Homework 4 solution

Thursday, April 21, 2016 10:22 AM

1 incompressible → $\nu = 0.5$ (poisson's ratio)

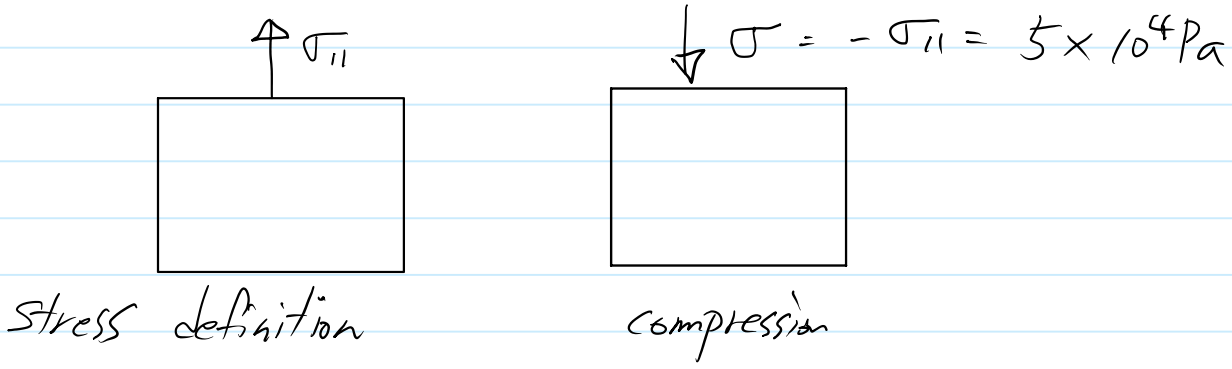
(a) 5% compression ~ strain $\varepsilon = -0.05$

Recall 1D Hooke's Law $\sigma_{11} = E\varepsilon_{11}$

$$\sigma_{11} = (10^6\ Pa) \times (-0.05) = -5 \times 10^4\ Pa$$

$$F = \sigma \times A = (-5 \times 10^4\ Pa) \times (0.01\ m)^2$$

$$= \boxed{-5\ N} \quad \text{(compression)}$$



stress definition compression

(b) $\nu = 0.5$ (from the assumption of incompressibility)

Again Hooke's Law, $\gamma_{21} = \frac{2(1+\nu)}{E}\sigma_{12}$, $E = 2G(1+\nu)$

and, $$G = \frac{\sigma_{12}}{\gamma_{21}} = \frac{\sigma_{12}}{\left(\frac{0.1h}{h}\right)}$$

therefore, $G = \frac{E}{3} = \boxed{3.33 \times 10^5\ Pa}$ (Shear Modulus)

$$\sigma_{21} = (3.33 \times 10^5 Pa) \times (0.1) = \boxed{3.33 \times 10^4\ Pa}$$

$$\boxed{F_{shear} = \sigma \times A = 3.33\ N}$$

$F_{shear} = \sigma \times A = 3.33\ N$

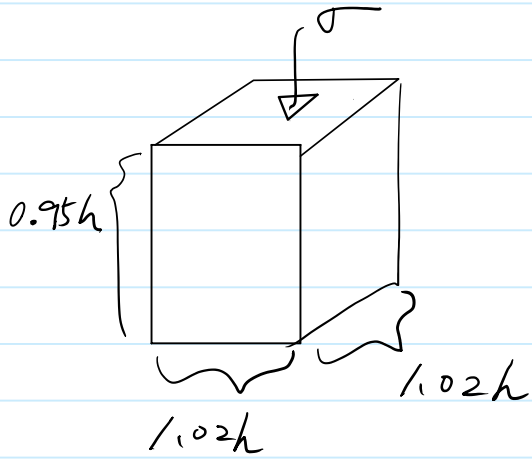
(c) Poisson Ratio $= -\frac{d\varepsilon_{trans}}{d\varepsilon_{axial}} \approx -\frac{\varepsilon_{22}}{\varepsilon_{11}}$ (for small strain)

$= -\frac{(0.02)}{(-0.05)} = 0.4$ compressible!

$V_{initial} = 1\ cm^3$

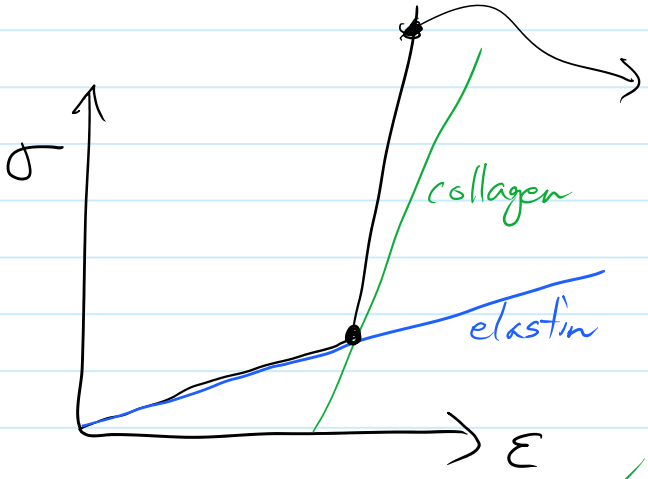
$V_{final} = (0.95)(1.02)^2$

$= 0.988\ cm^3$

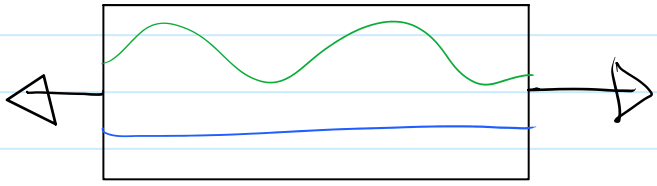


2

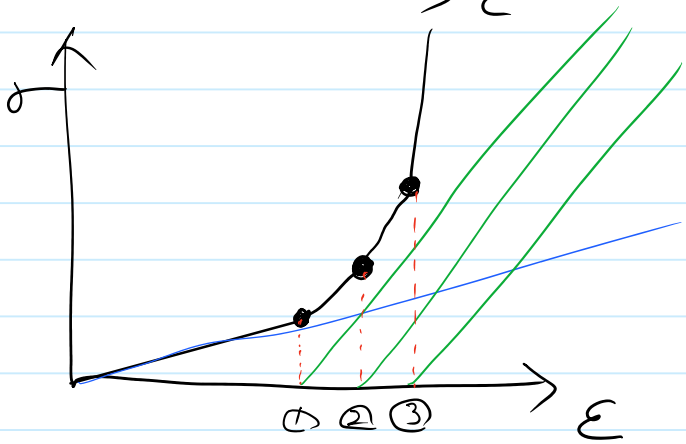
(a)



tissue made of one elastin and one collagen.



(b)



at the stiffness of collagen fiber

At each point, the stiffness of collagen fiber is added to the overall stiffness of tiss.

(c)

- Pressure is equivalent to stress and arterial diameter to strain, so, Figure 4 is similar to the 90-deg rotated stress-strain curve

1. We can see the linear relationship of stress and strain roughly up to 70-80 mmHg in pressure
2. Beyond the linear range, more than 80 mmHg, increasing stress (pressure) for less strain change can be seen that is similar to the curve we have drawn in 2(b)
3. As the percentage of elastin is increased, the specimen gets more "elastic", which means more deformable.
4. For the curve of the least elastin in Figure 4 shows the highest stiffness in overall: it is not shown in Figure 4, however, with less elastin fibers, collagen fibers no longer have the waviness. Therefore, the arterial mechanical property is shifted into the stiffer range of collagen fibers.

3

Any two references in the standard formats

General solution from Wikipedia is okay;

- The keywords are "basement membrane" and "basal lamina"
- It lies between the epidermis and the dermis
- Functions: anchoring, mechanical barrier, angiogenesis, so on
- Basal lamina is regarded as epithelium, while other layer is collagen connective tissue