# Mechanical Work, Energy, and Power

Office Hours
T: Green Alcove 2-4pm
W: Media: scape 2-4pm

**Goals:**

* Energy
* Work
* Power

} Energetics

Energetic variable contain the most info.
Joint Power considered the most discriminatory variable for pathologies! → clinical application.

**Mechanical Energy vs Work**

* Units (J) same
- different meanings!

Mechanical Energy is a measure of the state of a body at an instant of time as to its ability to do Work.

Work a measure of energy flow from one body to another and time must elapse for work to be done.

**Law of Conservation of Energy**

$$\Delta E_s = \cancel{0} - W$$
$$E_2 - E_1 = -W$$

$$PE_1 + KE_1 = PE_2 + KE_2$$

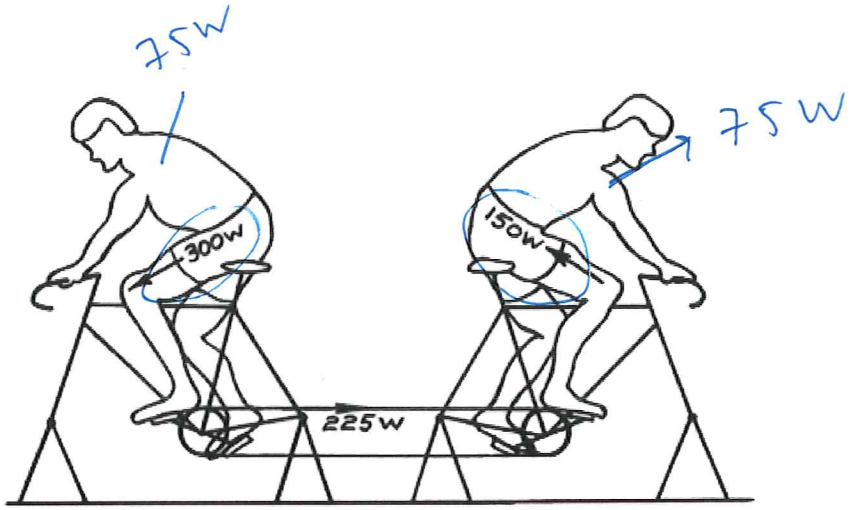

## Internal vs External Work

* only source of mechanical energy generation is muscles!
* major site of energy absorption
* Mechanical energy continously flows in/out of muscles from segment to

Work done segment (limbs) → internal work

Work done of external loads → external work

| Example |
| --- |
| Walk (internal) |
| Walk up hill (both) |
| cycling (both) |

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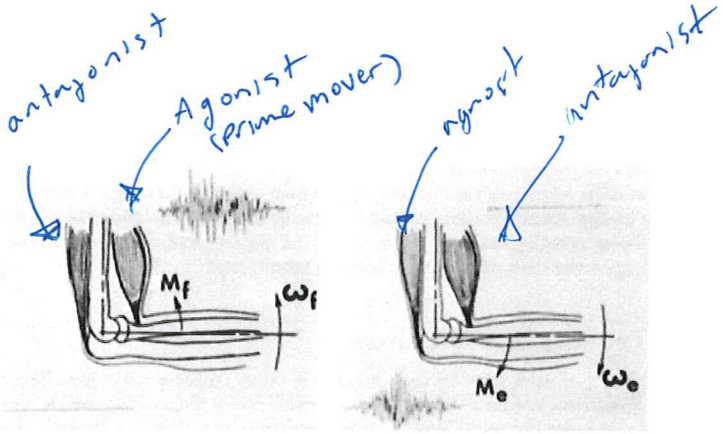
## Positive Work of Muscles

* concentric contraction
  * Moment ; Velocity same polarity (sign)

$$W_{net} = \int P\,dt$$

$$= \int M \cdot \omega\,dt$$

moment, angular velocity.

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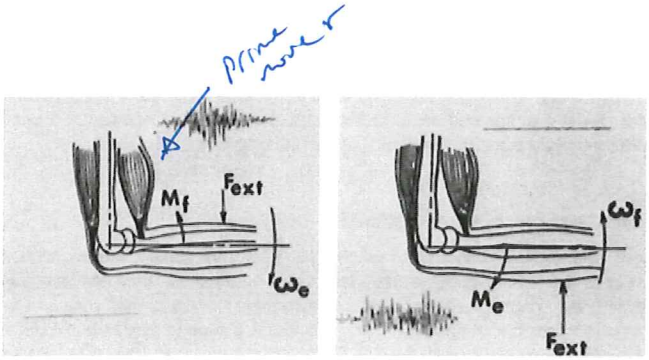
Flow energy

muscles → limbs

## Negative Work of Muscles

* eccentric contraction
  * Moment & angular velocity have opposite polarity.

$$W_{net} = \int M \cdot \omega \, dt$$

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Flow is from | limbs → muscles |

## Muscle Mechanical Power

* rate work is (almost) never constant during movt.

$$P(t) = M(t) \cdot \omega(t) \quad (W = J/s)$$

## Mechanical Work of Muscles

$P = \frac{dW}{dt}$ Power is the rate of doing work

$$W = \int P \, dt \quad (J)$$

## Mechanical Work Done on an External Load

* Work is only done if there movement.

$$\frac{dW}{ds} = F \Rightarrow W = \int_{s_1}^{s_2} F \cdot ds$$

$s_1$ ↑ Force of external load; $s_2$ ↑ change displacement

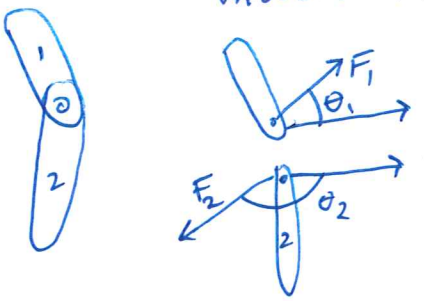
$$P = \frac{dW}{dt} = F\frac{ds}{dt} = \underline{F} \cdot \underline{V}$$

$\frac{ds}{dt} = V$

$$F \cdot V = F_x V_x + F_y \cdot V_y = \|F\| \|V\| \cos\theta$$

## Mechanical Energy Transfer between Segments

* each segment exerts forces on its neighbors
  - if there is a translational ~~movement~~ movement of joint → mechanical energy transfer




$V_1 = V_2$

$F_1 = -F_2$

$P_1 = \|F_1\| \|V_1\| \cos\theta_1$

$P_2 = \|F_2\| \|V_2\| \cos\theta_2$

$$P_1 = -P_2$$

## Efficiency

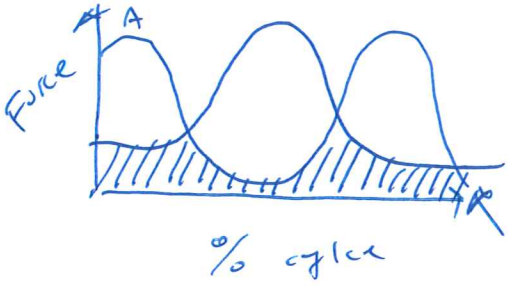
Two fundamental inefficiencies

1. conversion of metabolic energy to mechanical energy.
2. Neurological inefficiencies in control the mechanical energy.
   * Cerebral Palsy.

* Important mechanism of energy transfer between adj. segments.
* Passive process (no muscle activity)

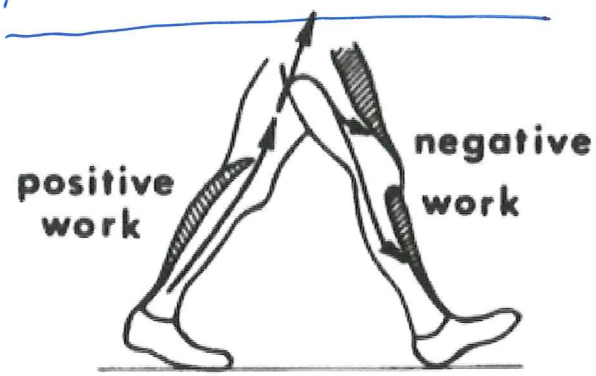
## Causes of Inefficient Movement

* Cocontractions




* isometric contraction against gravity

* generate energy @ one joint & absorb energy at another.

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## Forms of Energy Storage

(1) Potential Energy

$PE = mgh + \frac{1}{2} k x^2$

($x$: displacement; $k$: spring constant)

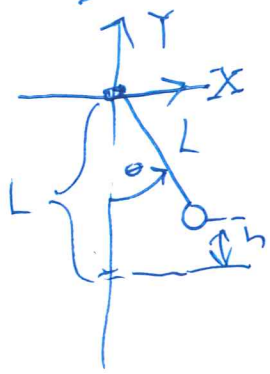
(2) Kinetic Energy

$KE = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$E_S = PE + KE \rightarrow mgh + \frac{1}{2} k x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

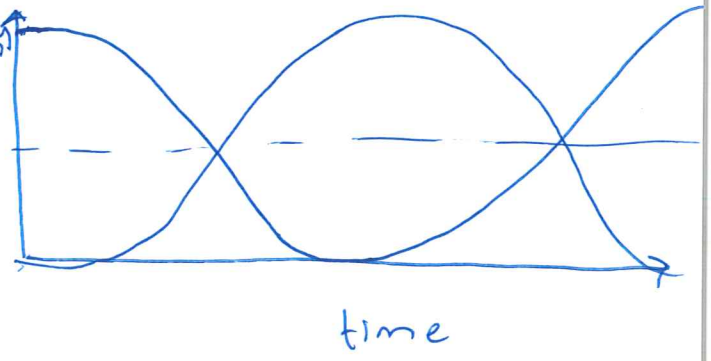
## Body Segment Energy and Exchange Within Segment

$\Delta E_S = 0$




$PE = L(1-\cos\theta)mg$

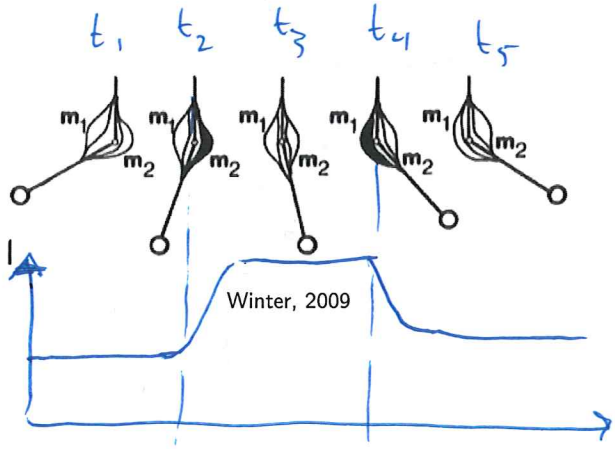
$KE = \frac{1}{2}(mL^2)\omega^2$



## Total Energy of Multisegment System

$$E_b = \sum_{i=1}^{B} E_{si}$$

* Energy transfer between segments results in no change.




① When muscles do positive work, they increase total body energy.

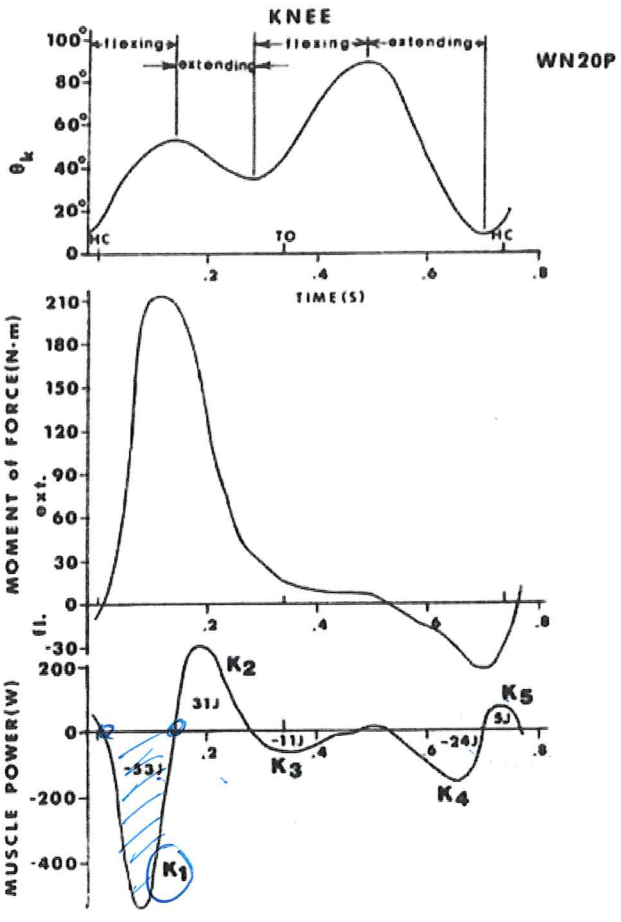
② When muscles do negative work they decrease total body energy.

③ During cyclic activity (constant velocity running)

net $\Delta E_b = 0$

$E^+ = E^-$

## Joint Power and Work

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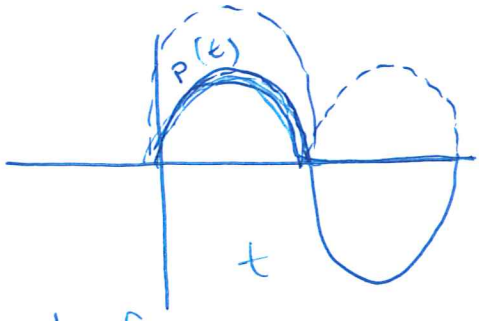
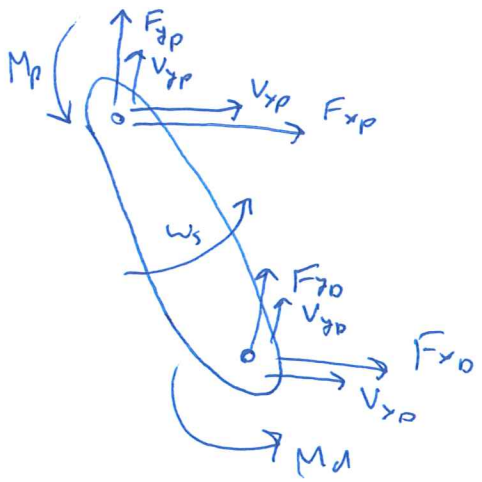
$$P^+ = \frac{1}{2}(P + |P|)$$




$$W^+ = \int P^+ \, dt$$

## Power Balance at Joints and Within Segments

# Energy transfer via Muscles




Passive

$P_p = F_{xp} \cdot V_{xp} + F_{yp} \cdot V_{yp}$.

$P_{m_d} = \underbrace{M_p \omega_s}_{\text{muscles}}$

$P_{pd} = F_{yd} \cdot V_{yd} + F_{xd} V_{xd}$.

$P_{m_d} = M_d \omega_s$

$$\frac{dE_s}{dt} = \sum P$$

$$= P_{pp} + P_{mp} + P_{pd} + P_{md}$$