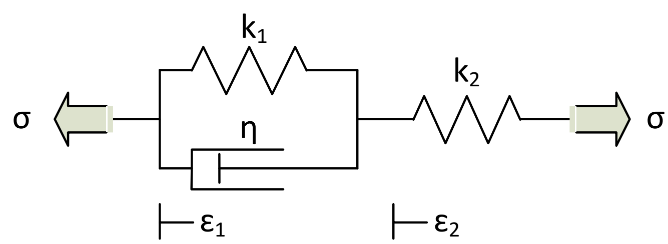
ME445/BioEN440 Spring 2016

Introduction to Biomechanics

Homework 5 (corrected)

1. The following model is another form of the Zener model (three-parameter model). And the stress-strain relationship of the Voigt model is given. Based on the Voigt model equation, find the stress-strain relationship for this model. (in the standard form)



Voigt Model Equation: $\sigma_1 = k_1\varepsilon_1 + \eta\dot{\varepsilon}_1$

2. We have derived the solutions for stress relaxation and creep tests for Maxwell, Voigt, and Kelvin(SLS) models. The constant strain for a stress relaxation test and constant stress for a creep test as well as the constants of spring and dashpot are given.

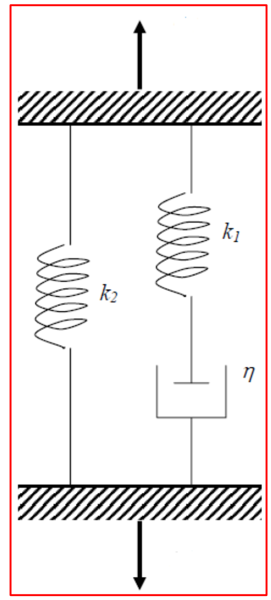
For a stress relaxation test, $\varepsilon 0 = 1$, and for a creep test, $\sigma 0 = 1$.

- Maxwell Model: $k = 1.5$, $\eta = 1$
- Voigt Model: $k = 1.5$, $\eta = 1$
- Kelvin(SLS) Model: $k1 = 1$, $k2 = 0.5$, $\eta = 0.3$, where the spring of k1 is connected in series to a dashpot.

a) Plot the stress responses of three models to the stress relaxation test (one figure)

b) Plot the strain responses of three models to the creep test (one figure)

3. Three-parameter Model (SLS with Maxwell)



a) In class, we stated that the three-parameter model for the standard linear viscoelastic solid in Figure. And it could be described by an ordinary differential equation of the form:

$$\sigma + \alpha\dot{\sigma} = k_2\varepsilon + \beta\dot{\varepsilon}$$

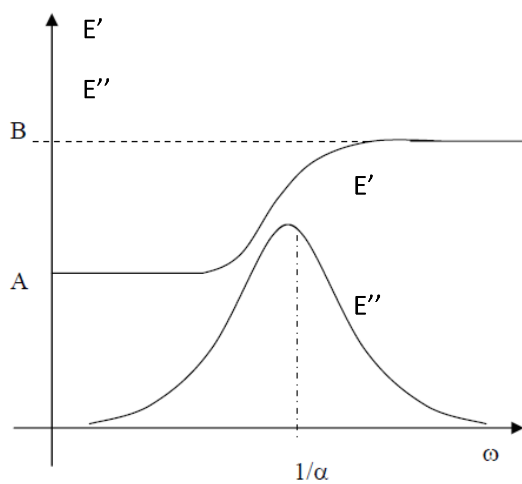
Show that this form is correct, and find the constant $\alpha$and βin terms of the element values k1, k2, and η.

b) The differential equation given above is useful for describing creep and stress relaxation of some viscoelastic materials. For the case of cyclic (sinusoidal) loading it is more convenient to describe the frequency dependence of the material stress-strain behavior using a "complex modulus". In class, we showed that the complex modulus has the form:

$$E^* = E' + iE''$$

where E* = (k2 + iωβ)/(1 + iωα). Find the real (E') and imaginary (E'') parts of the complex modulus in terms of α, β and the angular frequency ω.

c) Show that E'(ω) and E''(ω) have the frequency dependence shown qualitatively in Figure by reasoning the low and high frequency limits. Find the constants A and B in terms of the element values k1 and k2 based on physical (and/or mathematical) arguments.



4. In this question we will assess the strength of cortical and trabecular bone. Stress-strain curves in a compression test have been obtained for cortical and trabecular bone (Figure 3). The two samples of trabecular bone came from patients with healthy and osteopenic bone. From these results, we can determine the mechanical properties for each bone sample. These values are shown in Table 1.

Table 1. Mechanical Properties of Bone in Compression

	Cortical	**Trabecular (ρ=0.9 g/cm3)**	**Trabecular (ρ =0.3 g/cm3)**
Yield Strength (MPa)	165	35	5
Ultimate Strength (MPa)	180	60	5
Yield Strain (m/m)	0.01	0.03	0.04
Ultimate Strain (m/m)	0.025	0.235	0.23
Elastic Modulus (GPa)	16.5	1.2	0.125
Anelastic Modulus (MPa)	N/A	120	0
Strain-Energy Density (J/cm3)	?	?	?

1) The strain energy density U of a material gives a measure of the amount of energy it can absorb before fracture. This relationship is given by

$$\mathrm{U} = \int_0^{\varepsilon_u} \sigma \, d\varepsilon$$

where σ is the compressive stress, ε is the strain, and ε_u is the ultimate strain at failure. Estimate U for the three bone samples.

2) What does your result indicate about the function of healthy trabecular versus cortical bone in absorbing energy?

3) Describe the structure of the two bone types (trabecular and cortical bones) and how it governs the difference in strain energy density.

4) What does your result in part (1) indicate about the risks of osteoporosis?

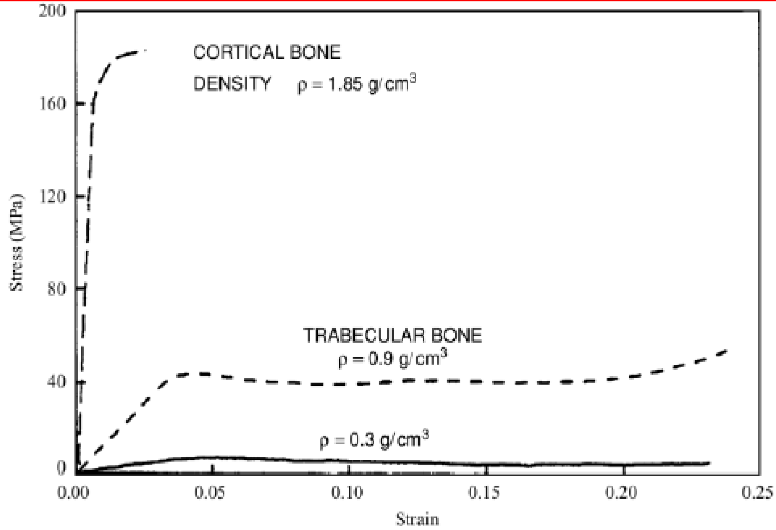


Figure 3. Stress-strain curves of bone under compression