Lecture #3: In Class Problems

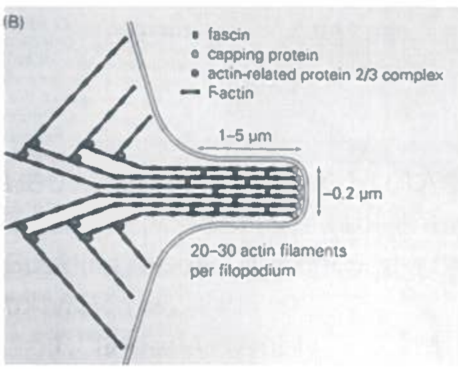
In lecture, we discussed the critical force a filopodium can bear based on Euler Buckling modes and derived a basic formula for critical length. Now let's apply this to two cases:

1. Loose Assembly
2. Tight Assembly

As a reminder, here is the formula:

$$L_{crit} = \frac{\pi}{2}\sqrt{\frac{EI}{5r_{actin}\sqrt{n}}} \text{ where } E = 1.9\,GPa \text{ and } r_{actin} = 3.5\,nm \text{ and } I = \frac{\pi r_{actin}^4}{4}$$

For the first case of loose assembly, this means the actin filaments are not compact, while the second case means that the filaments are so compact that you can almost treat many filaments as a one filament with a bigger radius.



There are two assumptions you can make based on these cases. Which one is valid for which case?

$$r_{bundle} = \sqrt{n}r_{actin} \quad \text{tight}$$

$$I_{bundle} = nI_{actin} \quad \text{loose.}$$

Can you derive where the first assumption comes from?

$A_{filopodia} = \pi r_{filopodia}^2$ $\qquad A_{fil} = A_{actin} \longrightarrow \boxed{r_{fil} = \sqrt{n}\, r_{act.}}$

$A_{actin} = n\pi r_{actin}^2$

Knowing both assumptions, state the two equations for critical length.

$$F_{fil} = \frac{\pi^2 EI}{4L_{crit}^2} = 5\sqrt{n}\, r_{act}$$

② $L_{crit} = \frac{\pi}{2}\sqrt{\frac{E n^2 \pi r_{act}^4/4}{5\sqrt{n}\, r_{act}}}$

$$L_{crit} = \frac{\pi}{2}\sqrt{\frac{EI}{5\sqrt{n}\, r_{act}}} \qquad \text{For ① } L_{crit} = \frac{\pi}{2}\sqrt{\frac{E\,n(\pi r_{act}^4/4)}{5\sqrt{n}\, r_{act}}}$$

Suppose I told you that this particular bundle has n = 30 filaments. Which model would is better and why?

① Loose

$L_{crit} \approx (0.17769\,\mu m)(n^{1/4})$

$L_{crit} \approx 0.416\,\mu m$ $\qquad$ TOO LOW

② Tight

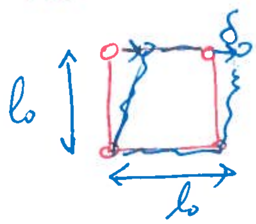
$L_{crit} \approx (0.17769\,\mu m)(n^{3/4})$

$L_{crit} \approx 2.278\,\mu m$ $\longleftarrow$ Better Model!

In lecture, we used a six-fold connectivity network model to derive the shear modulus. The final expression for the network is

$$\mu = K_s = \frac{\sqrt{3}k_{spring}}{4}$$

What is the shear modulus for a fourfold connectivity network? Here, the unit cell is a square lattice of springs with a spring constant of k_{sp}~~/2~~ and undeformed length of ~~R_0.~~ l_o



$$\sum_{i=1}^{4} W_i^{sprin} = 2\underbrace{\left(\tfrac{1}{2}k\delta^2\right)}_{\text{vertical}} + \underbrace{2\cdot 0}_{\text{horizontal}} = k\delta^2$$

In small strain limit

$$l = \sqrt{l_o^2 + \delta^2} \approx l_o \quad \text{so } \delta = 0$$

so $\sum W = 0 = W^{mic}$

Thus, $\boxed{\mu = 0!}$

Based on your result, what conclusion can you draw about both sixfold and fourfold connectivity with regards to red blood cells?

For sixfold, $\mu = \frac{\sqrt{3}}{4} k_{sp}$.

For fourfold, $\mu = 0$

This means ~~a~~ four fold network cannot store energy upon shearing. The microstructural network just collapses. There basically no resistence. Unlike 6-fold, 4-fold does not recover its original shape when sheared.

<u>With regard to RBC</u>: 6-fold can easily store and release energy. This is absolutely necessary for RBC to squeeze through capillaries.

The microstructure of RBC is just one elegant and energetically favorable design for a sell.

<u>Note</u>: for 4-fold in tension: $K = \frac{1}{2} k_{spring}$.