ME445/BioEN440 Spring 2016

Introduction to Biomechanics

Homework 6

1. Use Fung's strain energy density function of blood vessels below.

$$W = \frac{C}{2}\exp\left(a_1 E_{\theta\theta}^2 + a_2 E_{zz}^2 + 2a_4 E_{\theta\theta} E_{zz}\right)$$

a) Obtain an expression for **<u>Kirchhoff stress $S_{\theta\theta}$</u>** as a function of **<u>Green strains</u>**.

b) Use the following Fung data from the carotid artery and the aorta. As mechanical parameters in the above equation.

Artery	C [kPa]	a_1	a_2	a_4
Carotid	2.9	2.5	0.46	0.176
Upper Aorta	3.38	2.8	0.52	0.58

<u>Plot Kirchhoff stress $S_{\theta\theta}$ versus Green strain $E_{\theta\theta}$</u> for these vessels on the same plot when E_{zz} = 10% is held constant and $E_{\theta\theta}$ goes from 0 to 1.

c) Re-plot the aortic curve in b) for **<u>Cauchy stress versus Green strain</u>**. Note that

$$s_{\theta\theta} = \lambda_{\theta\theta}^2 \mathbf{S}_{\theta\theta} = (2E_{\theta\theta} + 1)\mathbf{S}_{\theta\theta}$$

, where s is Cauchy stress and **S** is Kirchhoff stress

d) At 120mm of Hg, compute **circumferential(hoop) stress in KPa** using the **thin-wall solution**. An average aorta has a radius of 12mm and a **(t/r)** ratio of 0.10, where r is the radius of an aorta and t is a vessel wall thickness.

e) At 120mm of Hg, compute **circumferential(hoop) stress in KPa** the **thick-wall solution**. Now, you will need to plot the curve with respect to 'r', the thickness of a wall. And what is an averaged circumferential stress over the thickness?

f) Find a strain point of the circumferential stress in (d) and averaged circumference stress (e) on the aortic curve in part (c).
(Note: Thin-wall and thick-wall equations in this application will be giving the true or **Cauchy stress**.)

2. In physiology, pressure is measured frequently with a mercury manometer. Show that the mean pressure distribution in the blood vessels of a standing man as shown in Figure is roughly correct. The atmospheric pressure is taken as zero.

   Hydrostatic pressure : $p_1 - p_0 = \rho g h$, where ρ is density, g is gravitational acceleration, h is height

   Note: The density of blood is 1000 kg/m^3. The density of mercury is 13,600 kg/m^3, and g is 9.8 m/s, 1 mmHg = 113 N/m^2

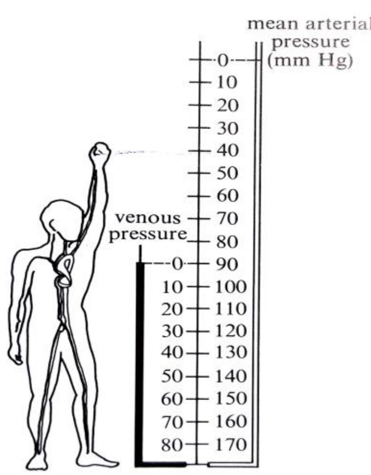

Figure. Mean arterial and venous pressures in human circulation, relative to atmospheric pressure.

3. In Fung's model, if Ezz = 0, then for a large a_1, the curve of $S_{\theta\theta}$ and $E_{\theta\theta}$ will leave the origin closer to the strain axis, and then rise more rapidly to the final point, $(S^*_{\theta\theta}, E^*_{\theta\theta})$. **Briefly discuss** how the magnitude of the strain energy density function will be changed for increasing a_1. (Note: think about the change of the coefficient C while the final point is fixed. I don't expect to see any mathematical analysis or details; you may just shortly explain what you think about the change of a curve in terms of W)

$$W = \frac{C}{2}\exp\left(a_1 E_{\theta\theta}^2 + a_2 E_{zz}^2 + 2a_4 E_{\theta\theta} E_{zz}\right)$$