Quantum Computer Systems: Research for Noisy Intermediate-Scale Quantum Computers

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Chapter 8
Noise Mitigation and Error Correction

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EE260 Spring 2021

8 Noise Mitigation and Error Correction

Classical Computers

- Intel Xeon Phi
- 100 errors per billion device hours (114,077 years) due to radiation

Quantum Computers

- IBM Q Melbourne device (with 14 qubits)
- Average single-qubit gate error rate is 4.7×10^{-3}.
- Average two-qubit gate error is 9.46×10^{-2}
- Qubit decoherence (loss of information in a qubit)
  - Phase/Spin decoherence average $60\mu s$
  - Amplitude decoherence average $50\mu s$

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Quantum Computers are sensitive and noisy

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Overview

- 8.1 Characterizing Realistic Noises **(What is noise and how do we define/measure it?)**
  - 8.1.1 Measurement of Decoherence
  - 8.1.2 Quantum-State Tomography
  - 8.1.3 Randomized Benchmarking
- 8.2 Noise Mitigation Strategies **(What are some strategies to decrease noise?)**
  - 8.2.1 Randomized Compiling
  - 8.2.2 Noise-Aware Mapping
  - 8.2.3 Crosstalk-Aware Scheduling
- 8.3 Quantum Error Correction **(What are ways to correct for when errors do occur?)**
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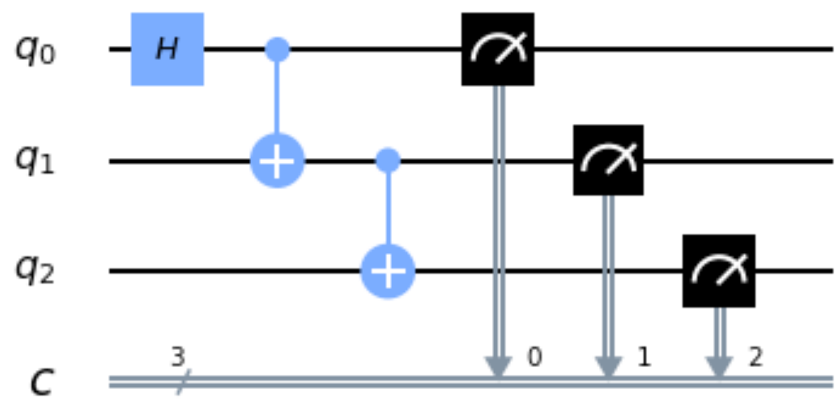
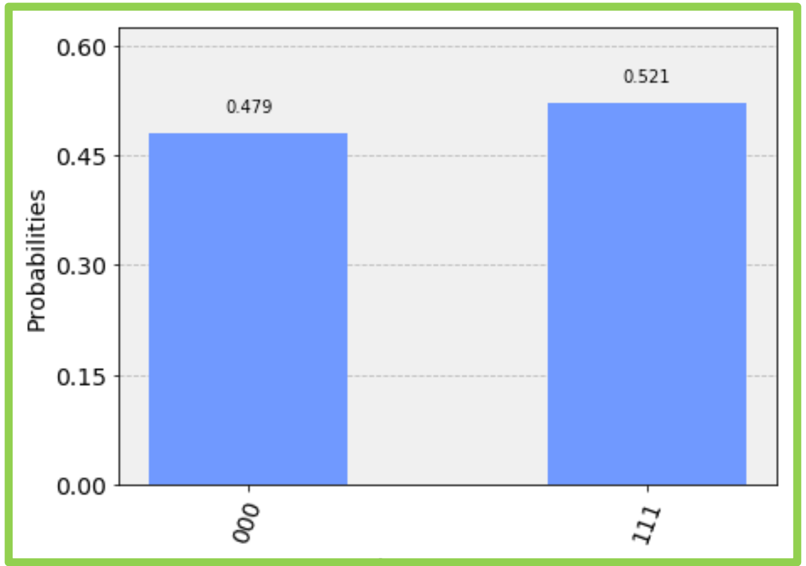
- 8.4 Summary, Outlook, Further Reading

8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

First we need to understand how to quantitatively study noise

- Environmental noise causes a probability distribution
  - $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
  - ρ is called the density matrix of a quantum state
- We care about the "distance" between the results in 2 scenario ideal and noisy





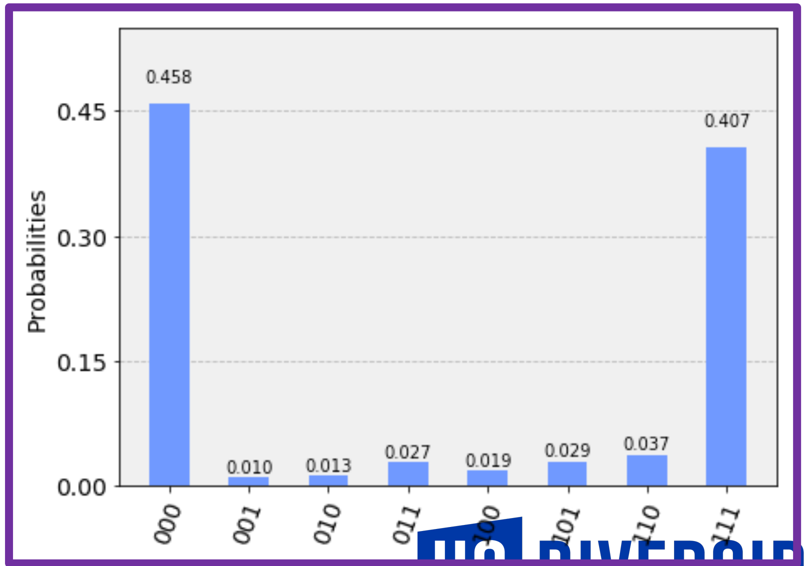


Image Source: https://github.com/qiskit-community/qiskit-community-tutorials/blob/master/Coding_With_Qiskit/ep8_Noise_and_Error_Mitigation.ipynb

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8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

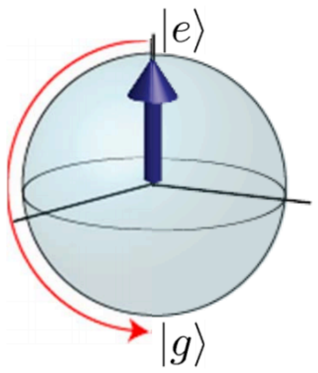
- How do we measure "distance" between the ideal and noisy scenarios?
- 2 ways to measure distance between two state (i.e. ρ and σ)
  - **Fidelity**: $F(\rho,\sigma) = ||\sqrt{\rho}\sqrt{\sigma}||_1^2$
  - **Trace Distance**: $\mathrm{D_{tr}}(\rho,\sigma) = \frac{1}{2}||\rho - \sigma||_1$
- 2 ways quantifying noise of a process (i.e. $\mathcal{U}$ and $\mathcal{E}$)
  - **Average Error Rate:** $r(\mathcal{U},\mathcal{E}) = 1 - \int d\psi \, \langle\psi|U^\dagger\mathcal{E}(\psi)U|\psi\rangle$
    - $\mathcal{U}$ is ideal and $\mathcal{E}$ is noisy
  - **Dimond Distance:** $D(\mathcal{P},\mathcal{Q}) = \sup_{\rho}\frac{1}{2}||[\mathcal{P}\otimes\mathcal{I} - \mathcal{Q}\otimes\mathcal{I}]||_1$
    - Measures the worst case difference between two channels ($\mathcal{P}$ and $\mathcal{Q}$) based on single shot measurement

8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

- What about qubits? How reliable are they?

Qubit decoherence (loss of information)

- T_1 is called the "spin-lattice relaxation time"
  - As time goes on, a qubit is more likely to undergo energy loss
  - a.k.a Amplitude Dampening noise or "T_1 coherence time"
- T_2 is called the "spin-spin relaxation time"
  - The loss of information without the loss of energy
  - a.k.a. Phase Dampening Noise or "T_2 coherence time".



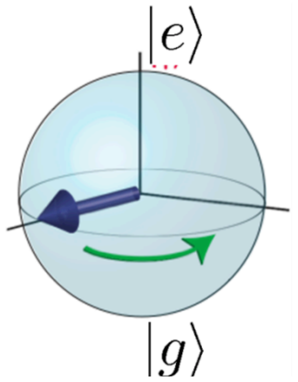


Image Source: https://qudev.phys.ethz.ch/static/content/courses/QSIT11/QSIT11_V08_slides.pdf

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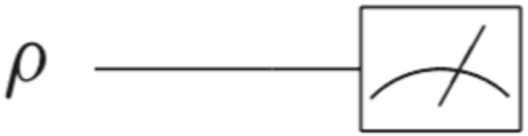
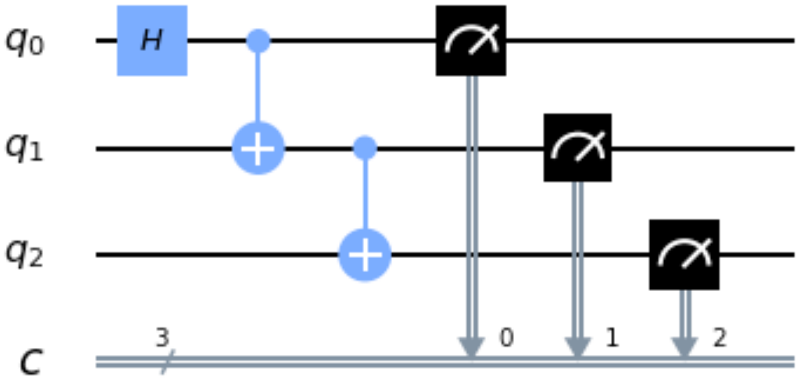
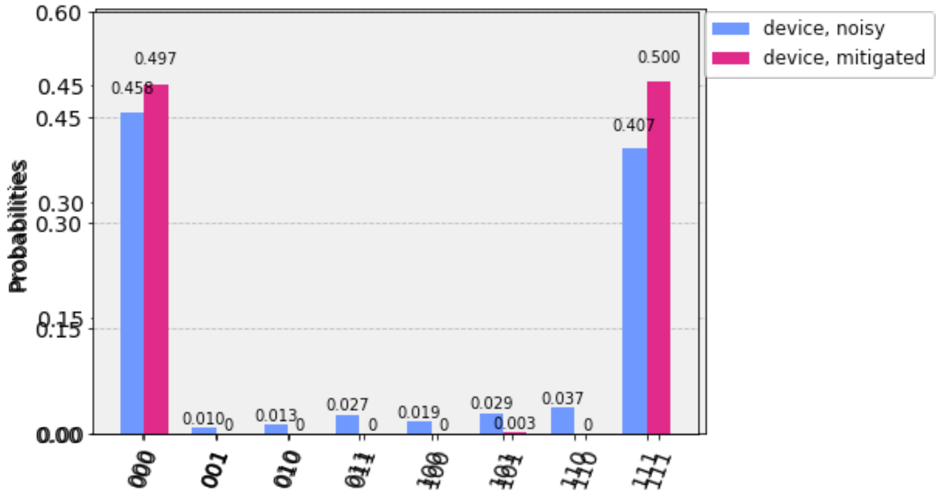
8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

Quantum State Tomography

- The goal of quantum state tomography [310, 311] is to reconstruct an unknown quantum state ρ based on the outcomes from a series of measurements.
- First obtain a probability distribution of measurement outcomes p, by sampling repeatedly. Use estimator p to reconstruct ρ.



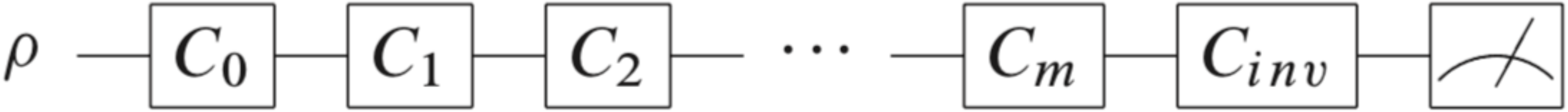




8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

Randomized Benchmarking

- Randomized benchmarking emphasizes on gate errors by applying a long sequence of gates and postponing measurements to the very end.

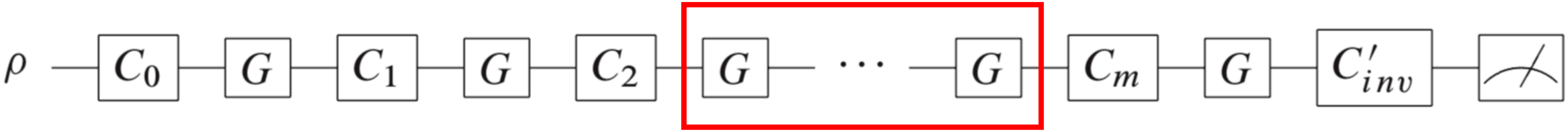


$$p = \Pr[E|\mathcal{C}, \rho]$$

8.1 Characterizing Realistic Noises
(What is noise and how do we define/measure it?)

Randomized Benchmarking

- **Interleaved Randomized Benchmarking**
- Now if we compare the RB decay of this interleaved RB sequence with that of the original standard RB sequence
- We expect to obtain the effect of noise from the extra application of G, and thus bound the fidelity of G.



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8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(1) Randomized compiling

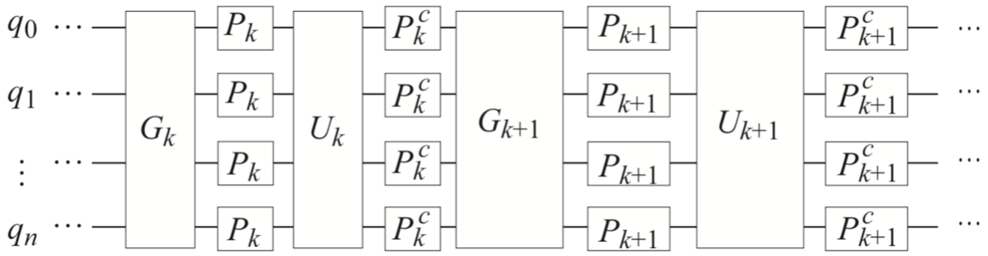
(2) Noise-Aware Mapping

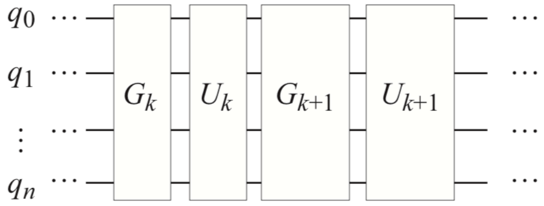
(3) Crosstalk-Aware Scheduling

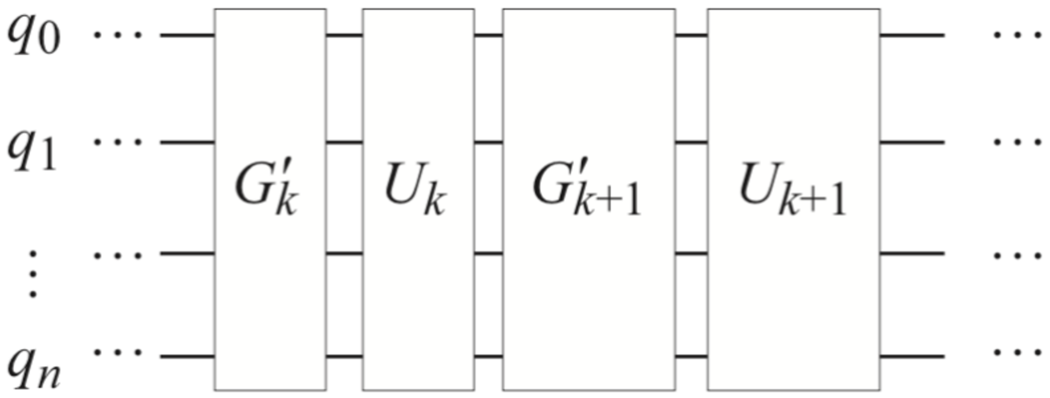
8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(1) Randomized compiling

- Utilize East & Hard Gates
  - Easy gates: The gates that can be implemented with relatively high precision or low resource cost (G_k)
  - Hard gates: any other gates (U_k)







8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(1) Randomized compiling

- Utilize East & Hard Gates
  - Easy gates: The gates that can be implemented with relatively high precision or low resource cost
  - Hard gates: any other gates
- Inserts random gates into a quantum circuit, and averages over many of those independently sampled random circuits.
- "[329] demonstrated that the randomized transformation can be viewed as a scrambling of noise on the quantum circuit "

8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(2) Noise-Aware Mapping

- Adjust for noisy equipment
- Use heuristics to optimize for specific program input, physical machine size and physical topology.
- Take daily variations into account and optimize to increase the probability of correct program output
- Princeton [228] and GATech [229] observing from IBM daily calibration data that qubits and links between qubits vary substantially in their error rate.

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8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(3) Crosstalk-Aware Scheduling

- Crosstalk is when a signal transmitted on one circuit or channel creates an undesired effect in another circuit or channel

<u>Ways to decrease crosstalk</u>

- (1) Connectivity Reduction
- (2) Qubit Frequency Tuning
- (3) Coupler Tuning

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8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(3) Crosstalk-Aware Scheduling

(1) Connectivity Reduction

- Simply build with sparse connections between qubits
- Works by building devices with sparse connections between qubits, hence reducing the number of possible crosstalk channel
- This greatly increases the circuit mapping and re-mapping overhead for executing a logical circuit
- Increases the need for an intelligent scheduler to serialize operations

8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(3) Crosstalk-Aware Scheduling

<u>(2) Qubit Frequency Tuning</u>

- Relies on actively tuning qubit frequencies to avoid crosstalk, featured in some prototypes [333] and by Google [334].
- Software can decide when to schedule an instruction and which frequency to operate the instruction at.

8.2 Noise Mitigation Strategies
(What are some strategies to decrease noise?)

(3) Crosstalk-Aware Scheduling

(3) Coupler Tuning

- A third class builds not only frequency-tunable qubits but also tunable couplers between qubits, termed “*gmon*” architectures [337].
- A different subset of connections are activated (via flux drives to the couplers) at different time steps.
- As such, a schedule for when to activate couplers is needed. For instance, Google proposes a tiling pattern in [66].

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8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

What is Quantum Error Correction (QEC)?

- A way to systematically detect and correct a quantum error

- **Basic Principles of QEC**
  - Quantum Error Correction Codes (QECC)
  - Stabilizer Codes
- **Implementing Logical operations fault-tolerantly**
  - Transversality and Eastin-Knill Theorem
  - Knill's Error Correction Picture

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8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
  - Quantum Error Correction Codes (QECC)

- Quantum Error vs. Classical Error

  - Classical: $0 \rightarrow 1$

  - Quantum: $|0\rangle \xrightarrow{X\ gate} \sqrt{\epsilon}\,|0\rangle + \sqrt{1-\epsilon}|1\rangle$ where ϵ is the error

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
  - Quantum Error Correction Codes (QECC)

- 2 Key Concepts that make quantum error correction possible
  - (1) Redundant Encoding
    - Use redundant encoding of information
    - Thus, noise in certain parts of the system can be tolerated and not effect the whole system
  - (2) Digitizing Quantum Error.
    - Digitize quantum errors since we know how to deal with digitized errors

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
  - Quantum Error Correction Codes (QECC)
- QECC is a mapping from k logical qubits to n physical qubits.
  - $n > k$
  - $|0_l\rangle = |000\rangle$ (0_l for logical qubit)
  - $|1_l\rangle = |111\rangle$ (1_l for logical qubit)
  - probability p one of the physical qubits flipped and we got $|0_l\rangle = |001\rangle$
  - Can be corrected i.e. through majority rule

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
  - Quantum Error Correction Codes (QECC)
- <u>Locate bit flip by using two-bit operators</u>
  - Z Gate: $\begin{cases} Z|0\rangle \rightarrow |0\rangle \\ Z|1\rangle \rightarrow -|1\rangle \end{cases}$
  - Suppose $|\psi\rangle = |100\rangle$ instead of expected $|000\rangle$
  - $|\psi\rangle = |100\rangle \rightarrow ZZI = -|100\rangle$
  - $|100\rangle \rightarrow IZZ = |100\rangle$

- $|000\rangle$ would produce (+1, +1) ∴ no bits flipped
- $|100\rangle$ would produce (-1, +1) ∴ first bit flipped
- $|010\rangle$ would produce (-1, -1) ∴ second bit flipped
- $|001\rangle$ would produce (+1, -1) ∴ third bit flipped

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
    - Quantum Error Correction Codes (QECC)
- <u>Check Matrix Formalism:</u> create a set of qubit operations using the stabilizer formalism, with enough permutations sequences of eigenvalues to determine which qubit is flipped

$$|\psi\rangle = |0 \quad 0 \quad 0\rangle$$

$$\begin{array}{c} ZZI \\ IZZ \end{array} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$|\psi\rangle = |0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\rangle$$

$$\begin{array}{c} ZZZZIIII \\ ZZIIZZII \\ ZIZIZIZI \end{array} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Basic Principles of QEC**
  - Stabilizer Codes

$$C(S) = \{|\psi\rangle \in \mathcal{H}, s.t.\ g|\psi\rangle = |\psi\rangle \forall g \in S\}$$

Theorem 8.7 *Given a set of Pauli errors $\mathcal{E}$, if for all $E_i, E_j \in \mathcal{E}$, $\exists g \in S$, s.t. $E_i^\dagger E_j g = -g E_i^\dagger E_j$, then the set of errors $\mathcal{E}$ is correctable by the stabilizer code $C(S)$.*

- The proof of this theorem can be found in [86].
- Effectively, if a Pauli error anti-commute with a stabilizer, then the stabilizer can detect and correct an occurrence of the error.

8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Implementing Logical operations fault-tolerantly**
  - Transversality and Eastin-Knill Theorem
- Transversality
  - For each error correction code, there is a class of gates whose logical gate operations (i.e., encoded gates) are easy to implement fault-tolerantly, namely the transversal quantum gates.
  - Impose restriction of one qubit to make a gate into a transversal gate
- Eastin-Knill Theorem
  - No-go theorem
  - "No quantum error correcting code can have a continuous symmetry which acts transversely on physical qubits".
  - In other words, no quantum error correcting code can transversely implement a universal gate set.
  - Can be circumvented by using gate teleportation, a.k.a. “Knill’s error correction picture“

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8.3 Quantum Error Correction
(What are ways to correct for when errors do occur?)

- **Implementing Logical operations fault-tolerantly**
  - Knill's Error Correction Picture
- Way to get around Eastin-Knill Theorem
- Quantum Teleportation: Technique for transferring quantum information from a sender at one location to a receiver some distance away.
- Quantum Gate Teleportation: Quantum gate teleportation is where quantum gates are applied to quantum states via quantum teleportation.

8.4 Summary, Outlook, Further Reading

- **Defining/Measuring Noise**
  - Fedelity,
  - Trace distance,
  - Average error rate,
  - Dimond Distance
- **Mitigating Noise**
  - Randomized compiling,
  - Noise aware mapping,
  - crosstalk-aware scheduling
- **Error Correction**
  - Quantum Error Correctio codes,
  - stabilizer codes,
  - Transversality and Eastin-Knill Theorem,
  - Knill's Error Correction Picture

8.4 Summary, Outlook, Further Reading

- **Physical Noise Mitigation**
  - composite pulses [298], dynamical decoupling [347]
- **Scalability remains a challenge**
  - characterize realistic quantum noises [328, 348], classically simulate quantum noises [349–351]
- **Quantum error correction (QEC)**
  - The capstone of QEC is the theorem called "threshold theorem" [352] states that once physical error rate is less than a certain threshold, we can preform quantum computation accurately with only a moderate increase in circuit size. Details of the theorem [23, 25, 86]

"In the near term, numerous efforts have been put in designing low-overhead quantum error correction codes adapted to different noise models and program characteristics, as well as aiming to reduce the cost of magic state distillation protocols."

Questions?