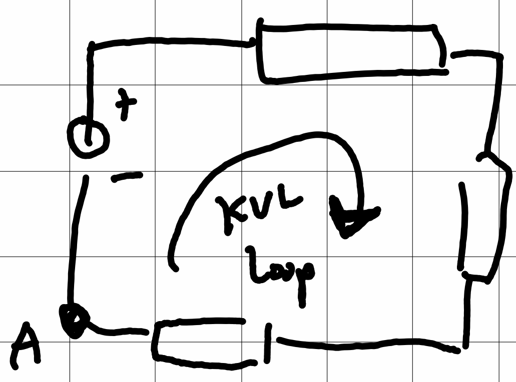
# ME 133 Lecture 3 — 1/11/2022

Last time:

- electrical elements
- I-V characteristics
- Kirchoff's laws

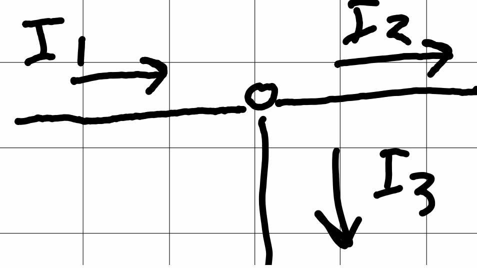
KVL: $\sum_{i}^{N} V_i = 0$

sum of V around closed loop



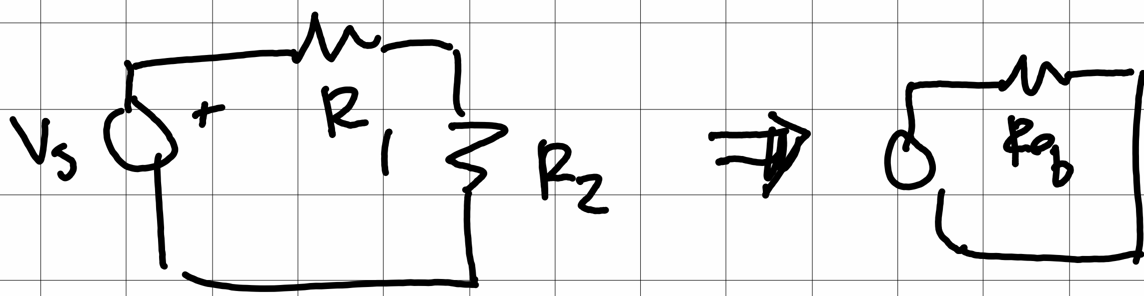

KCL: $\sum_{i}^{N} I_i = 0$

sum of I flowing into a node = 0




$I_1 = I_2 + I_3$

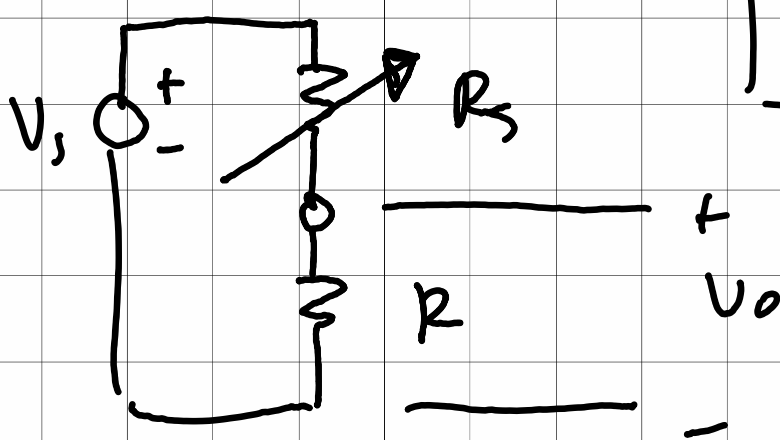
- Equivalent series resistance




$$R_{eq} = R_1 + R_2$$

- Voltage divider sensor circuit

$$\boxed{V_o = \frac{R}{R_s + R} V_s}$$



Today:

- Equivalent series & parallel circuits
- Sources & meters
- Thevenin & Norton Eq.
- Power
- Impedance

Thursday
- AC circuits
- Transformer

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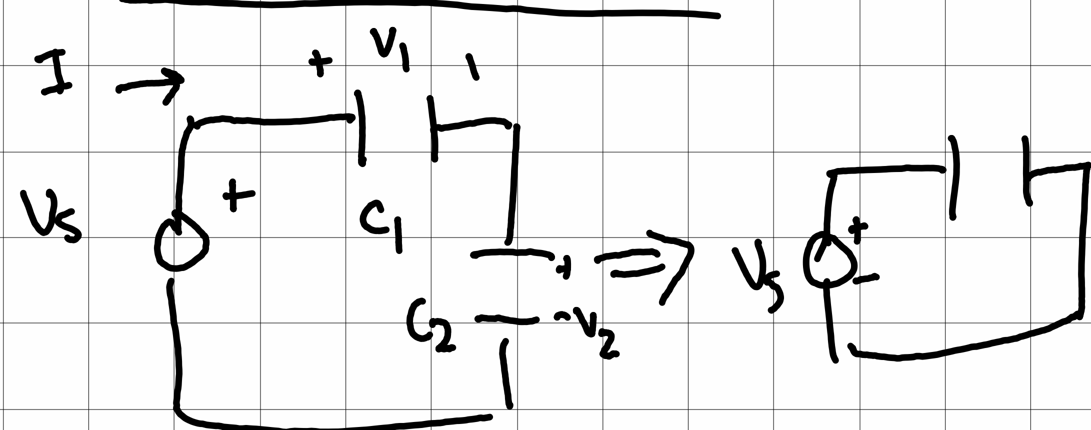
## Series Capacitors




KVL: $V_s = V_1 + V_2$

$$\rightarrow \frac{d}{dt} V_s = \frac{d}{dt} V_1 + \frac{d}{dt} V_2$$

$$\dot{V}_s = \dot{V}_1 + \dot{V}_2$$

$I = C\dot{V}$ $\quad \dot{V}_s = \frac{I}{C_1} + \frac{I}{C_2} = \left(\frac{C_1 + C_2}{C_1 C_2}\right) I$

$$\dot{V}_s = \left(\frac{C_1 + C_2}{C_1 C_2}\right) I$$

$$\Rightarrow I = C_{eq} \dot{V}_s$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

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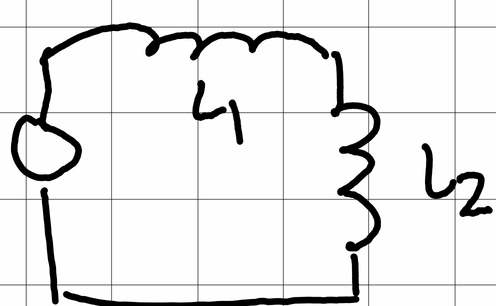
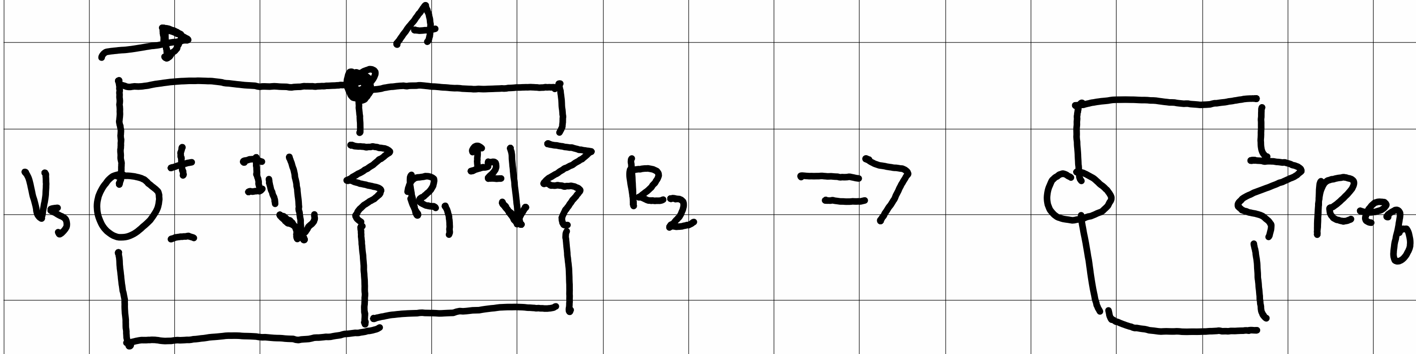
Series Inductors




$$L_{eq} = L_1 + L_2$$

# Parallel Resistance




KCL: $I = I_1 + I_2$

KVL: $V_s = V_1 = V_2$

$V = IR$

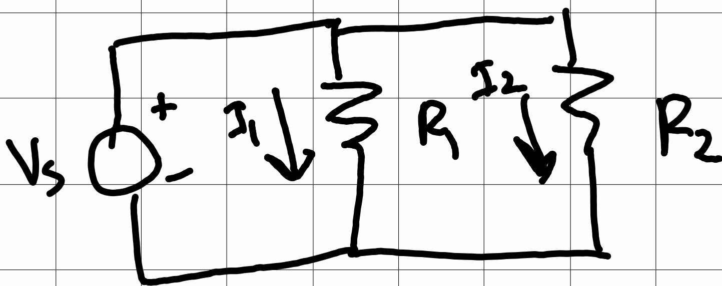
$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V_s}{R_1} + \frac{V_s}{R_2}$$

$$I = \left(\frac{R_2 + R_1}{R_1 R_2}\right) V_s$$

$$V_s = \underbrace{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}_{R_{eq}} I$$

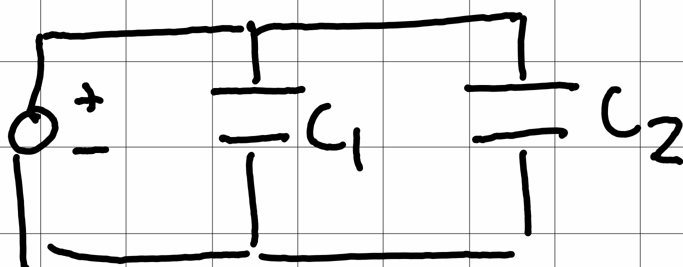
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

# Current Divider Circuit



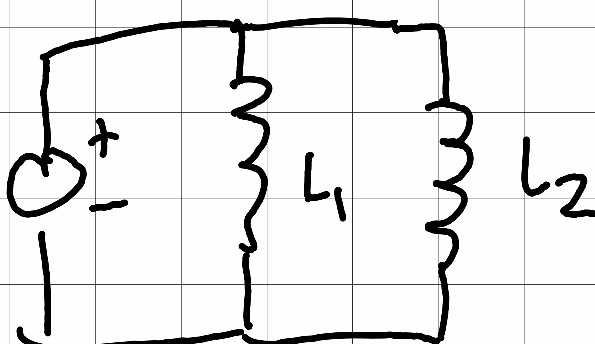

$$I_1 = \frac{\overset{R_2}{\cancel{R_1}}}{R_1 + R_2} I$$

# Capacitor in Parallel




$$\boxed{C_{eq} = C_1 + C_2}$$

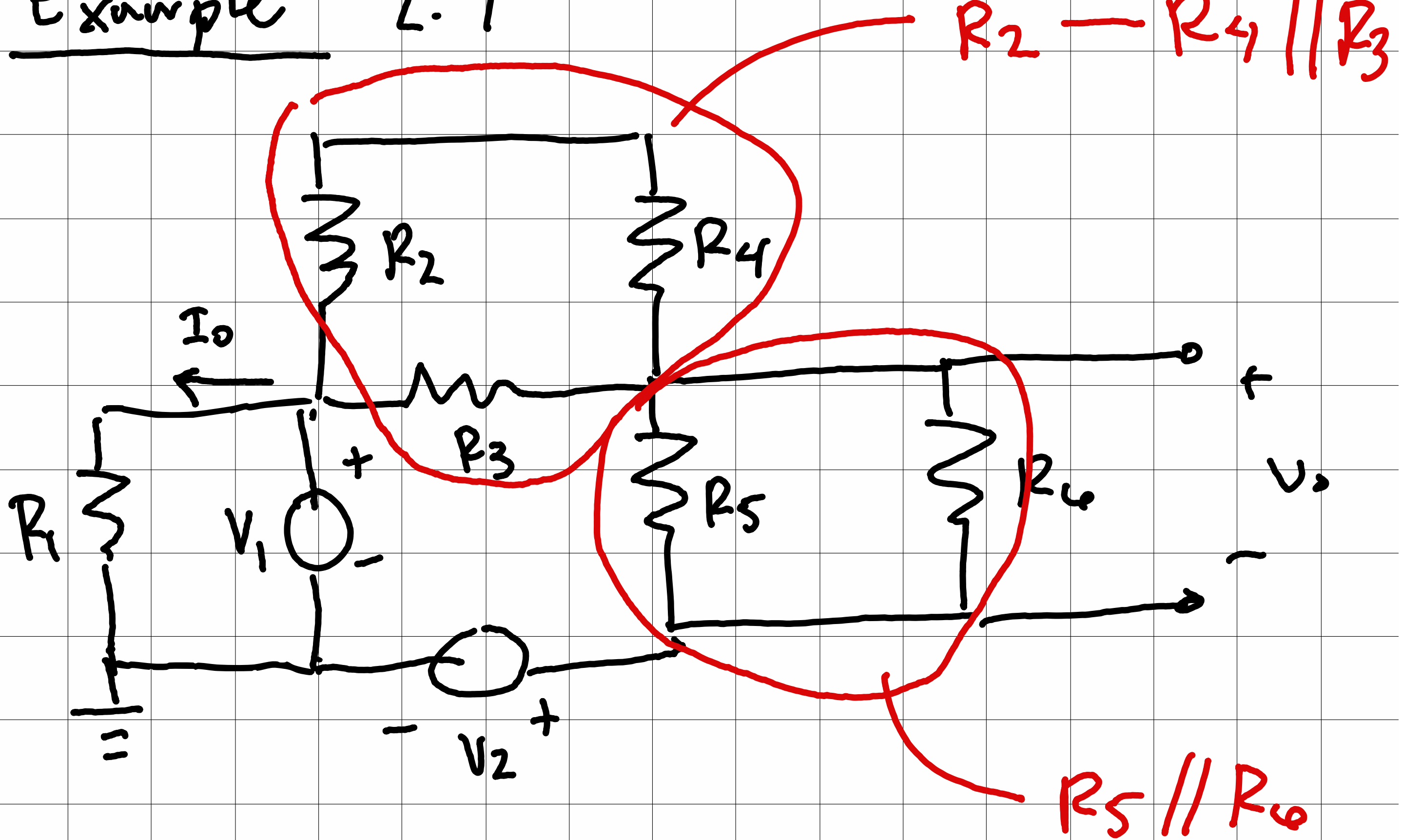
# Inductor in Parallel




$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$$

$$= \frac{L_1 L_2}{L_1 + L_2}$$

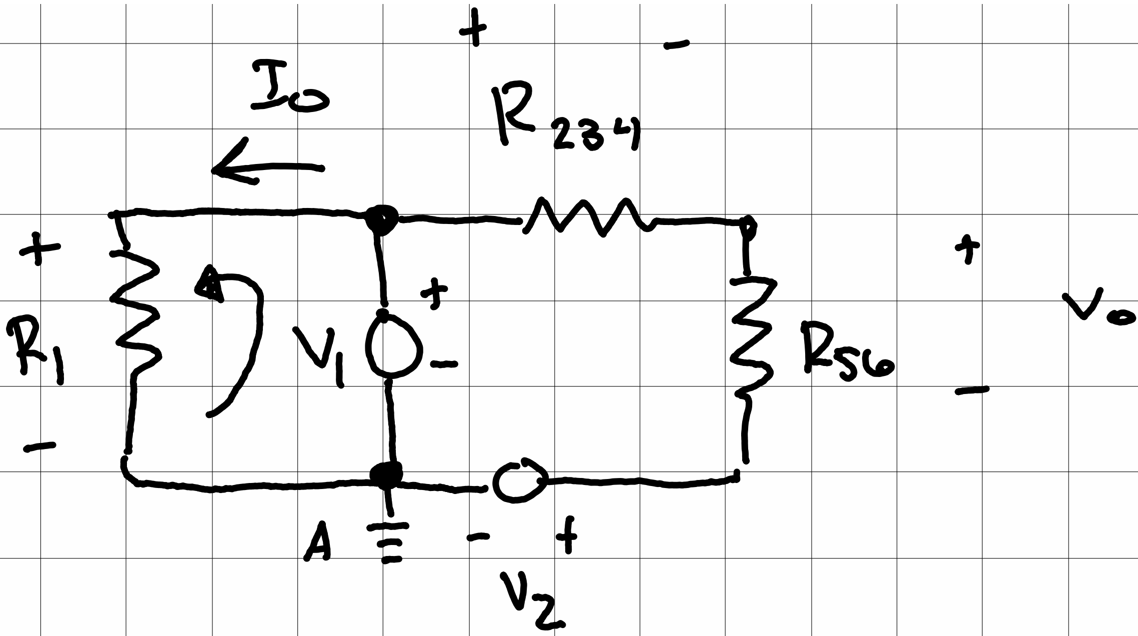
# Example 2.4




Find $I_o$ & $V_o$

- $(R_2 - R_4) // R_3$ : $R_{234}$
- $(R_5 // R_6)$ : $R_{56}$

$$R_{234} = \frac{(R_2 + R_4) \cdot R_3}{R_2 + R_4 + R_3}$$

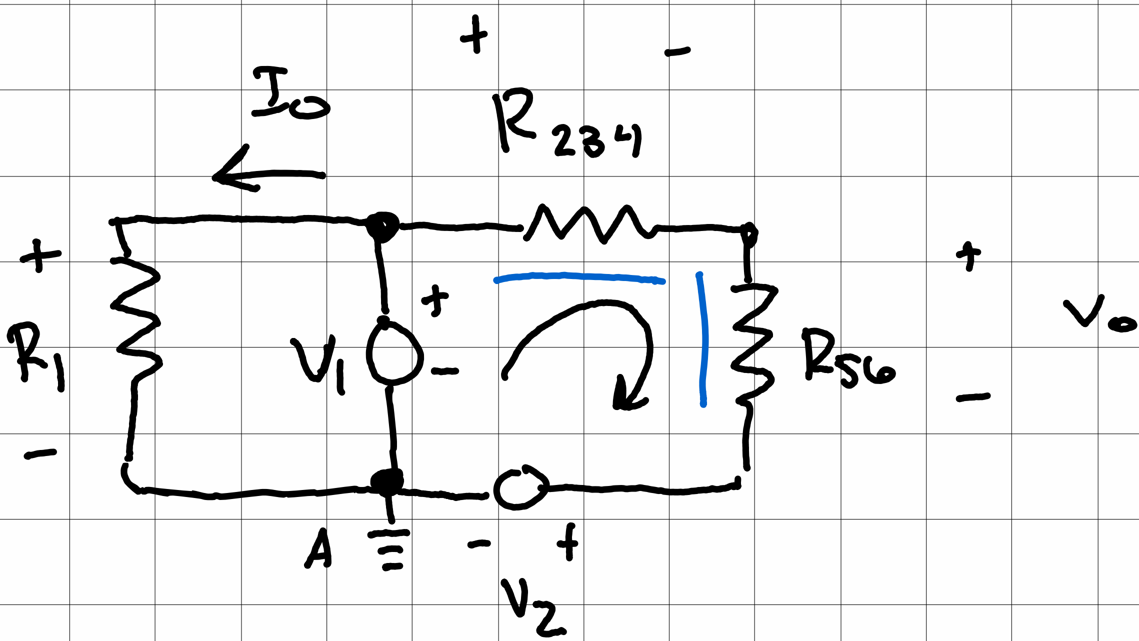



Find $I_o$ & $V_o$

KVL (Left Loop): $-V_1 + V_{R_1} \Rightarrow V_1 = V_{R_1}$

$$V_{R_1} = I_o R_1 = V_1$$

$$\boxed{I_o = \frac{V_1}{R_1}}$$




Find $V_o$:

KVL (right loop): $-V_1 + \overbrace{V_{234} + V_{56}}^{V_{23456}} + V_2 = 0$

$$V_{23456} = V_1 - V_2$$

Voltage divider:

$$V_{234} = \left(\frac{R_{234}}{R_{234} + R_{56}}\right)(V_1 - V_2)$$

$V_o$ is equal to $V_1$ minus voltage drop from $R_{234}$:

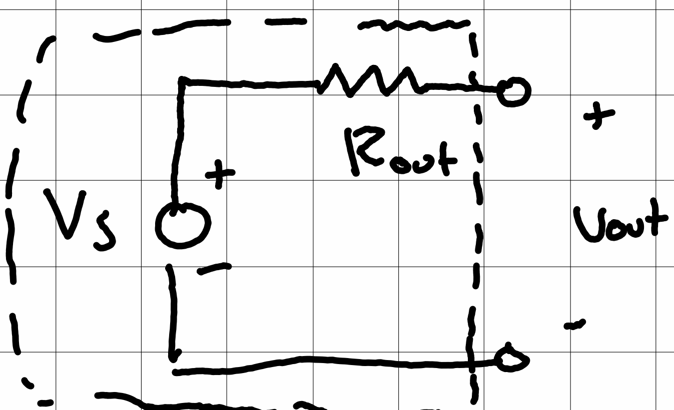
$$\boxed{\therefore V_o = V_1 - V_{234}}$$

# Sources & Meters

## Voltage source

- ideal source → generalize resistance
  - zero output impedance
  - ∞ current

- real source




real voltage source

Rout: output impedance

Vs: ideal voltage

$$\boxed{V_{out} \neq V_s}$$

$R_{out} << 1$

# current source

- ideal current source
  - $\infty$ output impedance
  - $\infty$ voltage


$R_{out} << \infty$

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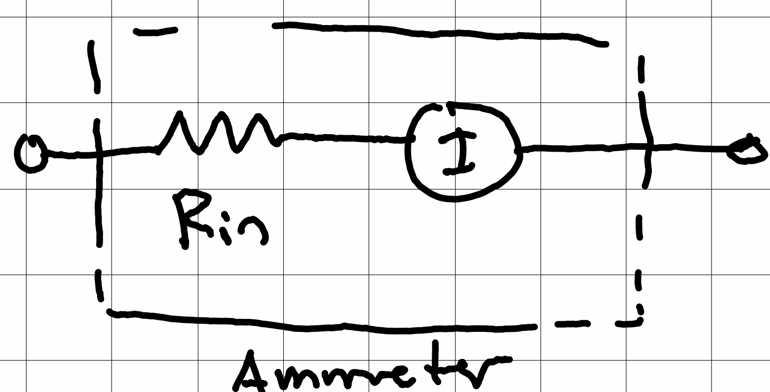
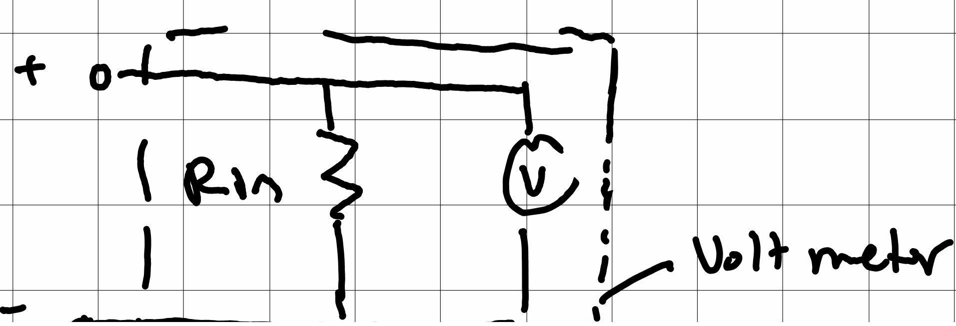
## TRUE FOR METER!




Rin: input impedance



Why should we care about input/output impedance?

→ Loading effect:

the effect a measurement system (or source) impacts the rest of the circuit

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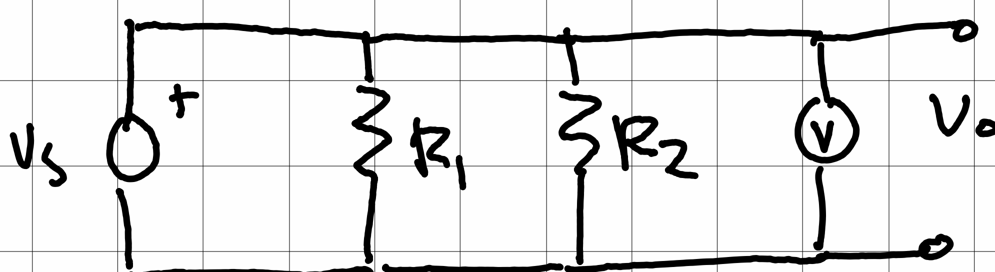
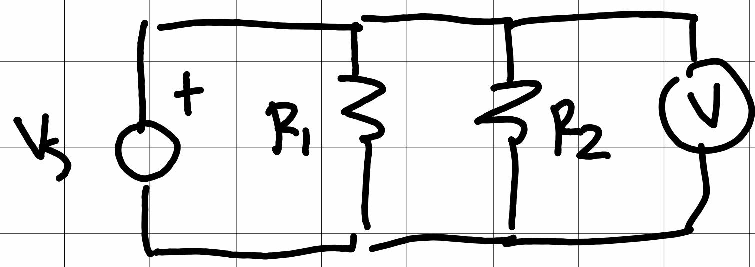
Example 2.5

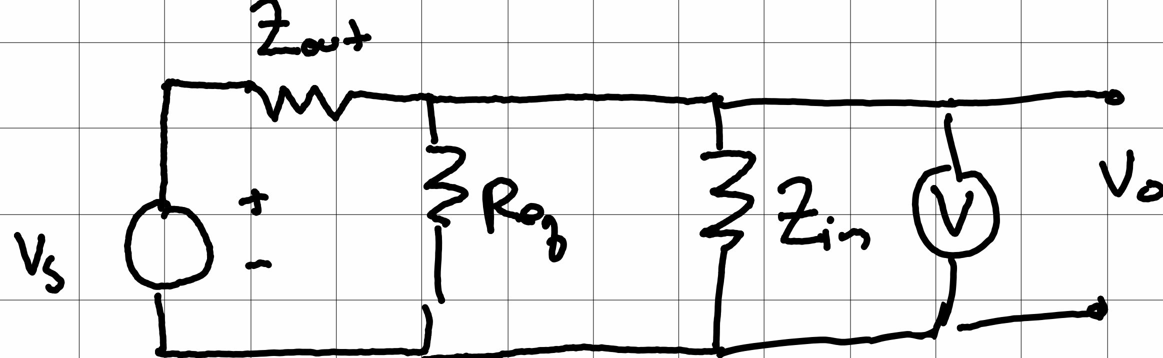


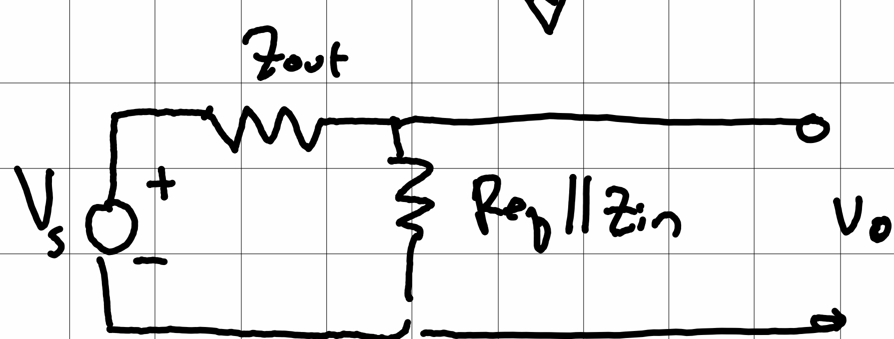

· ideal case $\boxed{V_o = V_s}$

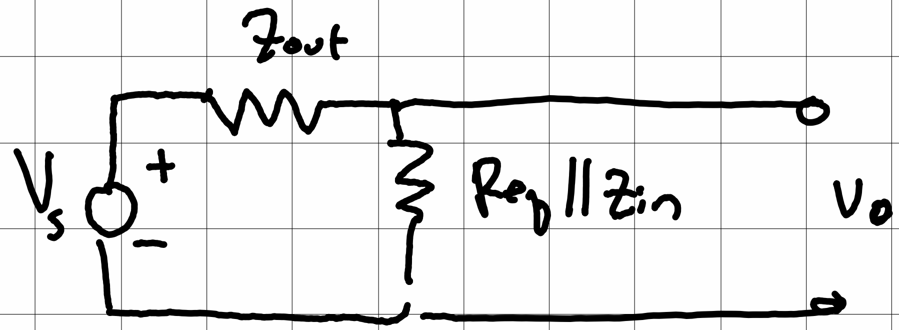



$$R_{eq} = R_1 \,||\, R_2$$


Non-ideal:





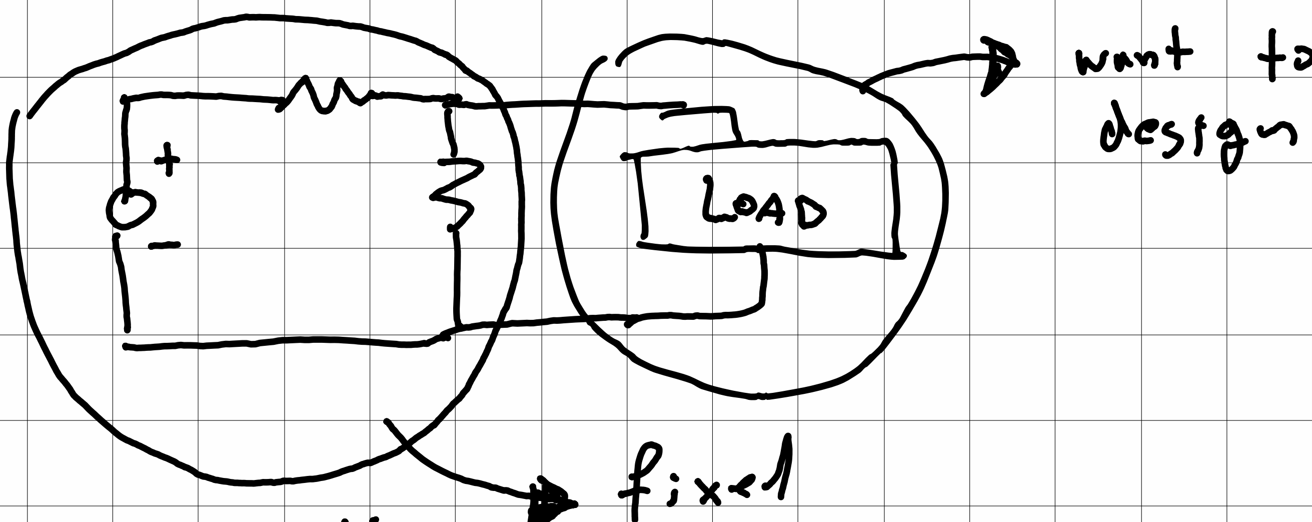

$$V_o = \underbrace{\left( \frac{R_{eq} \| Z_{in}}{Z_{out} + R_{eq} \| Z_{in}} \right)}_{\text{loading effect}} V_s$$

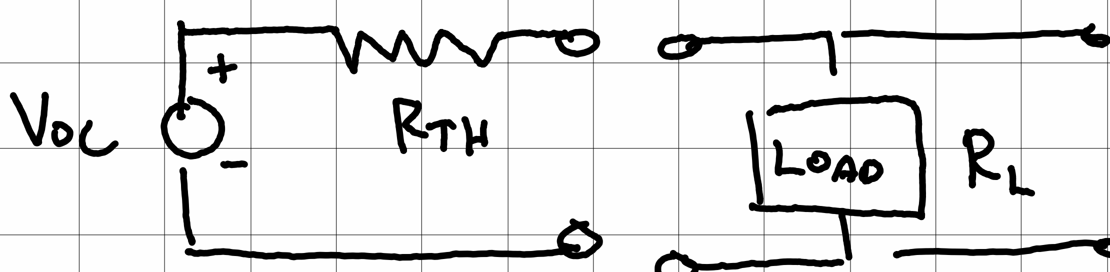
<u>Take away:</u> designer/engineer must be cognizant of input/output impedance characteristics

# Thevenin & Norton Equivalent Circuits

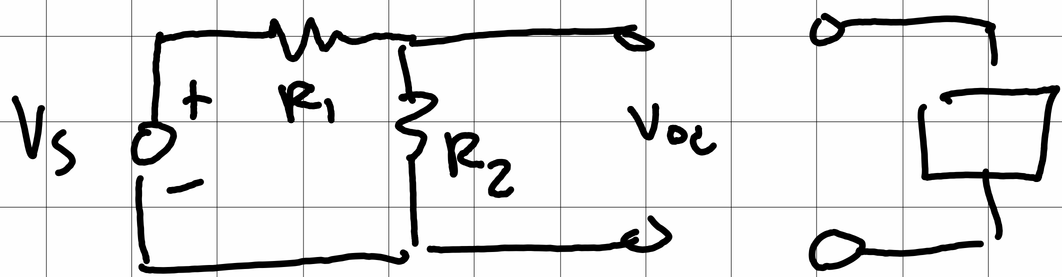
## Thevenin Eq:

- to simplify complex circuits by replacing voltage source & part of the network with an equivalent V source & series resistor



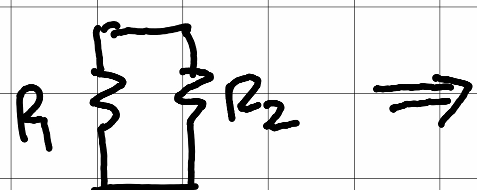



1. Find open circuit voltage $V_{oc}$
   - disconnect "rest of circuit" & find voltage across open terminal)



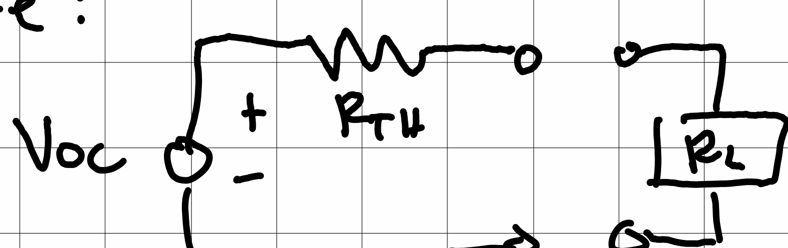

$$V_{oc} = \frac{R_2}{R_1 + R_2} V_s$$

2. To find Thevenin eq. resistance
   - short $V_s$ ($V_s = 0$)



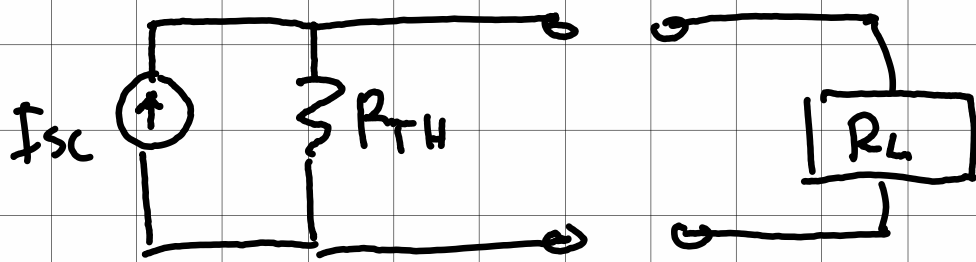

$$\Rightarrow \quad R_{TH} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

3. Analyze:



# Norton Equivalent

- Network is replaced by ideal current source $I_{sc}$ & Thevenin Resistance $R_{TH}$ in parallel

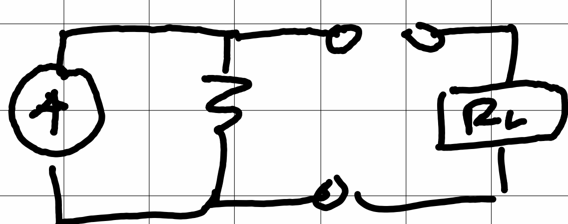



① Find $I_{sc}$

- calculating the current when short the load terminals

② Find $R_{TH}$ (same as previous)

③ Analyze circuit :




$$\boxed{I_L = \left(\frac{R\_th}{R_{TH}+R_L}\right) I_{sc}}$$

- Why all the trouble?

    - Thevenin / Norton equivalents are independent of the load
    - we can make changes (e.g. design) without reanalyzing whole circuit

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## Power in Circuits

$$\boxed{P = VI}$$

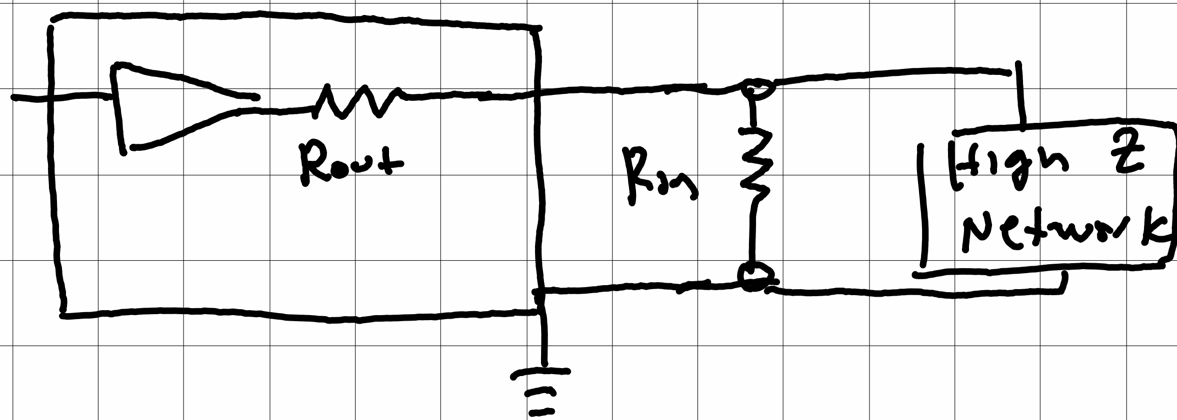
- for any element power consumed or generated is the product of voltage across & current through that element

P negative: dissipation / storage

P positive: generation

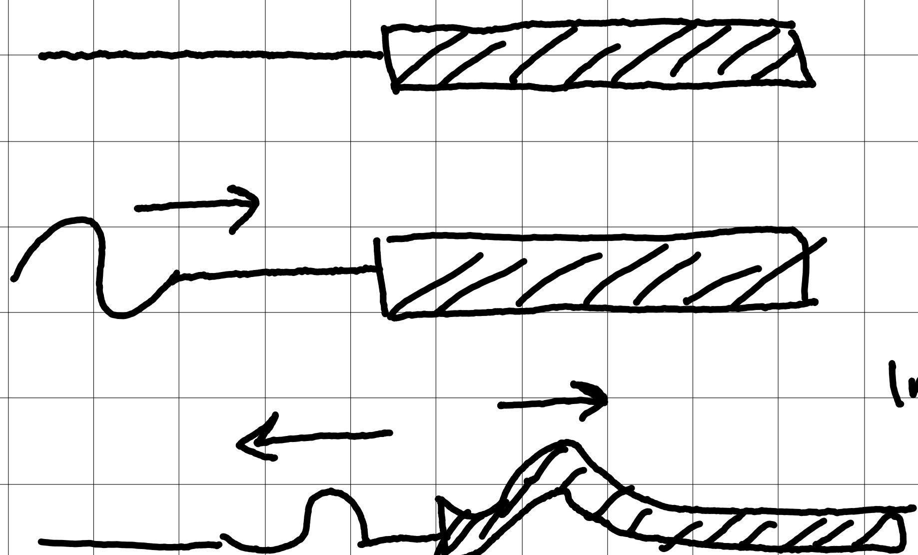
For resistor : $P = VI = I^2R = V^2/R$

## Impedance Matching



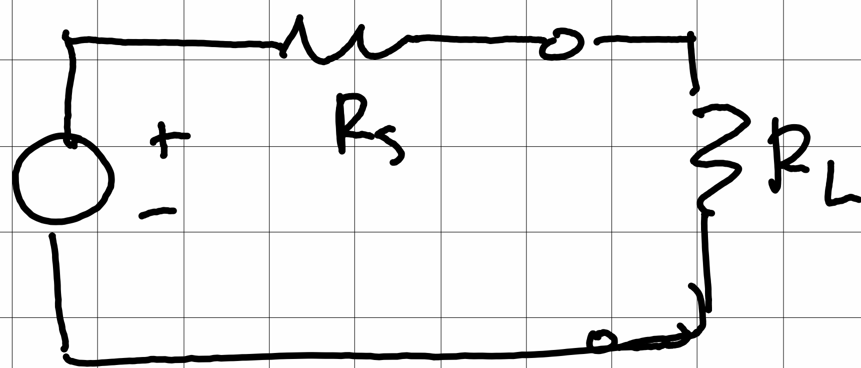

$\underline{\underline{R_{out} = R_{in}}}$ matching impedance

Why? string analogy



Impedance mismatch

- impedance matching is important for transmitting maximum power




$R_L = R_S$ maximizes power

How? homework

Hint: $\boxed{\dfrac{d P_L(R_L)}{d R_L} = 0}$

Nexttime:
- AC circuits
- transformers
- CH. 3