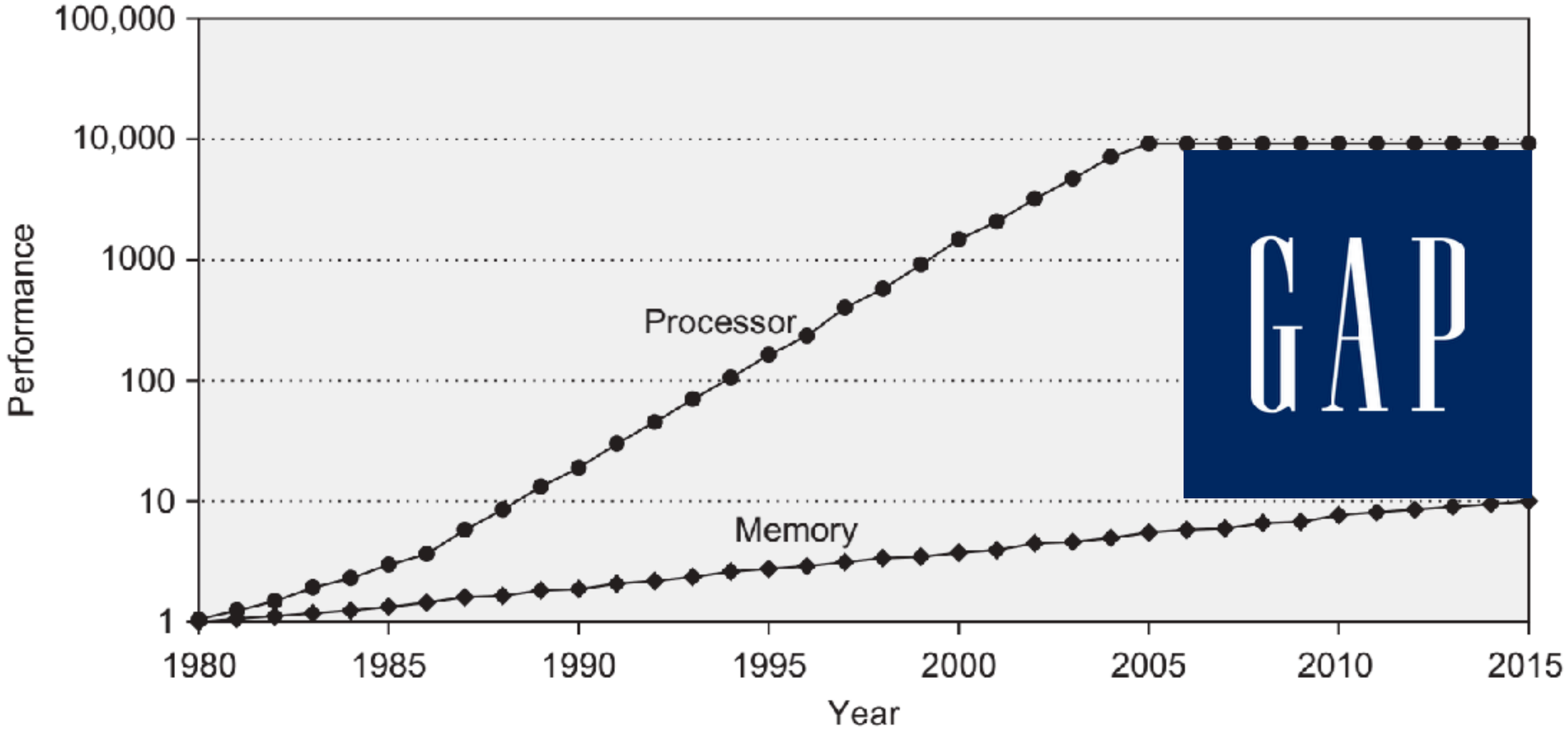
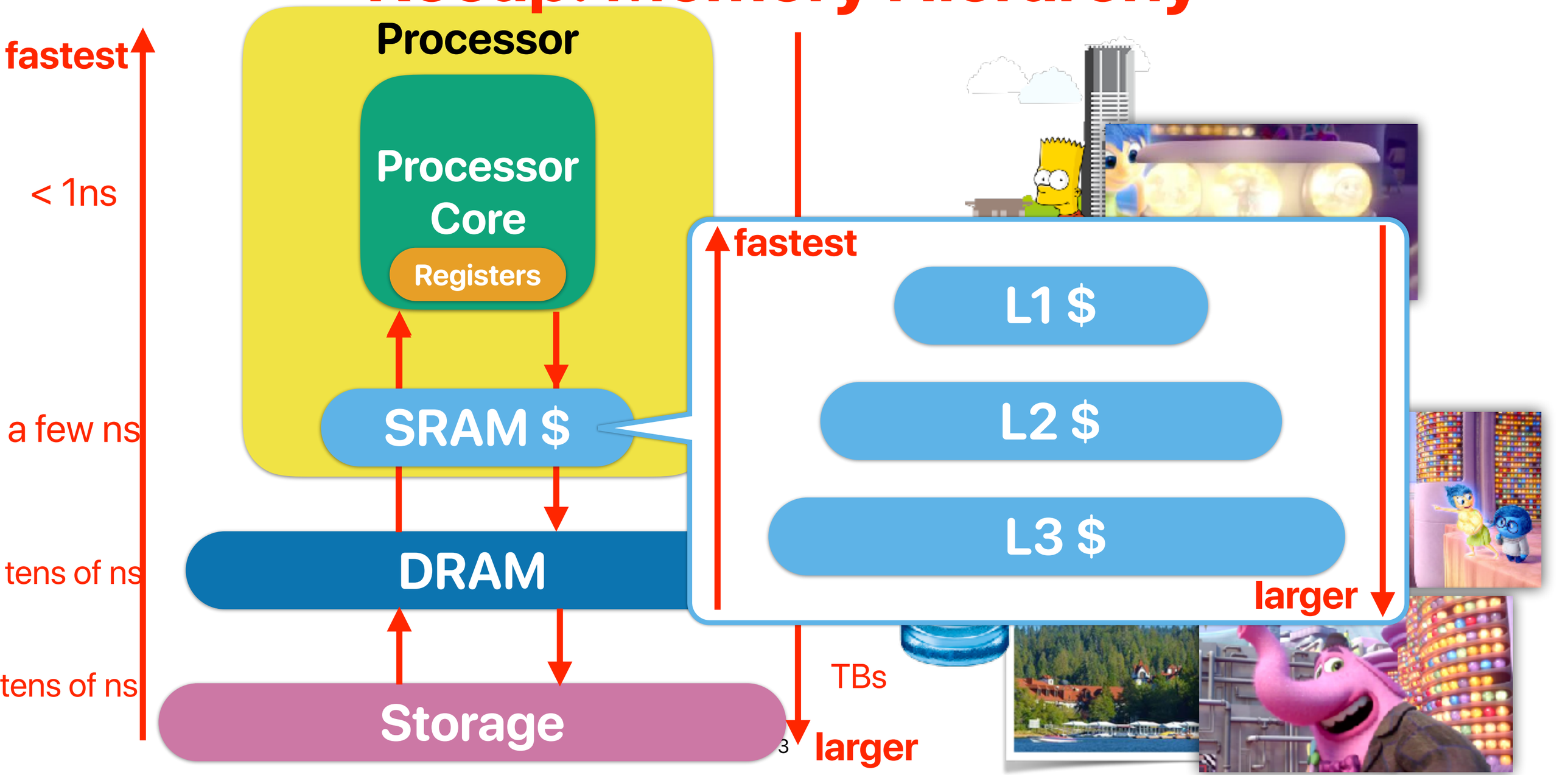
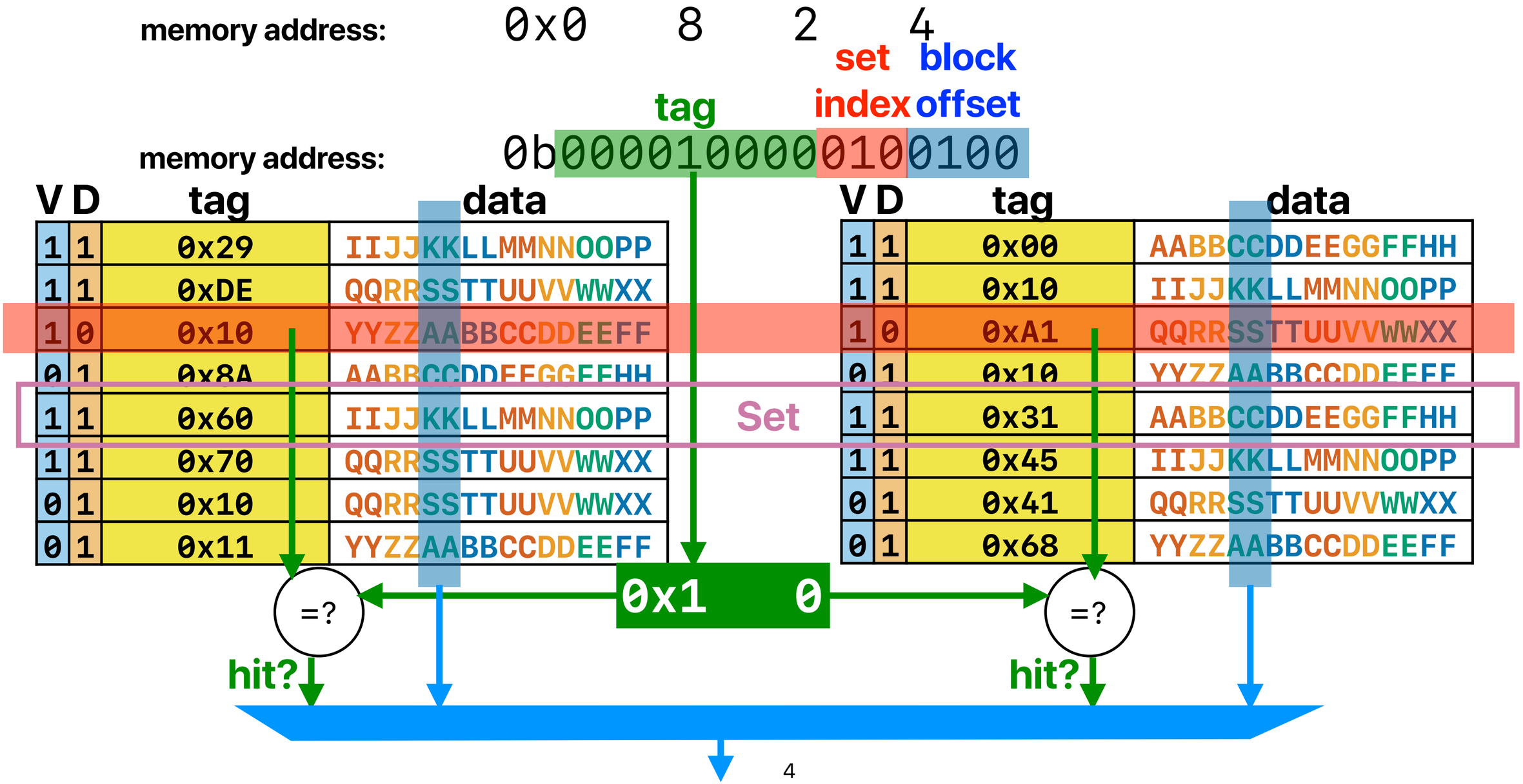
Recap: Performance gap between Processor/Memory



Recap: Memory Hierarchy



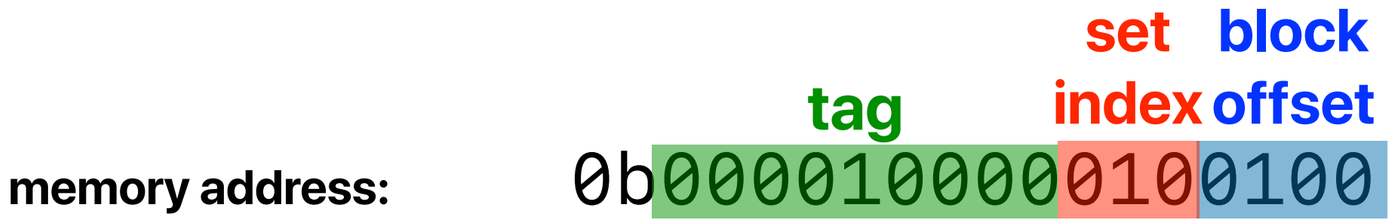
Recap: Way-associative cache



C = ABS

- **C**: **C**apacity in data arrays
- **A**: Way-**A**ssociativity — how many blocks within a set
  - N-way: N blocks in a set, A = N
  - 1 for direct-mapped cache
- **B**: **B**lock Size (Cacheline)
  - How many bytes in a block
- **S**: Number of **S**ets:
  - A set contains blocks sharing the same index
  - 1 for fully associate cache

Corollary of C = ABS



- number of bits in **b**lock offset — lg(**B**)
- number of bits in **s**et index: lg(**S**)
- tag bits: address_length - lg(S) - lg(B)
  - address_length is 32 bits for 32-bit machine
- (address / block_size) % S = set index

AMD Phenom II

100% miss rate!

- Size 64KB, 2-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 48-bit address.

```
int a[16384], b[16384], c[16384];
/* c = 0x10000, a = 0x20000, b = 0x30000 */
for(i = 0; i < 512; i++)
    c[i] = a[i] + b[i]; /*load a[i], load b[i], store c[i]*/
```

C = ABS
64KB = 2 * 64 * S
S = 512
offset = lg(64) = 6 bits
index = lg(512) = 9 bits
tag = the rest bits

	address in hex	address in binary (tag / index / offset)	tag	index	hit? miss?
load a[0]	0x20000	0b10 0000 0000 0000 0000	0x4	0	compulsory miss
load b[0]	0x30000	0b11 0000 0000 0000 0000	0x6	0	compulsory miss
store c[0]	0x10000	0b01 0000 0000 0000 0000	0x2	0	compulsory miss, evict
load a[1]	0x20004	0b10 0000 0000 0000 0100	0x4	0	conflict miss, evict 0x6
load b[1]	0x30004	0b11 0000 0000 0000 0100	0x6	0	conflict miss, evict 0x2
store c[1]	0x10004	0b01 0000 0000 0000 0100	0x2	0	conflict miss, evict 0x4
⋮	⋮	⋮	⋮	⋮	⋮
load a[15]	0x2003C	0b10 0000 0000 0011 1100	0x4	0	miss, evict 0x6
load b[15]	0x3003C	0b11 0000 0000 0011 1100	0x6	0	miss, evict 0x2
store c[15]	0x1003C	0b01 0000 0000 0011 1100	0x2	0	miss, evict 0x4
load a[16]	0x20040	0b10 0000 0000 0100 0000	0x4	1	compulsory miss
load b[16]	0x30040	0b11 0000 0000 0100 0000	0x6	1	compulsory miss
store c[16]	0x10040	0b01 0000 0000 0100 0000	0x2	1	compulsory miss, evict

3Cs and A, B, C

- Regarding 3Cs: compulsory, conflict and capacity misses and A, B, C: associativity, block size, capacity
  How many of the following are correct?
  - ① Increasing associativity can reduce conflict misses
  - ② Increasing associativity can reduce hit time
  - ③ Increasing block size can increase the miss penalty
  - ④ Increasing block size can reduce compulsory misses

  A. 0
  B. 1
  C. 2
  D. 3
  E. 4

Increases hit time because your data array is larger (longer time to fully charge your bit-lines)

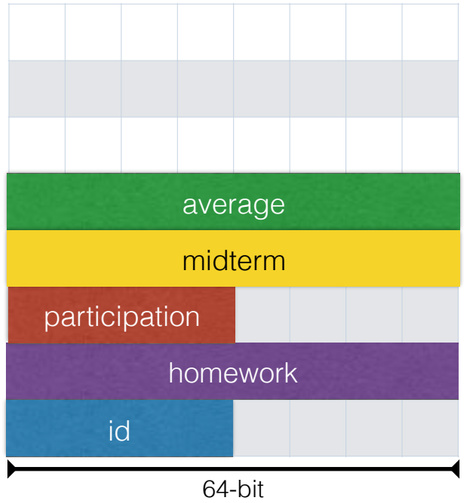
You need to fetch more data for each miss

You bring more into the cache when a miss occurs

The result of `sizeof(struct student)`

- Consider the following data structure:

```
struct student {
    int id;
    double *homework;
    int participation;
    double midterm;
    double average;
};
```



What's the output of `printf("%lu\n",sizeof(struct student))`?

A. 20
B. 28
C. 32
D. 36
E. 40

Outline

- Software techniques in improving cache performance
- Hardware optimizations in improving cache performance

Programming and memory performance (cont.)

Loop interchange/fission/fusion

Demo — programmer & performance

A

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

B

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

$O(n^2)$	Complexity	$O(n^2)$
Same	Instruction Count?	Same
Same	Clock Rate	Same
Better	CPI	Worse

AMD Phenom II

- D-L1 Cache configuration of AMD Phenom II
  - Size 64KB, 2-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 32-bit address.

```
int a[16384], b[16384], c[16384];
/* c = 0x10000, a = 0x20000, b = 0x30000 */
for(i = 0; i < 512; i++) {
    c[i] = a[i] + b[i];
    //load a, b, and then store to c
}
```

What's the data cache miss rate for this code?

A. 6.25%
B. 56.25%
C. 66.67%
D. 68.75%
E. 100%

C = ABS

64KB = 2 * 64 * S

S = 512

offset = lg(64) = 6 bits

index = lg(512) = 9 bits

tag = 64 - lg(512) - lg(64) = 49 bits

What if the code look like this?

- D-L1 Cache configuration of AMD Phenom II
  - Size 64KB, 2-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 32-bit address.

```
int a[16384], b[16384], c[16384];
/* c = 0x10000, a = 0x20000, b = 0x30000 */
for(i = 0; i < 512; i++)
    c[i] = a[i]; //load a and then store to c
for(i = 0; i < 512; i++)
    c[i] += b[i]; //load b, load c, add, and then store to c
```

What's the data cache miss rate for this code?

A. 6.25%
B. 56.25%
C. 66.67%
D. 68.75%
E. 100%

Loop Fusion

```
/* Before */
for (i = 0; i < N; i = i+1)
    for (j = 0; j < N; j = j+1)
        a[i][j] = 1/b[i][j] * c[i][j];
for (i = 0; i < N; i = i+1)
    for (j = 0; j < N; j = j+1)
        d[i][j] = a[i][j] + c[i][j];


/* After */
for (i = 0; i < N; i = i+1)
    for (j = 0; j < N; j = j+1)
    {
        a[i][j] = 1/b[i][j] * c[i][j];
         d[i][j] = a[i][j] + c[i][j];
    }
```

2 misses per access to a & c vs. one miss per access

When should we use fission/fusion?

- If you have many ways — fusion
- If you have very limited ways — fission

Blocking

Case study: Matrix Multiplication

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

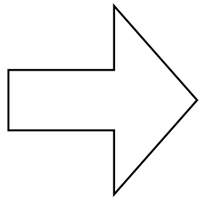
Algorithm class tells you it's $O(n^3)$

If n=512, it takes about 1 sec

How long is it take when n=1024?

Block algorithm for matrix multiplication

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

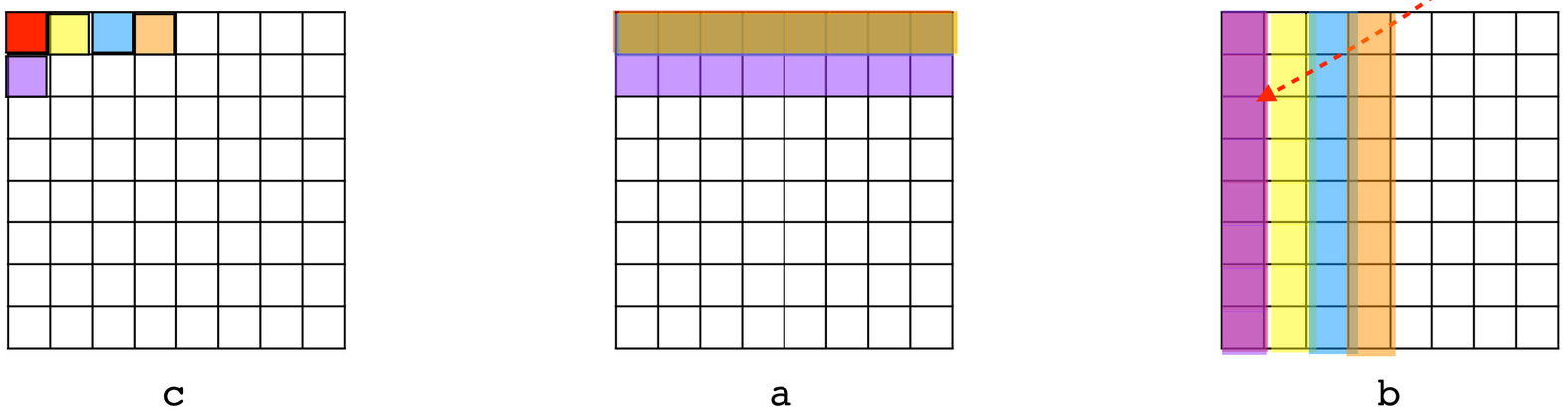


```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
            for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
              c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```

Matrix Multiplication

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

Very likely a miss if array is large

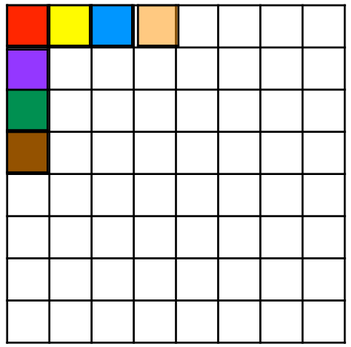


- If each dimension of your matrix is 1024
  - Each row takes 1024*8 bytes = 8KB
  - The L1 $ of intel Core i7 is 32KB, 8-way, 64-byte blocked
  - You can only hold at most 4 rows/columns of each matrix!
  - You need the same row when j increase!

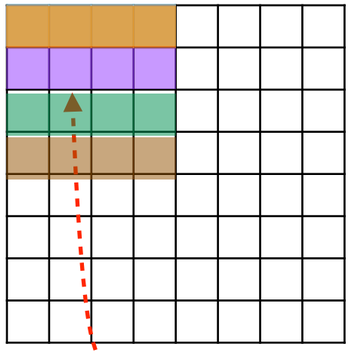
Block algorithm for matrix multiplication

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

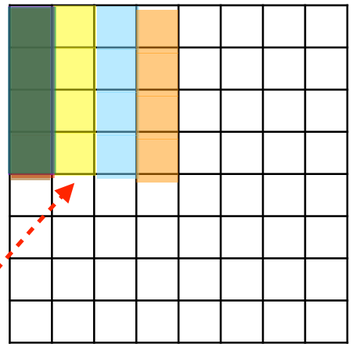
```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
            for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
              c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```



c



a



b

You only need to hold these sub-matrices in your cache

What kind(s) of misses can block algorithm remove?

- Comparing the naive algorithm and block algorithm on matrix multiplication, what kind of misses does block algorithm help to remove? (assuming an intel Core i7)

Naive

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

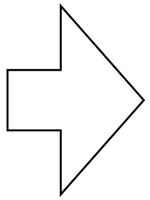
Block

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
        for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
            c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```

A. Compulsory miss
B. Capacity miss
C. Conflict miss
D. Capacity & conflict miss
E. Compulsory & conflict miss

Matrix Transpose

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
            for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
              c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```



```
// Transpose matrix b into b_t
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
      b_t[i][j] += b[j][i];
  }
}
```

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
            for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
              // Compute on b_t
              c[ii][jj] += a[ii][kk]*b_t[jj][kk];
    }
  }
}
```

What kind(s) of misses can matrix transpose remove?

- By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

Block

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
        for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
            c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```

A. Compulsory miss
B. Capacity miss
C. Conflict miss
D. Capacity & conflict miss
E. Compulsory & conflict miss

Block + Transpose

```
// Transpose matrix b into b_t
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
      b_t[i][j] += b[j][i];
  }
}


for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
            for(kk = k; kk < k+(ARRAY_SIZE/n); kk++
              // Compute on b_t
              c[ii][jj] += a[ii][kk]*b_t[jj][kk];
    }
  }
}
```

Column-store or row-store

- If you're designing an in-memory database system, will you be using

RowId	EmpId	Lastname	Firstname	Salary
1	10	Smith	Joe	40000
2	12	Jones	Mary	50000
3	11	Johnson	Cathy	44000
4	22	Jones	Bob	55000

  - column-store — stores data tables column by column

```
10:001,12:002,11:003,22:004;
Smith:001,Jones:002,Johnson:003,Jones:004;
Joe:001,Mary:002,Cathy:003,Bob:004;
40000:001,50000:002,44000:003,55000:004;
```

if the most frequently used query looks like —

```
select Lastname, Firstname from table
```

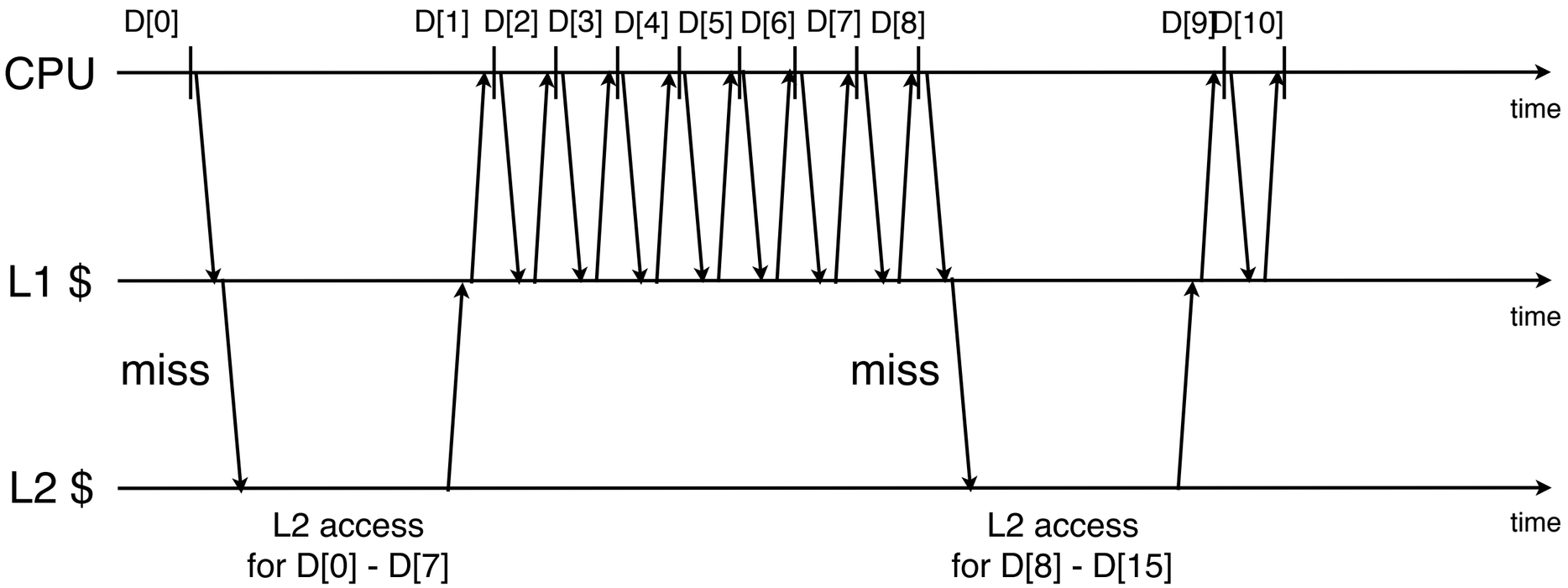
  - row-store — stores data tables row by row

```
001:10,Smith,Joe,40000;
002:12,Jones,Mary,50000;
003:11,Johnson,Cathy,44000;
004:22,Jones,Bob,55000;
```

Prefetching

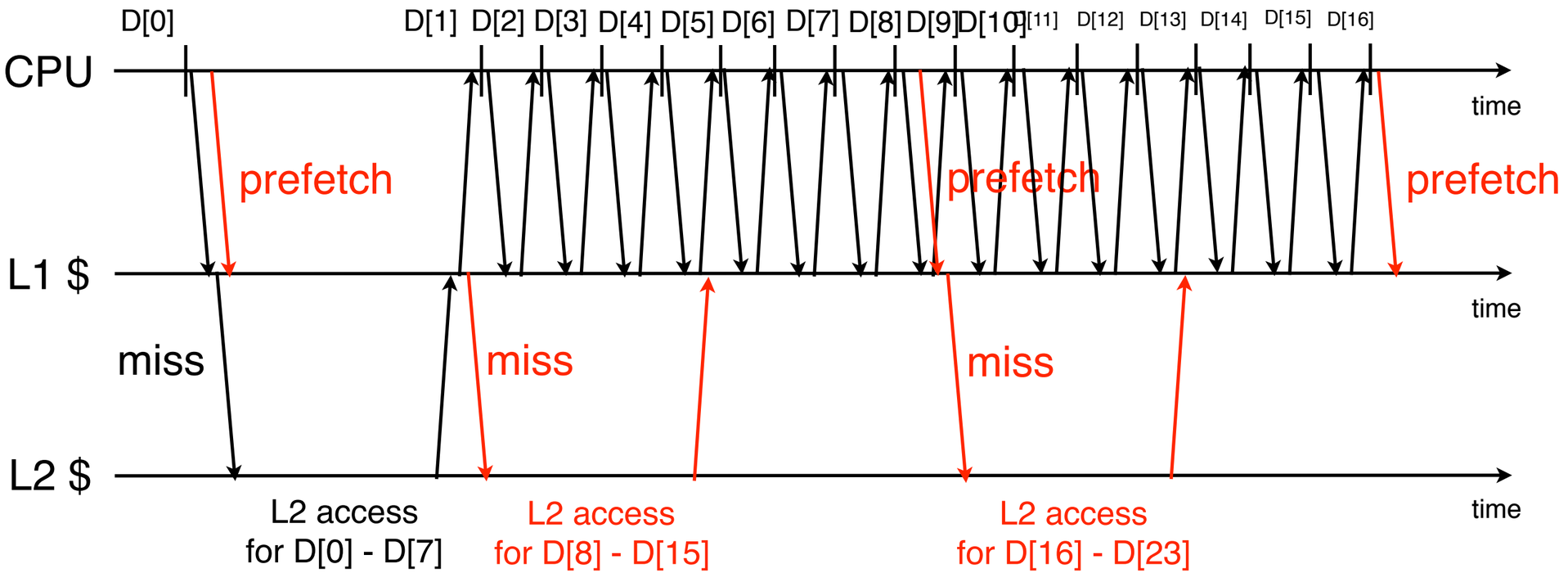
Characteristic of memory accesses

```
for(i = 0;i < 1000000; i++) {
     D[i] = rand();
}
```



Prefetching

```
for(i = 0;i < 1000000; i++) {
      D[i] = rand();
      // prefetch D[i+8] if i % 8 == 0
}
```



Prefetching

- Identify the access pattern and proactively fetch data/instruction before the application asks for the data/instruction
  - Trigger the cache miss earlier to eliminate the miss when the application needs the data/instruction
- Hardware prefetch
  - The processor can keep track the distance between misses. If there is a pattern, fetch miss_data_address+distance for a miss
- Software prefetch
  - Load data into X0
  - Using prefetch instructions

Where can prefetch work effectively?

- How many of the following code snippet can "prefetching" effectively help improving performance?

(1)
```
while(node){
    node = node->next;
}
```
— where the next pointing to is hard to predict

(2)
```
while(++i<100000)
    a[i]=rand();
```

(3)
```
while (root != NULL){
        if (key > root->data)
            root = root->right;

        else if (key < root->data)
            root = root->left;
        else
            return true;
}
```
— where the next node is also hard to predict

(4)
```
    for (i = 0; i < 65536; i++) {
      mix_i = ((i * 167) + 13) & 65536;
      results[mix_i]++;
    }
```
— the stride to the next element is hard to predict…

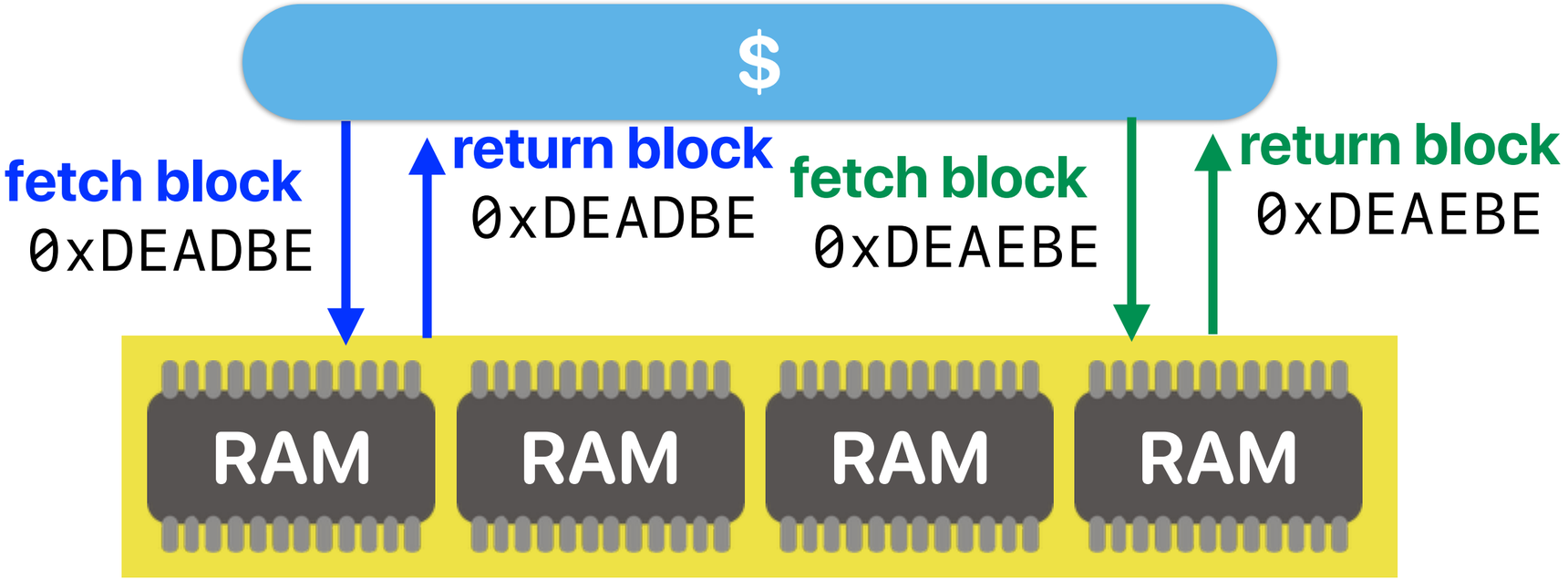
A. 0
B. 1
C. 2
D. 3
E. 4

Demo

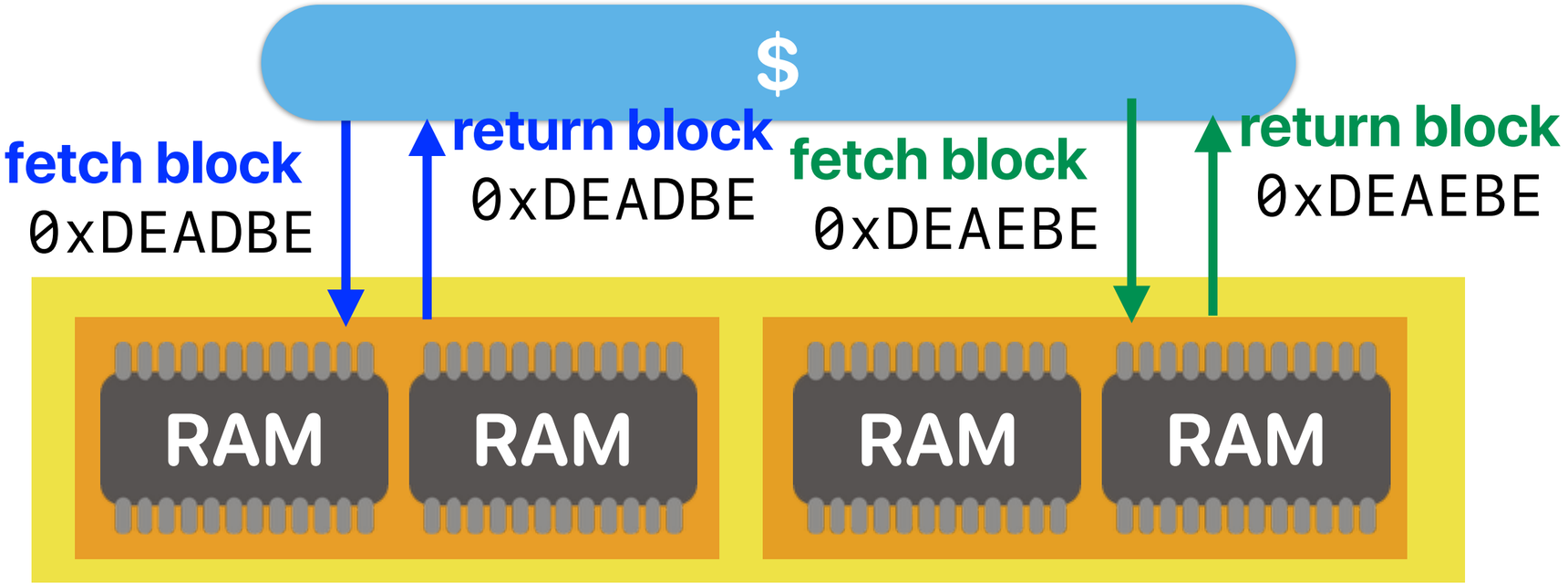
- x86 provide prefetch instructions
- As a programmer, you may insert `_mm_prefetch` in x86 programs to perform software prefetch for your code
- gcc also has a flag "-fprefetch-loop-arrays" to automatically insert software prefetch instructions

Advanced Hardware Techniques in Improving Memory Performance

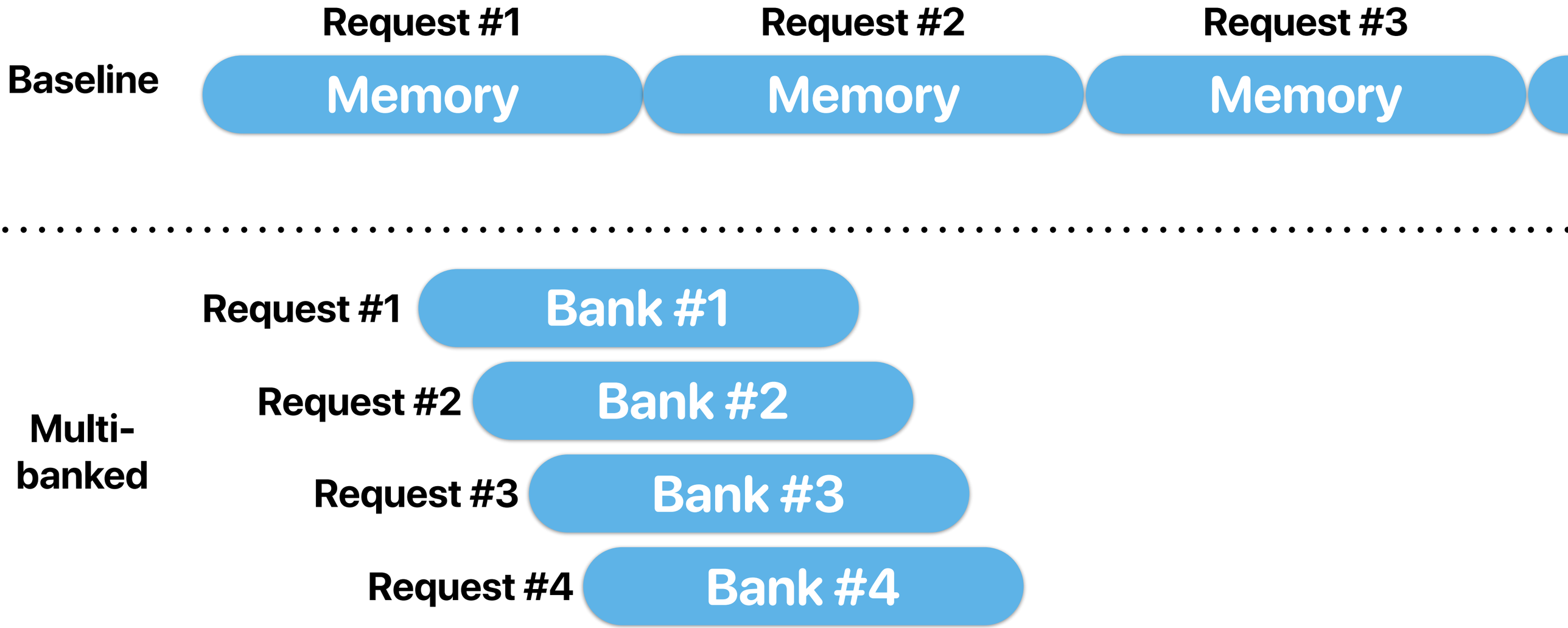
Without banks



Multibanks & non-blocking caches



Pipelined access and multi-banked caches



Pipelined access and multi-banked caches

- Assume each bank in the $ takes 10 ns to serve a request, and the $ can take the next request 1 ns after assigning a request to a bank — if we have 4 banks and we want to serve 4 requests, what's the speedup over non-banked, non-pipelined $? — pick the closest one

  A. 1x — no speedup
  B. 2x
  C. 3x
  D. 4x
  E. 5x

$$ET_{baseline} = 4 \times 10\ ns = 40\ ns$$

$$ET_{banked} = 10\ ns + 3 \times 1\ ns = 13\ ns$$

$$Speedup = \frac{Execution\ Time_{baseline}}{Execution\ Time_{banked}}$$

$$= \frac{40}{13} = 3.08\times$$

Announcement

- Make up lecture **7pm tonight @ WCH 143** — will be midterm review
  - Will use 70 minutes to highlight the important concepts for midterm
  - Will present a sample midterm
- Assignments
  - We will drop the lowest assignment grade — Josep's talk summary treated as a bonus one (technically we drop two)
  - Please make sure you have CLEAR description on what formula you're applying and state why this is a good one if necessary.
  - If you have any assumption on numbers/terms, please write them down. You won't get any grade if you did not do so.