Homework #2: Polymer Networks

Problem #1- Sixfold and Fourfold connectivity network model

In class, we explored the shear modulus of a sixfold and fourfold connectivity network model. Now, let's examine the network response of these two models for a normal stress. Is there an advantage to one versus the other under normal stress? You may assume the normal stress results in similar strains in both cases.
One thing that might help is the definition of elastic modulus:

$$K_a = \frac{\partial^2 W}{\partial \varepsilon^2}$$

To help: Start with fourfold connectivity and draw the unit cell with and without uniaxial compression. Use this to find strain energy density of one cell in terms of strain so you can use the definition above. Do the same thing for six-fold connectivity (you will have to do similar geometry as what we did in class).

Problem #2 – Micropipette Aspiration

In class, we used these equations to explain how micropipette aspiration works. Read the review *Micropipette Aspiration of Living Cells*. The review points out several different experiments to illustrate modeling a cell as a liquid drop or an elastic solid.

1. Explain the difference between the liquid drop vs. elastic solid model with respect to micropipette aspiration. Draw a graph to help your explanation.
2. Choose one of the papers this review highlights and describe:
    a. What was the cell of examination?
    b. What were the mechanical properties found about said cell?
    c. Was this cell modeled as a liquid drop or an elastic solid?
3. What are the limitations of micropipette aspiration?

Problem #3 – Measuring 3D mechanical forces with droplet mechanics

We derived many equations to understand membrane dynamics in class and problem 4. Now read *Quantifying cell-generated mechanical forces within living embryonic tissues* and see how it is used!

1. Explain in around 150 words, the gist of this paper. Pretend this is your project proposal and address all the points you would address in your presentation.
2. Why is this method more advantageous than other methodologies for measuring force? What are some of the disadvantages?
3. How were the measurements taken and calculated?

Problem #4 – Derivation of Kirchhoff plate equation **(Print and do work on here).**

$$p_z = K_B\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \quad where\ K_B = \frac{Eh^3}{12[1-\nu^2]}$$

Recall the equation I stated in class as shown above. For a red blood cell this number was very small, on the order of 10^{-19} Nm. So where does this equation come from? Let's begin with the kinematic assumptions. What 3 assumptions have been making throughout this class?

Based on these assumptions, we typically derive at three equations for deformation (in each axis). What are they (u, v, w)?

Now, recall that there is a constant term and a term that varies linearly with thickness. Only the terms that vary linearly with thickness can produce a bending moment. Thus, you can drop the constant terms. This simplifies your deformation equations to

From this, calculate the strains from their continuum definition. (Hint: look at lecture notes).

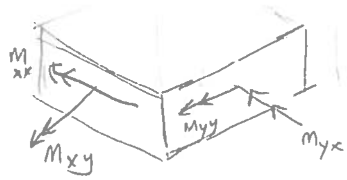
As before, we can use the definition of curvature to simplify these equations to

Alright then! Kinematic equations done, now let's move to the constitute model. We will assume generalized Hookean material behavior (linear and elastic). We will assume the material properties and thickness are constant (homogeneous). Because of our kinematic assumptions, what can we say about our z-direction strains and why?

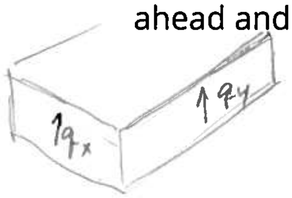
Write the simple form of the constitutive model that describes normal stress in x and y and shear stress. (In your lecture notes!)

Now substitute the definitions of curvature to replace your expressions of strain.

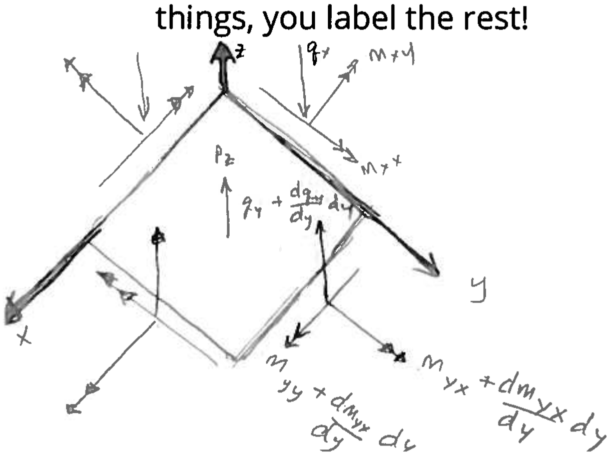
Alright, kinematic assumptions done. Constitutive model in place. What's next? Equilibrium conditions! We need to consider the resultant forces first. In this particular situation, we are considering bending only, so we need to derive the stress moment resultants. For a plate, we need three moments {xx, yy, xy}. Write what the moment equations would be (keep it in integral form).



With moment resultants defined, consider the drawing below. What's missing? We still need to define shear resultants in order to satisfy equilibrium conditions. Go ahead and write the integral form for q_x and q_y.



Now we can apply equilibrium equations. In class, we used force equilibrium to derive the governing equations and moment equilibrium about the z-axis to show a symmetry condition. For the plate bending, we need to add the moment about x and y axes. Note: force equilibrium in x and y does not apply since there are no forces in those directions. Let's start with a free body diagram. I labeled a few things, you label the rest!



Using the FBD, let's write the force equilibrium condition in the z direction.

Now write the moment equilibrium equations in the x and y direction. The z axis moment condition illustrates that two quantities are equal to each other. Write this down as well.

Wonderful! Now use these equations to form the classical equilibrium equation for the plate.

Now let's combine the original definition of moments with the constitutive law.

Now use these expressions for the moments in terms of curvature into the equilibrium equation you derived and voila! You have now derived the fourth order differential equation for plate bending. You can write it in terms of curvature or in terms of displacement, like below.

$$p_z = K_B\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \quad where\ K_B = \frac{Eh^3}{12[1 - \nu^2]}$$