Stability and Conceptual Walking Models

Goals:

* Walking & Running
* Limit cycles & stability
* Conceptual Models of Walking -

Mechanics

Motivations

Why do we want models?

* simple representation

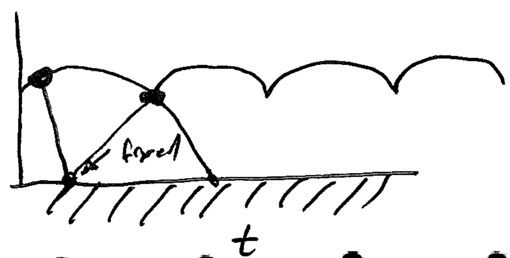
* Answer question.

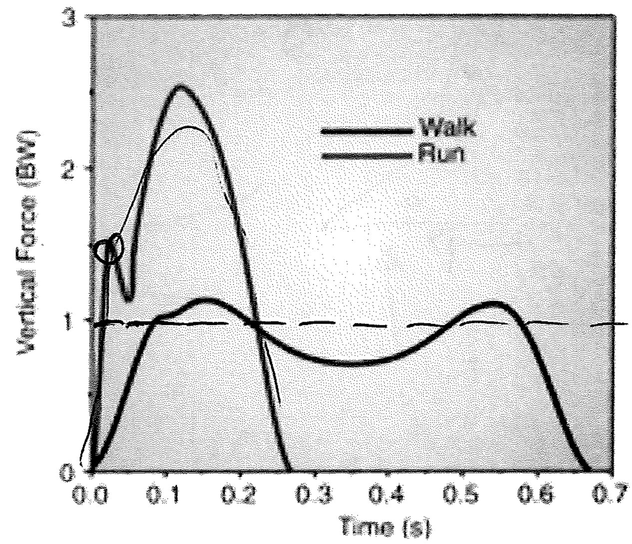
* design system that mimic biology.

Running vs Walking

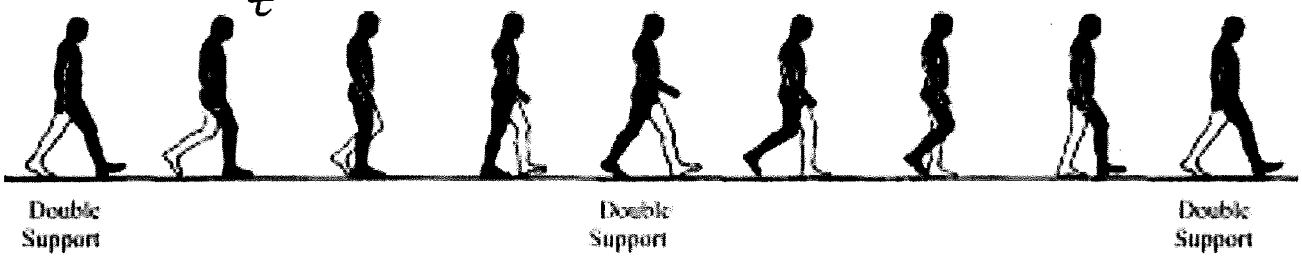
* Walking : double support
* Running : double float

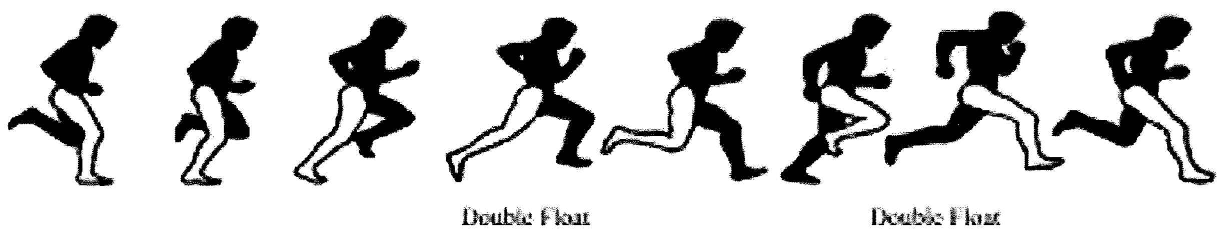
Center of Mass



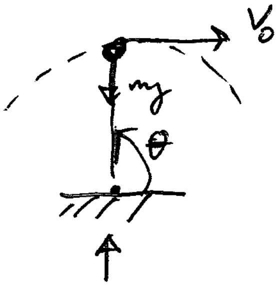


Winter, 2009





Walking has a Speed Limit



$V_{max} \approx 3$ m/s

$\ell\omega = v$

$\omega = \frac{v}{\ell}$

$\omega^2 = \frac{v^2}{\ell^2}$

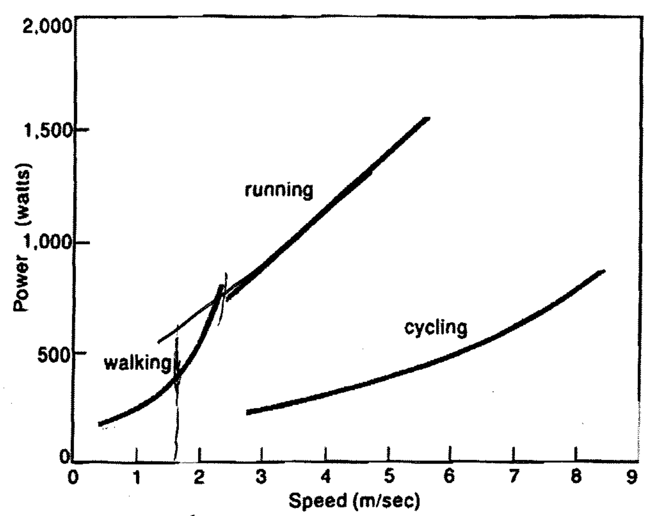
$\sum F_y = m a_y$

$F - mg = \underbrace{-m\ell\omega^2}_{\text{centripetal force}} = -\frac{m}{\ell}v^2$

$\frac{m}{\ell}v^2 \le mg$

$V_{max} = \sqrt{g\ell}$

Froude $= \frac{V}{\sqrt{g\ell}} \le 1$



Margaria, 1938

J. Realmuto, 2016

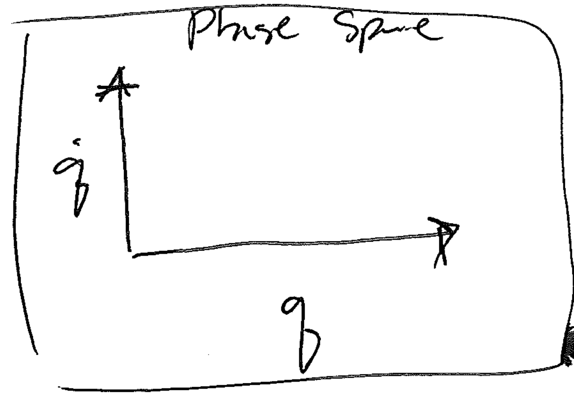
Stability of Walking

Phase Space

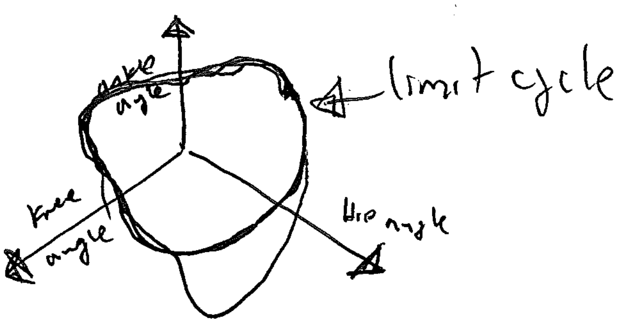
* represent all possible states of a system.

~~Conso~~ consider $\dot{x}(t) = f(x(t))$

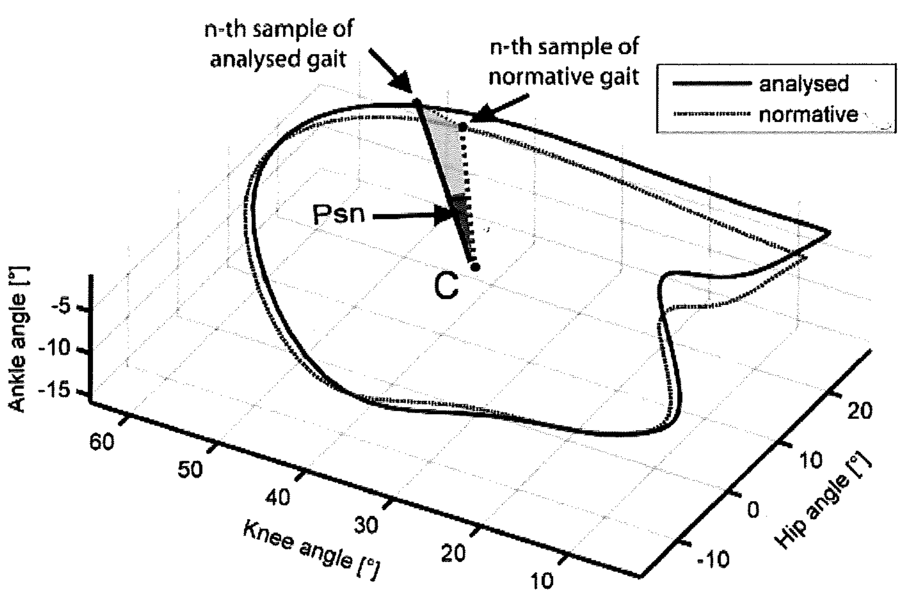
$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$



Limit Cycles

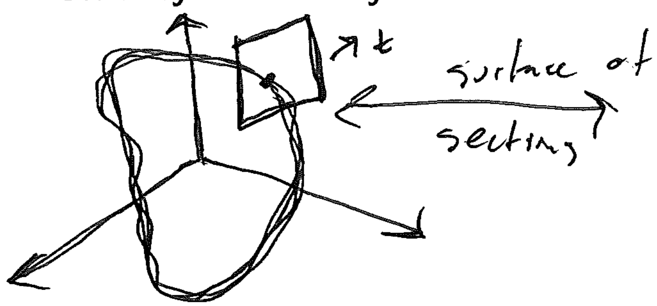


* closed trajectory in phase space w/ atleast one other trajectory ~~sprints~~ spirals into it.



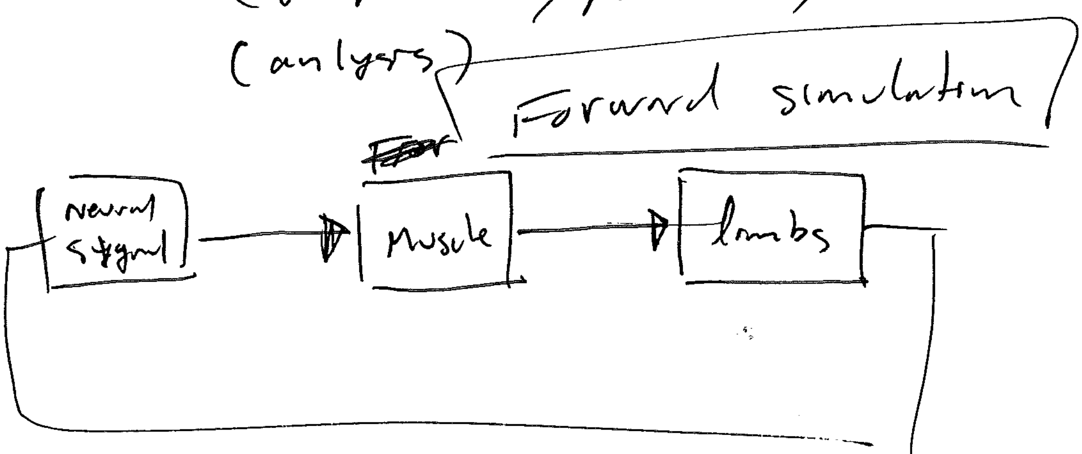
Stancic, 2012

Stability of Limit Cycles



Modeling Bipedal Locomotion

* Synthesis of Human Locomotion
  (e.g. robotics, prosthetics)
  (analysis)



Mathematical Formulation

* Newtonian Mechanics is good!
  but it is cumbersome.
  finally derive differentiation/

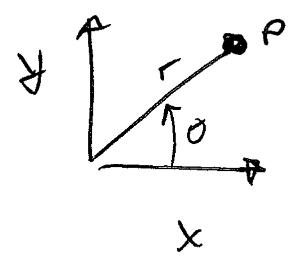
* Energy

(1788) Joseph-Louis Lagrange.

Lagrangian Mechanics

* ideal for conservative systems (e.g. no external forces, except gravity)
* lead to deep understanding of physics
* Principle of least action ⟶ calculus of variations

Generalized Coordinates



$q_1 = [y, x]$

$q_2 = [r, \theta]$

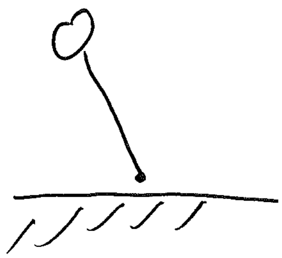
→ describe the configuration (state) of a system.

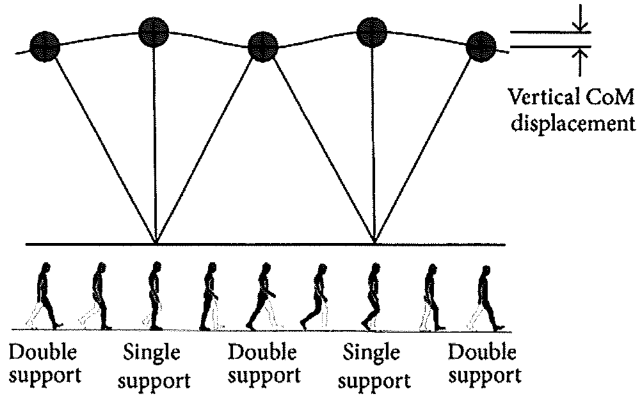
Lagrangian

$$L = T - V$$

- L: Lagrangian
- T: Kinetic energy
- V: Potential Energy (including strain energy)

Inverted Pendulum Mechanics





Tedrake, 2009

Rimless Wheel

$T = \frac{1}{2} m (l \dot{\theta})^2$

$V = mgl\cos\theta$

$L = \frac{1}{2} m (l\dot{\theta})^2 - mgl\cos\theta$

* legs massless
* mass only @ center

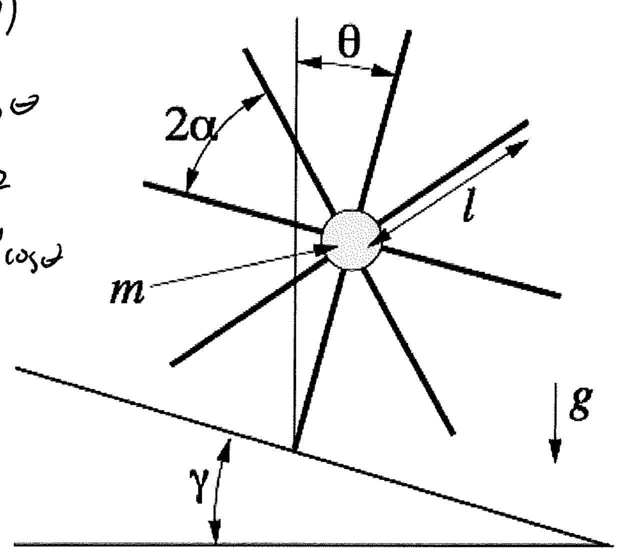
~~$q = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$~~ Error

$q = \theta$

Let's derive EOM.

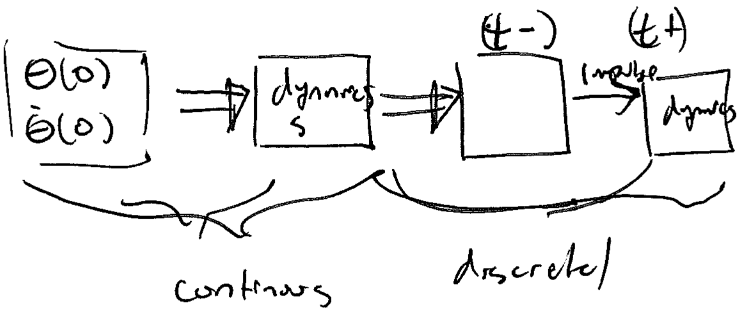
$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}$ $\qquad \frac{\partial L}{\partial \theta} = mgl\sin\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$ $\qquad \ddot{\theta} = \frac{g}{l}\sin\theta$



Tedrake, 2009

Rimless Wheel Simulation Procedure

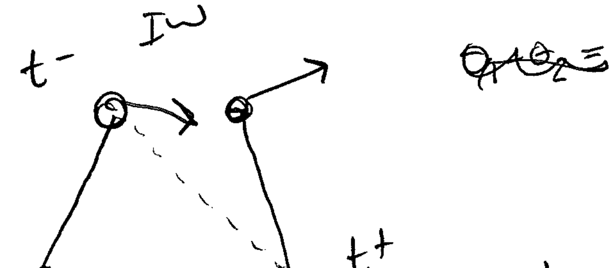


Hybrid System.

Transition.

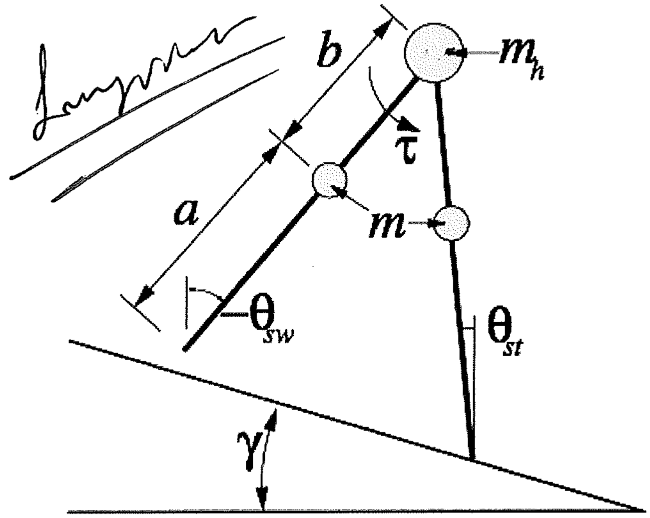
angular Momentum is conserved

$I\omega$



$\dot{\theta}(t^+) = \dot{\theta}(t^-)\cos 2\alpha$ Impulse

Passive Dynamic Walker



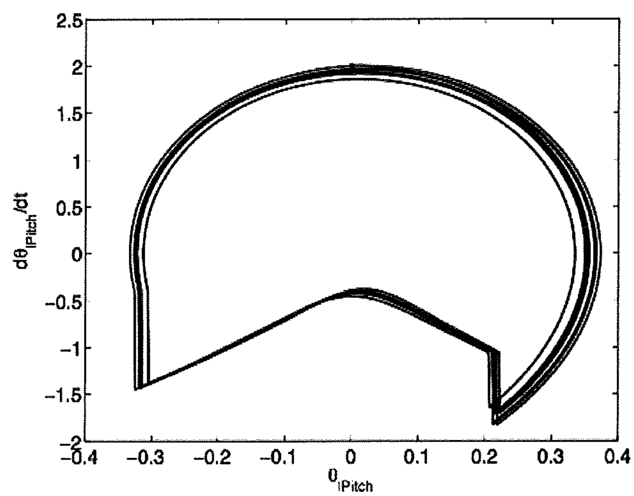
Compass Gait Model

$q = [\theta_1, \theta_2]^T$

~~Disctedt~~

Discrete dynamics

$Q_p \dot{q}(t^-) = Q_m \dot{q}(t^-)$



Tedrake, 2009

Kneed Walker

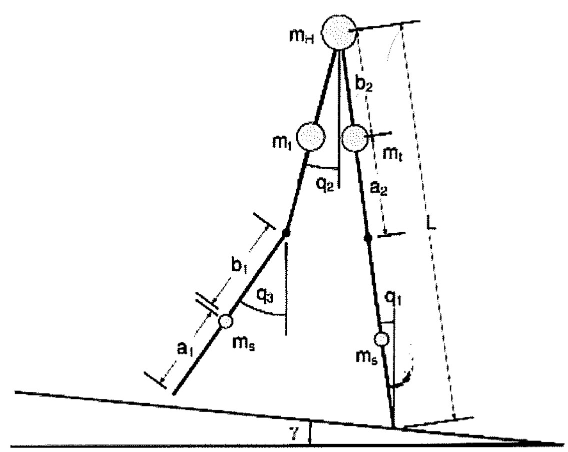
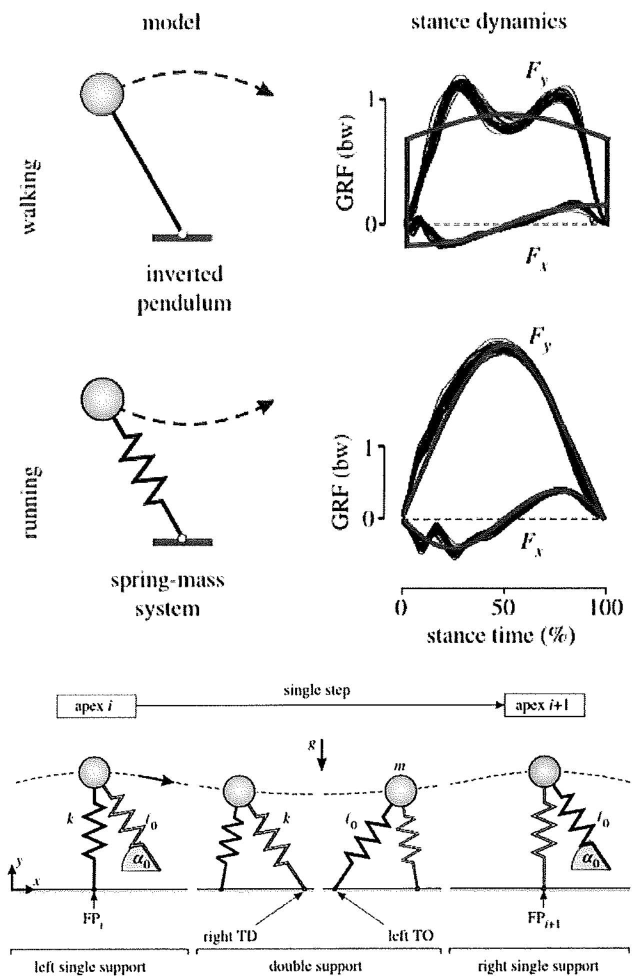


FIGURE 5.8 The Kneed Walker

Tedrake, 2009

Spring Mass Running/Walking



Geyer, 2006