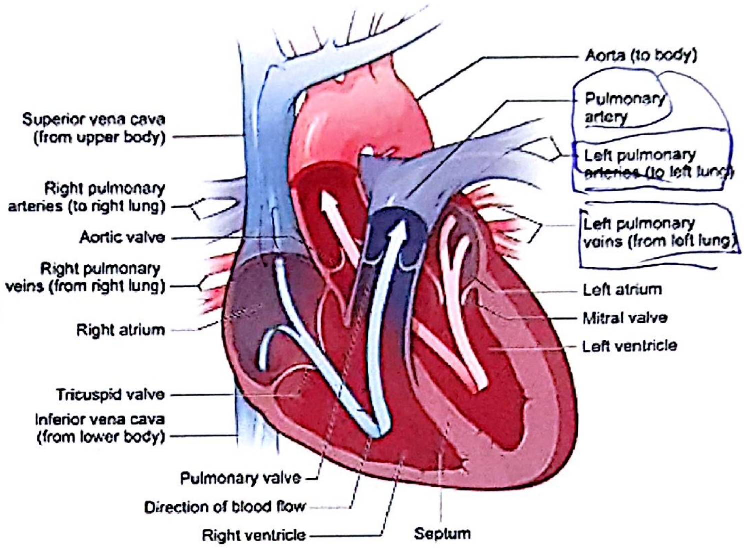
Session 2, Lecture 5, Blood Vessels 5/6

## Blood Vessels

- Arteries, capillaries, veins
- High pressure to low
- Adult vascular system ~ 60,000 miles ~40,000 km

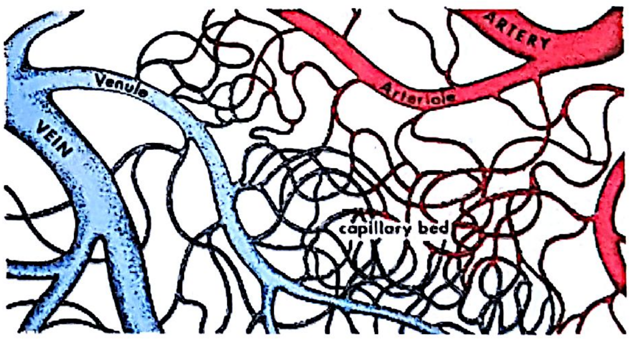


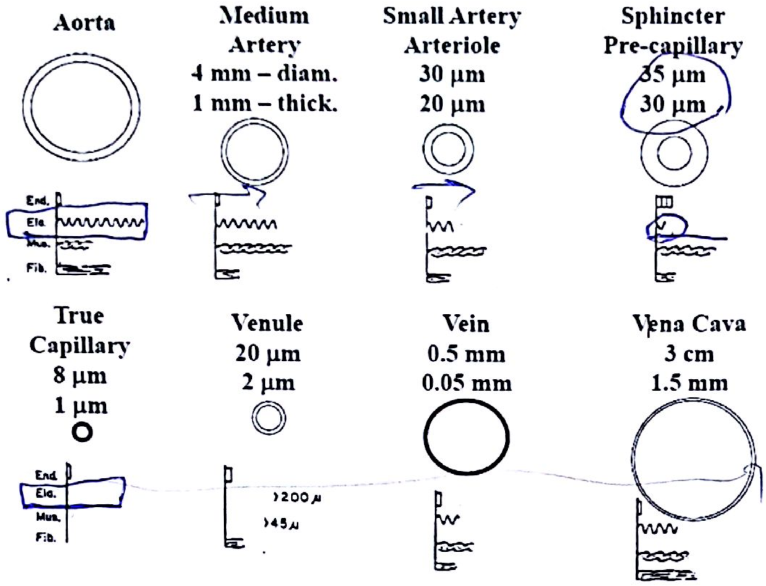

## Nomenclature

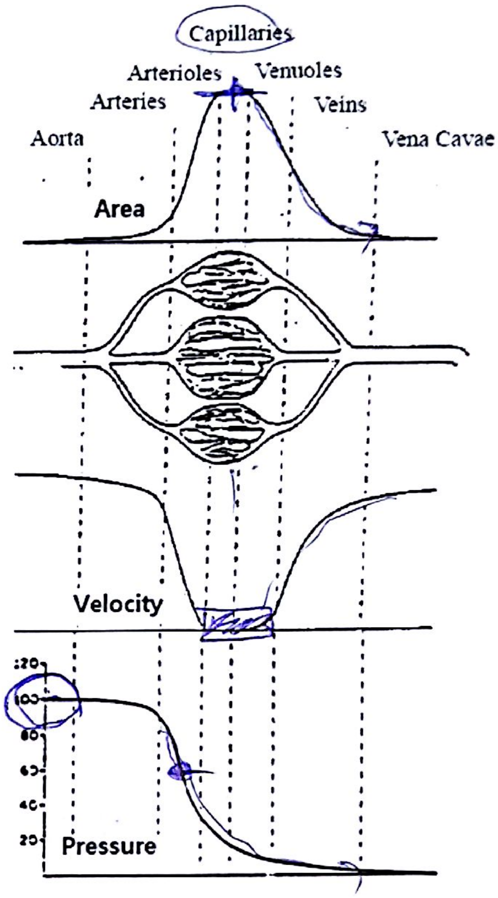
- Arteries:
  Blood flows away from heart
  to organs
  Usually oxygenated (except for pulmonary artery)
- Veins:
  Blood flows from organs to heart
  Usually deoxygenated (except for pulmonary artery)

## Hierarchy

- Arterial side
  Series of branching vessels
  Aorta-Arteries-Arterioles-Capillaries
- Venous side
  Series of coalescing vessels
  Capillaries-Venules-Veins-Superior and
  Inferior Vena Cavae






**Due to branching**

$$V = Q / A$$

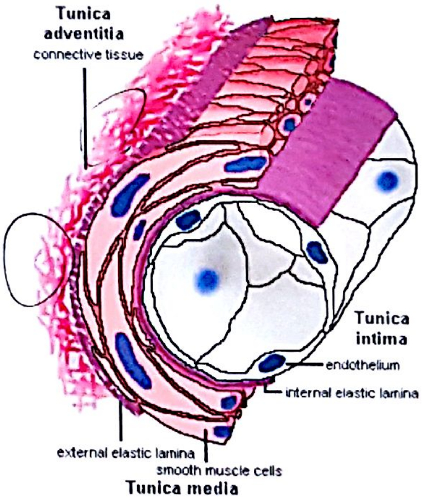
where V = velocity
Q = flow
A = cross sectional area

## Pressure variation

- Systolic: ventricle is contracting, ~120mm Hg in aorta [16,000 Pa]
- Diastolic: ventricle is filling, ~80mm Hg in aorta [10,640 Pa]

### Microstructure

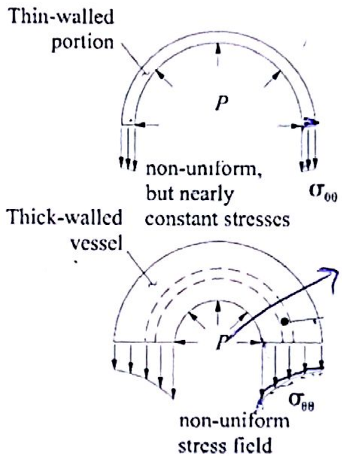
- Intima: Inner most layer, basal lamina, endothelial cells, Smooth wall and selective permeability to $H_2O$, electrolytes, sugars and other substances
- Media: Middle layer, elastic sheets (no elastin in capillaries), mechanical strength
- Adventitia: Outermost layer, tethering



## Stress-strain relationships for blood Vessels

1) **Cylindrical vessels** $\boxed{t/r \leq 0.1}$

1. Thin wall vessels
2. Thick wall vessels



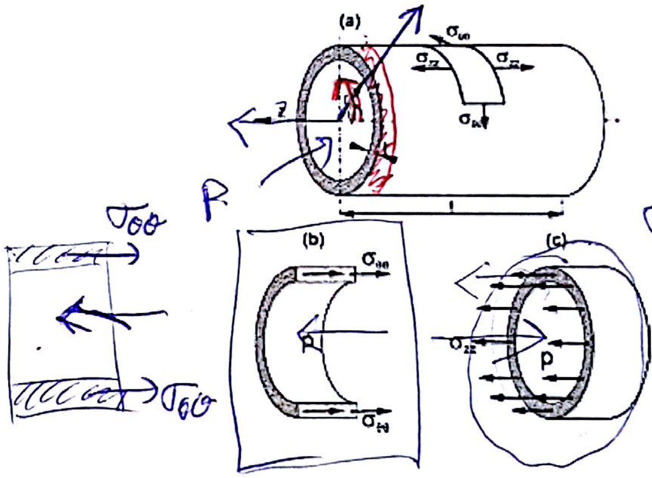

$\sigma_{\theta\theta} = C.$ //

$\sigma_{\theta\theta} = f(r)$

**Thin wall vessels (Laplace's Equation)**

Assumptions:

① No transverse shear - 2D problem.
② Not consider bending, nonaxisymmetric, localized, or concentrated loads.
③ No curved or branching vessels
④ $t \ll r$, $t/r < 0.1$




r: radial coordinate
θ: angular coordinate
z: axial coordinate
t: thickness

Circumferential(Hoop) stress

$\sigma A = F$ $(\sigma_\theta = \sigma_{\theta\theta})$

$2(\sigma_\theta \, dA) = p \, dA$

$2\sigma_\theta (t \cdot \Delta z) = p(2r \cdot \Delta z)$

$\sigma_\theta t = pr$

$\boxed{\sigma_\theta = \frac{pr}{t}}$

Longitudinal(Axial) stress

$\sigma_z [\pi(r+t)^2 - \pi r^2] = p\pi r^2$

$\sigma_z 2\pi t r = p\pi r^2$

$\boxed{\sigma_z = \frac{pr}{2t}}$

$\boxed{\sigma_\theta = 2\sigma_z}$ //

~~Valida~~ Validity

- Homogeneous (in thickness)
- Small strain
- Simple ~~aorta~~ analysis for experimental application.

Circumferential strain

$$\varepsilon_\theta = \frac{\Delta r}{r}$$

$$\boxed{E = \frac{\sigma}{\varepsilon}} \quad \boxed{\varepsilon_\theta = \frac{\sigma_\theta}{E} = \frac{Pr}{Et}}, \quad \boxed{\Delta r = \frac{P}{Et}}$$

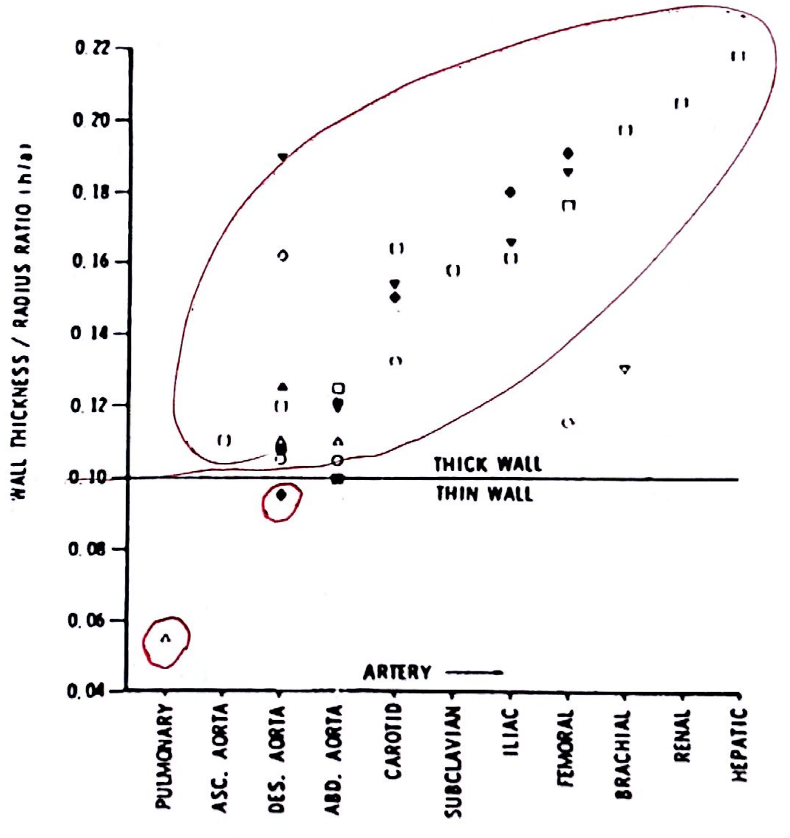
Young's Modulus (thin walls, small strain)

$$E = \frac{\sigma_\theta}{\varepsilon_\theta} = \frac{\frac{Pr}{t}}{\frac{\Delta r}{r}} = \boxed{\frac{P \cdot r^2}{\Delta r \cdot t}}$$

$$\varepsilon_{rr} = \frac{\cancel{E\Delta r}\,P}{E}$$

However,
Most blood vessels have $t/r > 0.1$
$(0.105 \sim 1.5)$

$t/r$

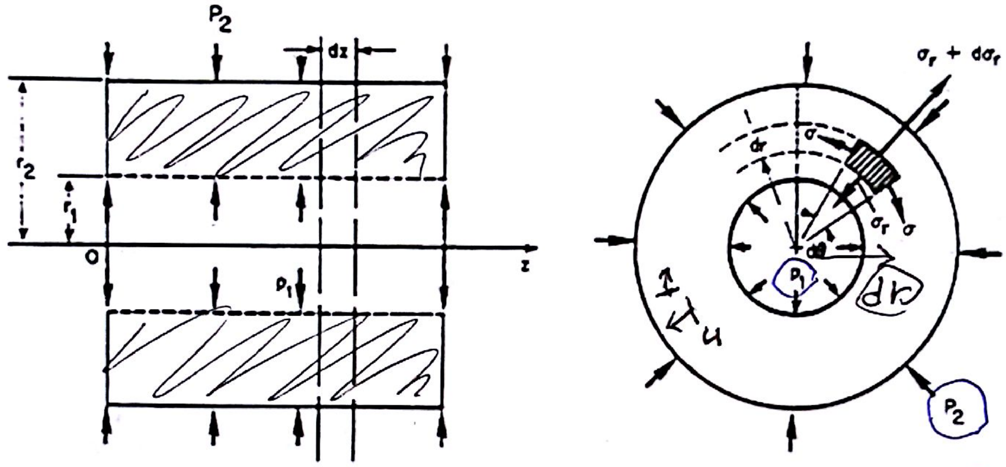


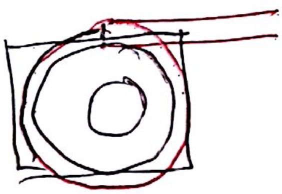
**Thick wall vessels** (Look up a handout for derivation)

$r_1$ = internal radius $\quad$ $p_1$ = internal pressure

$r_2$ = external radius $\quad$ $p_2$ = external pressure

Thickness dz, Arc dθ, axisymmetric geometry and loading, no shear stress



**Strains**

displacement

$$\varepsilon_r = \frac{du}{dr} \qquad \varepsilon_\theta = \frac{u}{r}.$$

**Stresses (Lame's equations)**

$$\sigma_r = P_1\left(\frac{r_1^2}{r_2^2 - r_1^2}\right)\left(1 - \frac{r_2^2}{r^2}\right) \qquad \sigma_\theta = P_1\left(\frac{r_1^2}{r_2^2 - r_1^2}\right)\left(1 + \frac{r_2^2}{r^2}\right)$$

If Plane stress ($\sigma_z$=0),

$$P_2 = 0$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_r + \sigma_\theta), \qquad \varepsilon_z = -\frac{2\nu}{E}\cdot\frac{P_1}{\left[(r_2/r_1)^2 - 1\right]}.$$

strain.

**Young's modulus**

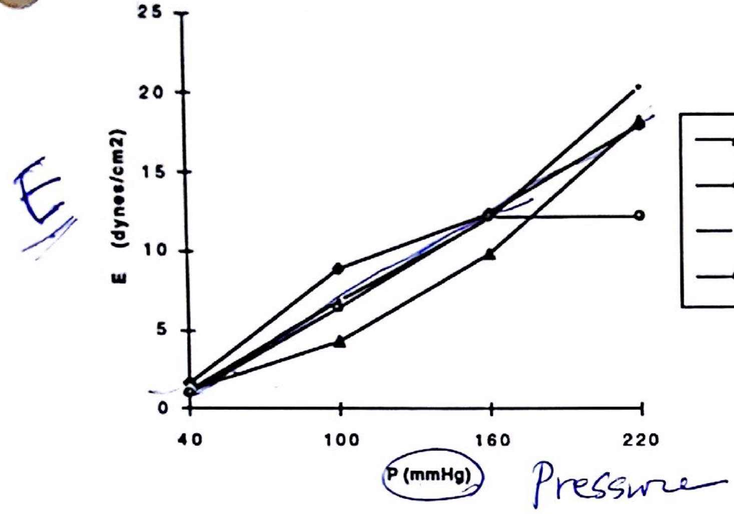
$$\sigma_\theta = \frac{E}{(1-\nu^2)}(\varepsilon_\theta - \nu\varepsilon_r) \quad \text{set } P_1 = \Delta P, \ \boxed{r = r_2}, \ u = \Delta r_2$$

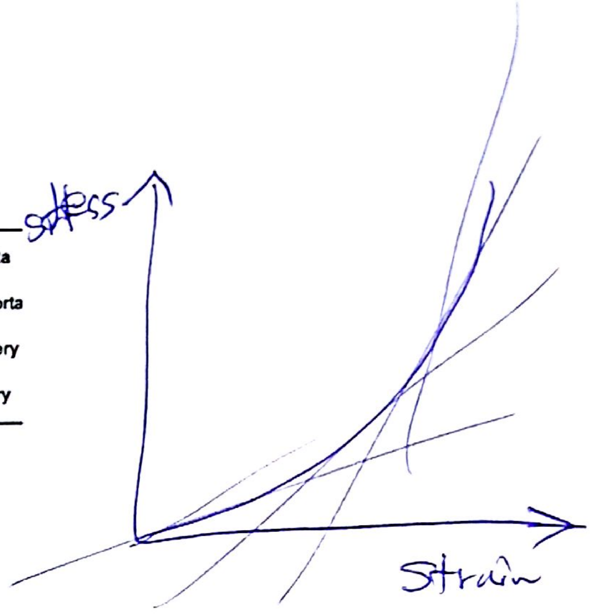
$$\varepsilon_r = 0$$

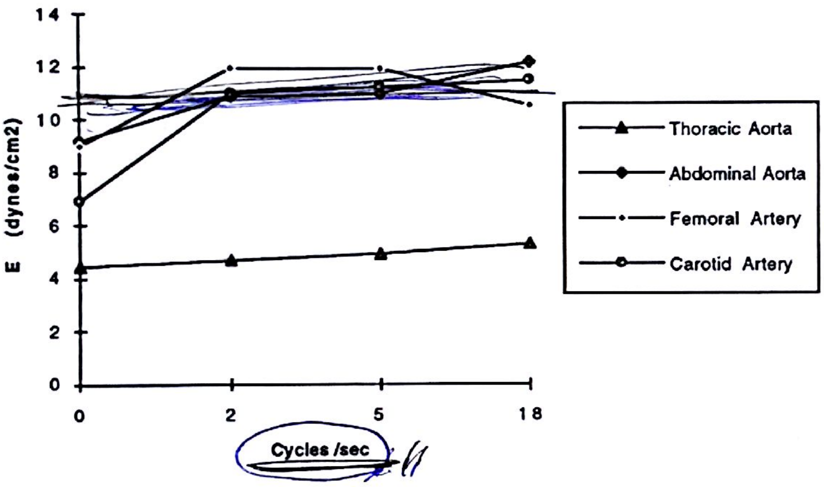
$$\boxed{E_{inc} = \frac{2(1-\nu^2)r_1^2 r_2}{(r_2^2 - r_1^2)\Delta r_2}\Delta P}$$

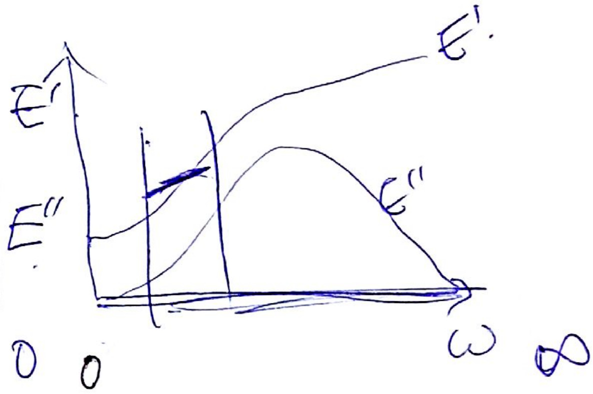
"incremental"

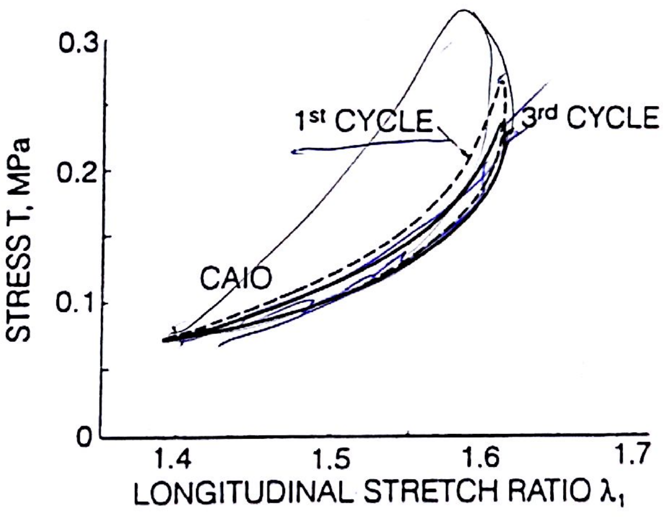
## Elastic Modulus



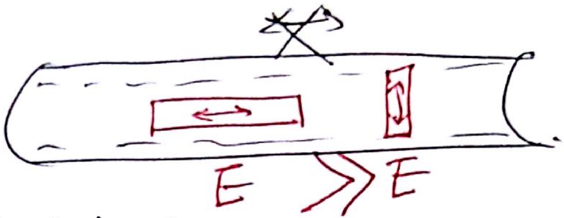



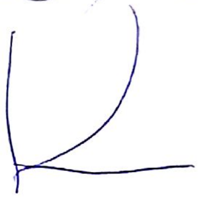



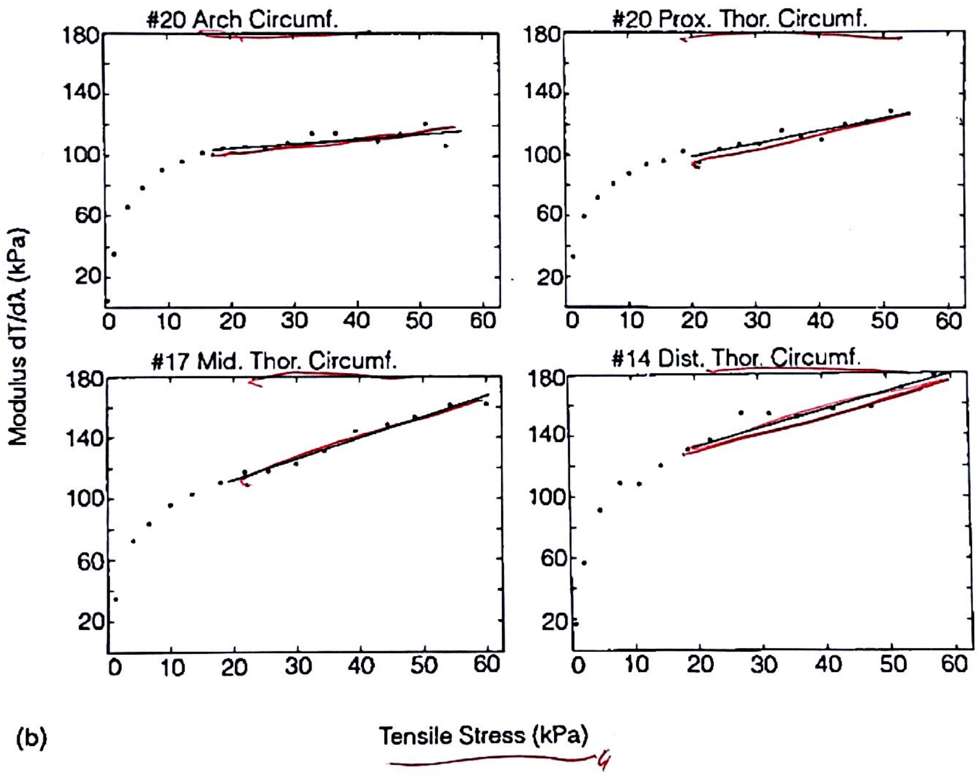
**Mechanical Properties**




- Hysteresis loop is insensitive to strain rate
- Longitudinal movement (extension) is less than 1% - highly tethered
- Due to tethering expect effective *in situ* $E_{long} > E_{radial}$, $E_{circ}$, so that tissue behaves anisotropically.
- Dependence on strain rate can be ignored
- In general, dT/dgamma = alphaT+E0, where alpha is the rate of increase of the Young's modulus
- Thus, exponential stress-strain relationship.

$$\boxed{T = (T^* + B)e^{a(\lambda-\lambda^*)} - B}$$



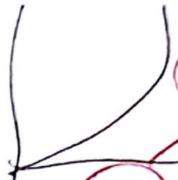


Pseudo-elasticity ($\approx$ hyper-).

① Pseudo-strain energy formulation

② $W$. strain energy density per unit volume of the tissue.

③ $\boxed{W,\ E_{\theta\theta},\ E_{zz},\ E_{rr},\ E_{\theta z},\ E_{z\theta},\ E_{rz}\cdots}$



Note

$\sigma$ : [MPa] $\rightarrow$ $N/m^2$ 2D

$\varepsilon$ : [m/m]

$W = \int \sigma\, d\varepsilon = [MPa]$

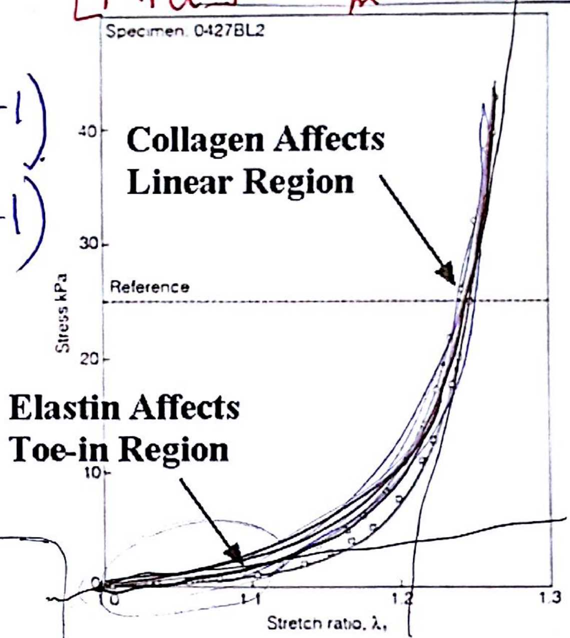
$[MPa] = \frac{MN}{m^2} = \frac{MN \cdot m}{m^2\, m} = MJ/m^3$

$= J/cm^3$ 3D

$$S_{ij} = \frac{\partial W}{\partial E_{ij}}$$

$$\boxed{S_{\theta\theta} = \frac{\partial W}{\partial E_{\theta\theta}}}, \quad E_{\theta\theta} = \frac{1}{2}(\lambda_\theta^2 - 1)$$

$$S_{zz} = \frac{\partial W}{\partial E_{zz}} \quad E_{zz} = \frac{1}{2}(\lambda_z^2 - 1)$$




Fung's model

$$\boxed{W = \underline{C} \cdot \exp[\underline{a_1} E_{\theta\theta}^2 + \underline{a_2} E_{zz}^2 + 2\underline{a_4} E_{\theta\theta} E_{zz}]}$$

Constants.

Note: ~~Ex~~ Experimental data can be fit accurately, but the parameters themselves have little meaning. And coefficients are sensitive to small changes in data

Note: $S_{\theta\theta} = \frac{\partial W}{\partial E_{\theta\theta}} = \frac{s_{\theta\theta}}{\lambda_\theta^2}$, $S_{zz} = \frac{\partial W}{\partial E_{zz}} = \frac{s_{zz}}{\lambda_z^2}$

$S$ : Kirchhoff stress, $s$ : Cauchy stress.

Note: Experimental data can be fit accurately, but the parameters themselves have little meaning. Coefficients are sensitive to small changes in data.