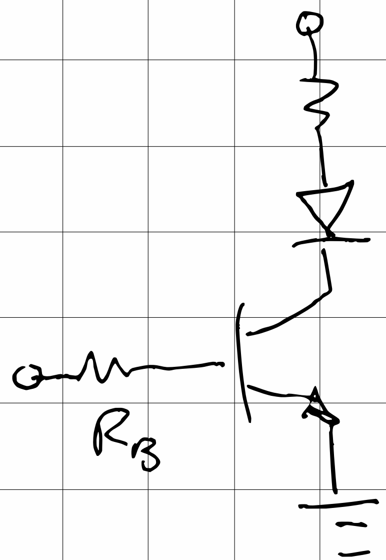
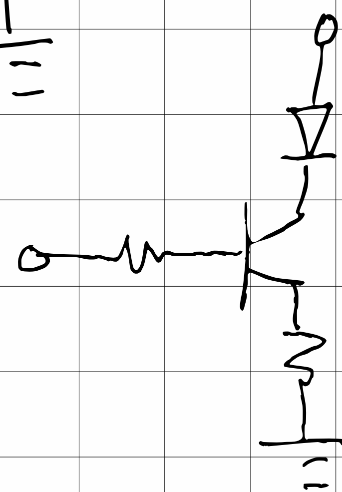
ME 133 Lecture 8 1/27/2022

Last time:

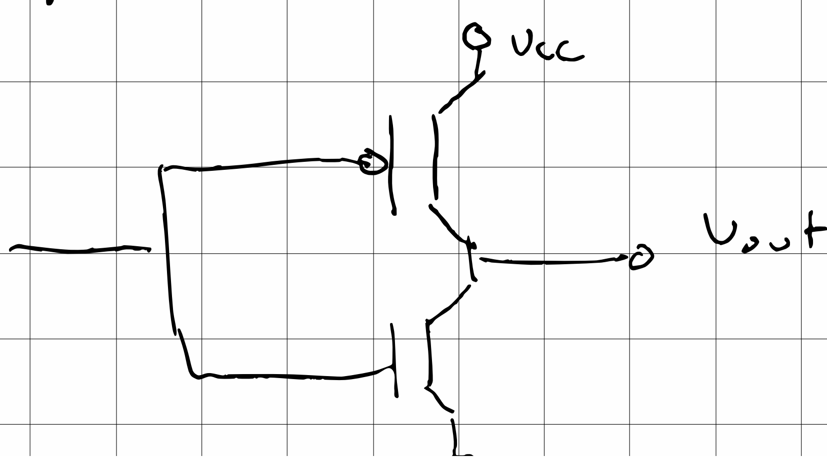
- 

BJT switch

For your lab:



- FET overview



Question last time:

Can you put too much current at Base of the BJT switch.

→ short answer, probably <u>not</u>.

→ main considerations:

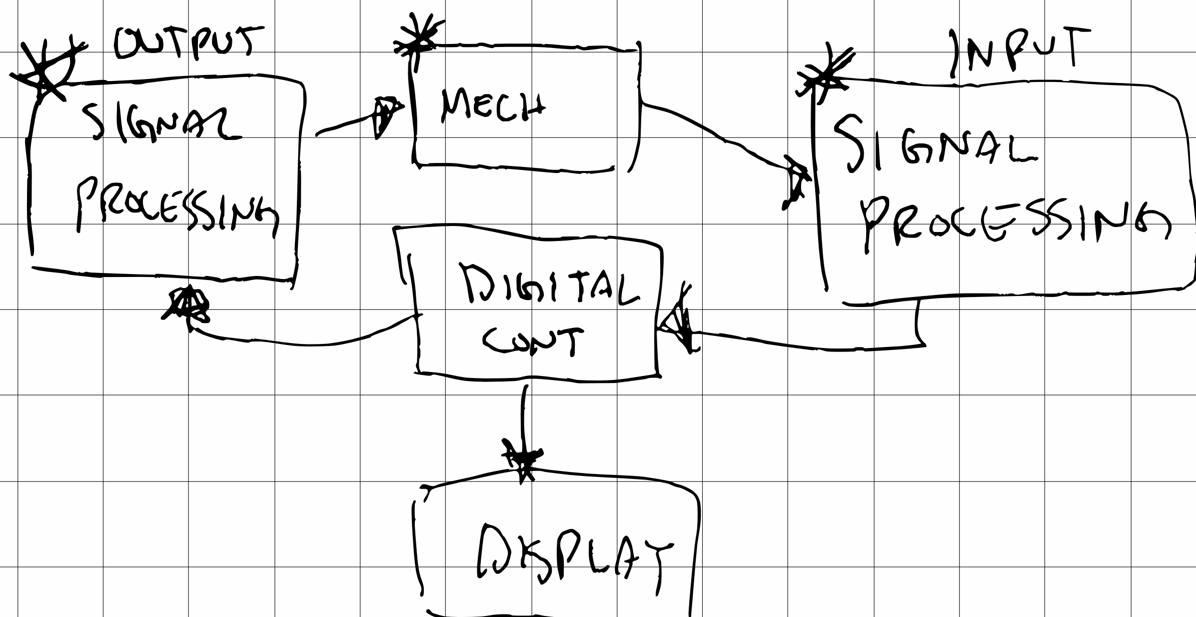
(1) power consumption

(2.) Time to switch off depends I_B

Today: Chapter 4

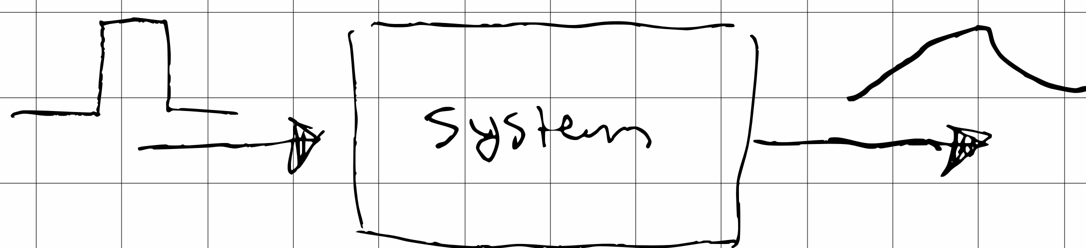
1. Amplitude linearity
2. Bandwidth

Ch. 4: System Response



Goal: mathematically model physical systems & characterize the response to dynamic inputs.

Response : given a input what does the output look like



We will consider "measurement system" for our discussion, but theory applies to any system.

What make a good measurement system?

1. Amplitude linearity
2. Adquate Bandwidth
3. Phase linearity

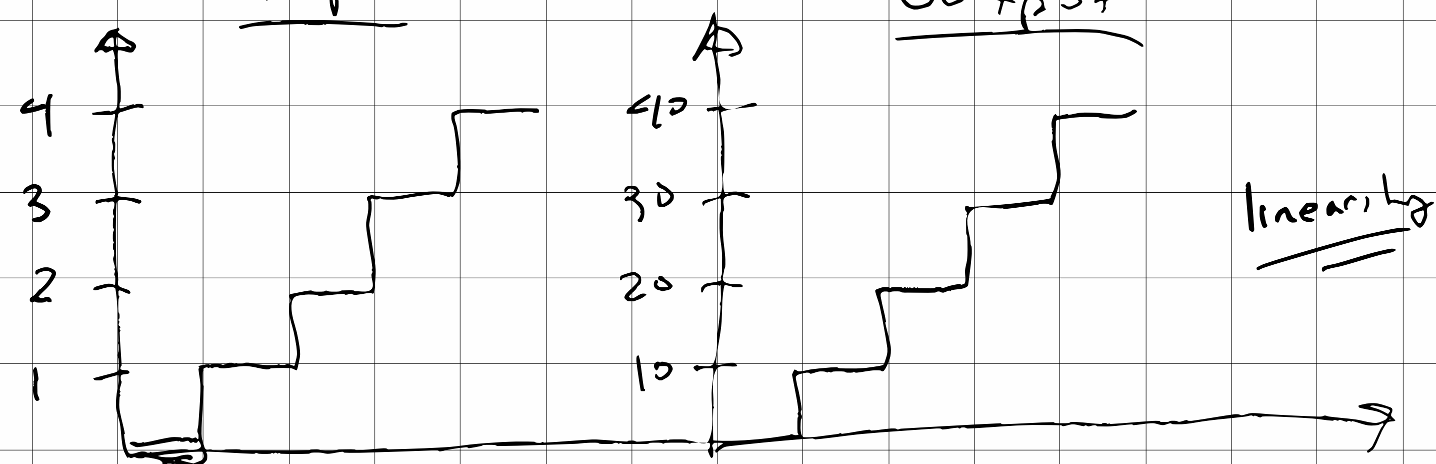
today

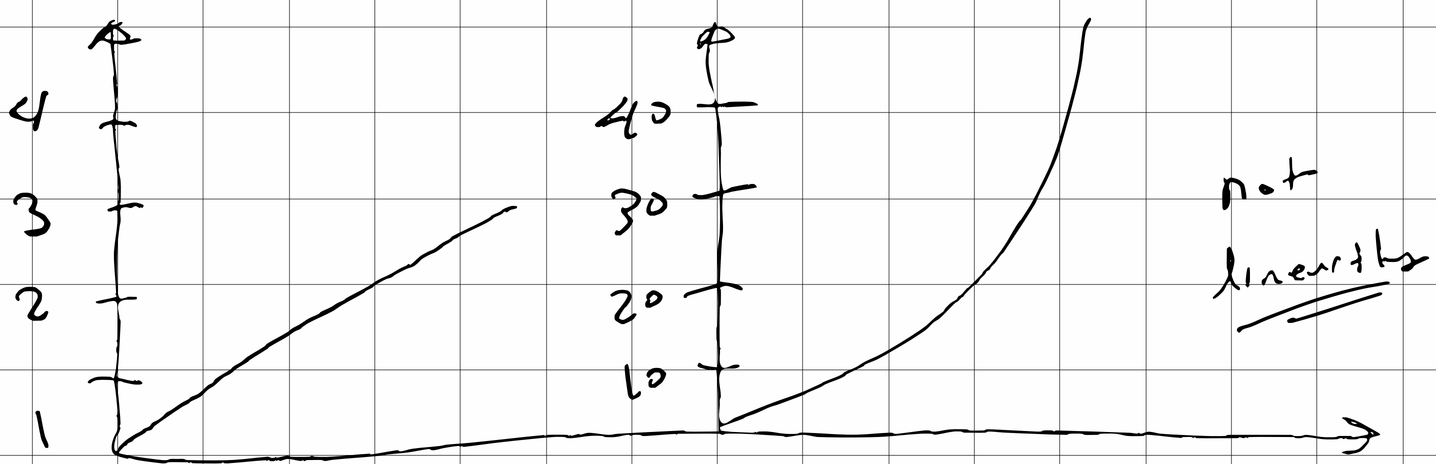
Amplitude linearity

$$V_{out}(t) - V_{out}(0) = \alpha \left[V_{in}(t) - V_{in}(0) \right]$$

Constant of proportionality

Input　　　　Output





Fourier Analysis (steady state)

Main Idea: Any periodic signal can be represented as an infinite sum of sine & cosine waveforms of different amplitude & freq.

Fundamental Harmonic: $\underbrace{\omega_0}_{rad/s} = \frac{2\pi}{T} = \underbrace{2\pi f_0}_{Hz}$

period

→ all other frequencies in the Fourier representation are integer multiples of ω_0

Fourier Series Representation

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

DC component

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

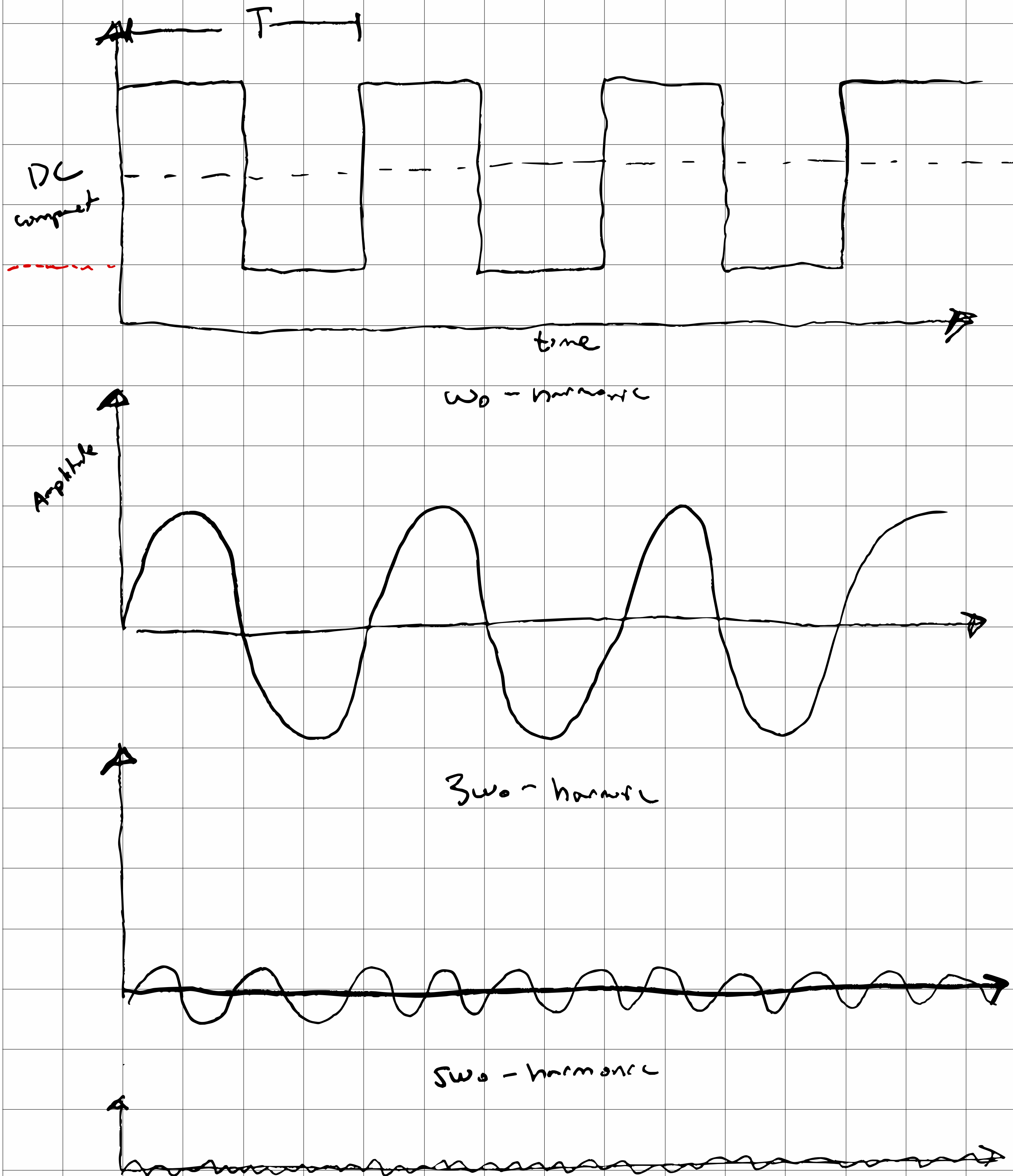
$$C_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{A_0}{2}$$

$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi_n)$$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

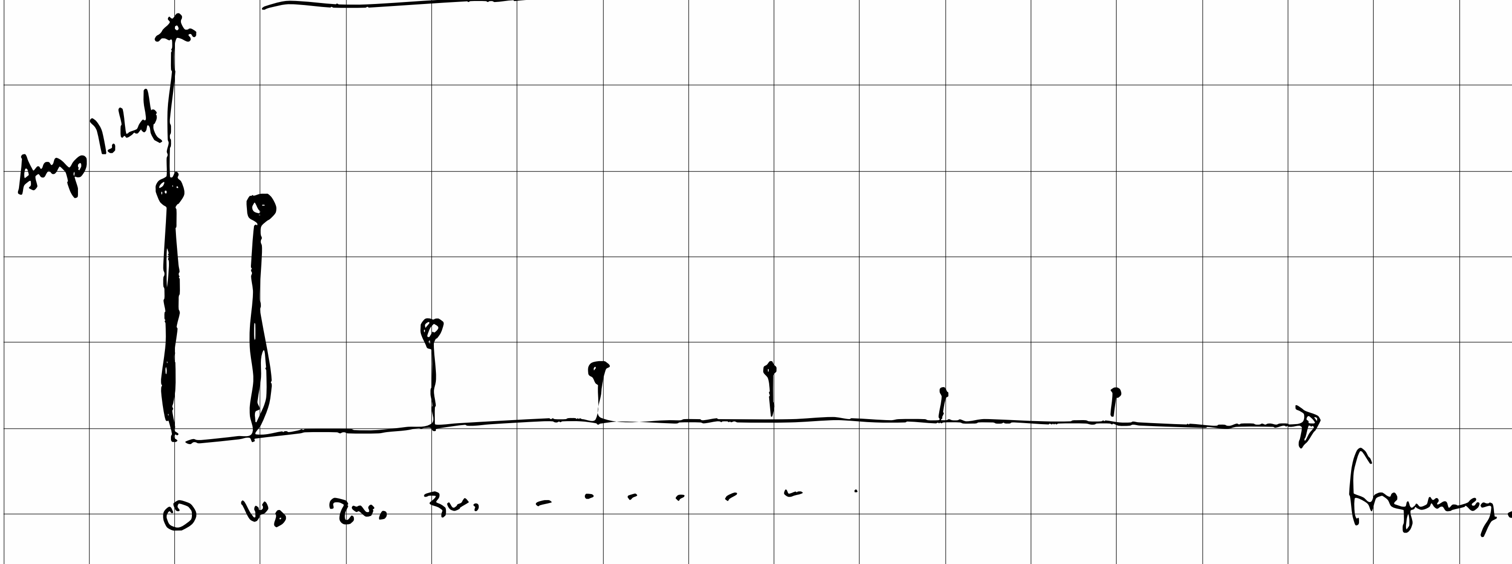
$$\phi_n = -\tan^{-1}\left(\frac{B_n}{A_n}\right)$$

Single amplitude version.


T
DC component
time
ω_0 - harmonic
Amplitude
$3\omega_0$ - harmonic
$5\omega_0$ - harmonic

ringing from Gibb's phenonomenon

Spectrum of Signal



* must use single-amplitude version of Fourier Series.

Bandwidth & Frequency Response

- Ideally a measurement system should replicate <u>all</u> frequency components of the input.

→ Decibels:

$$dB = 20 \log_{10} \left(\frac{A_{out}}{A_{in}} \right)$$

computed at each frequency!

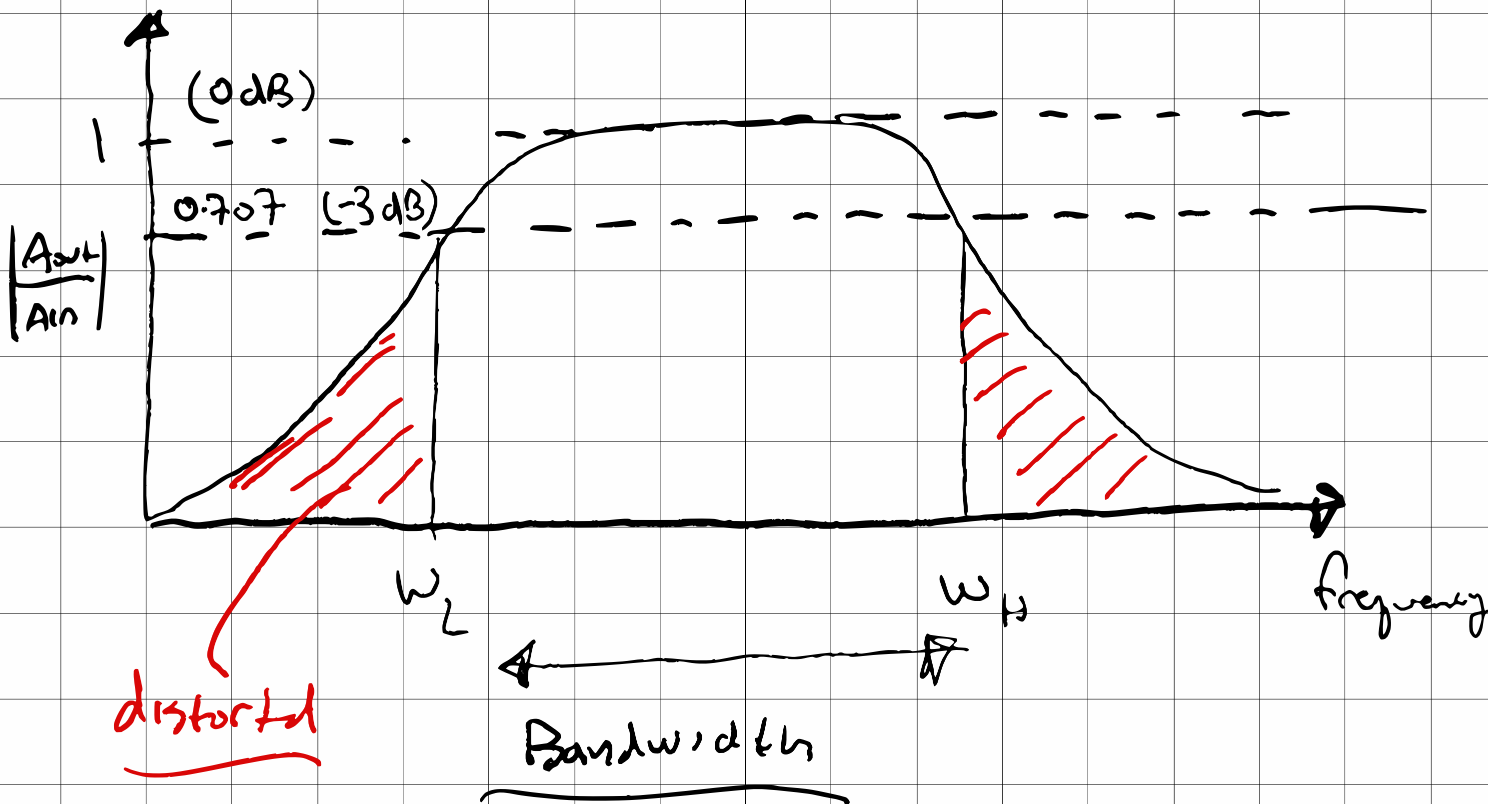
$0 \text{ dB} \rightarrow A_{out} = A_{in}$

$20 \text{ dB} \rightarrow A_{out} = 10 \cdot A_{in}$

$40 \text{ dB} \rightarrow A_{out} = 100 \cdot A_{in}$

$60 \text{ dB} \rightarrow A_{out} = 1000 \cdot A_{in}$

Frequency Response Curve



Bode Plot

Bandwidth is the range $w_L \rightarrow w_H$

- Why $-3\,dB$?

"half-power points"

$$\frac{P_{out}}{P_{in}} = \frac{1}{2} \rightarrow P = V^2 \cdot R$$

$$\therefore P \sim V^2$$

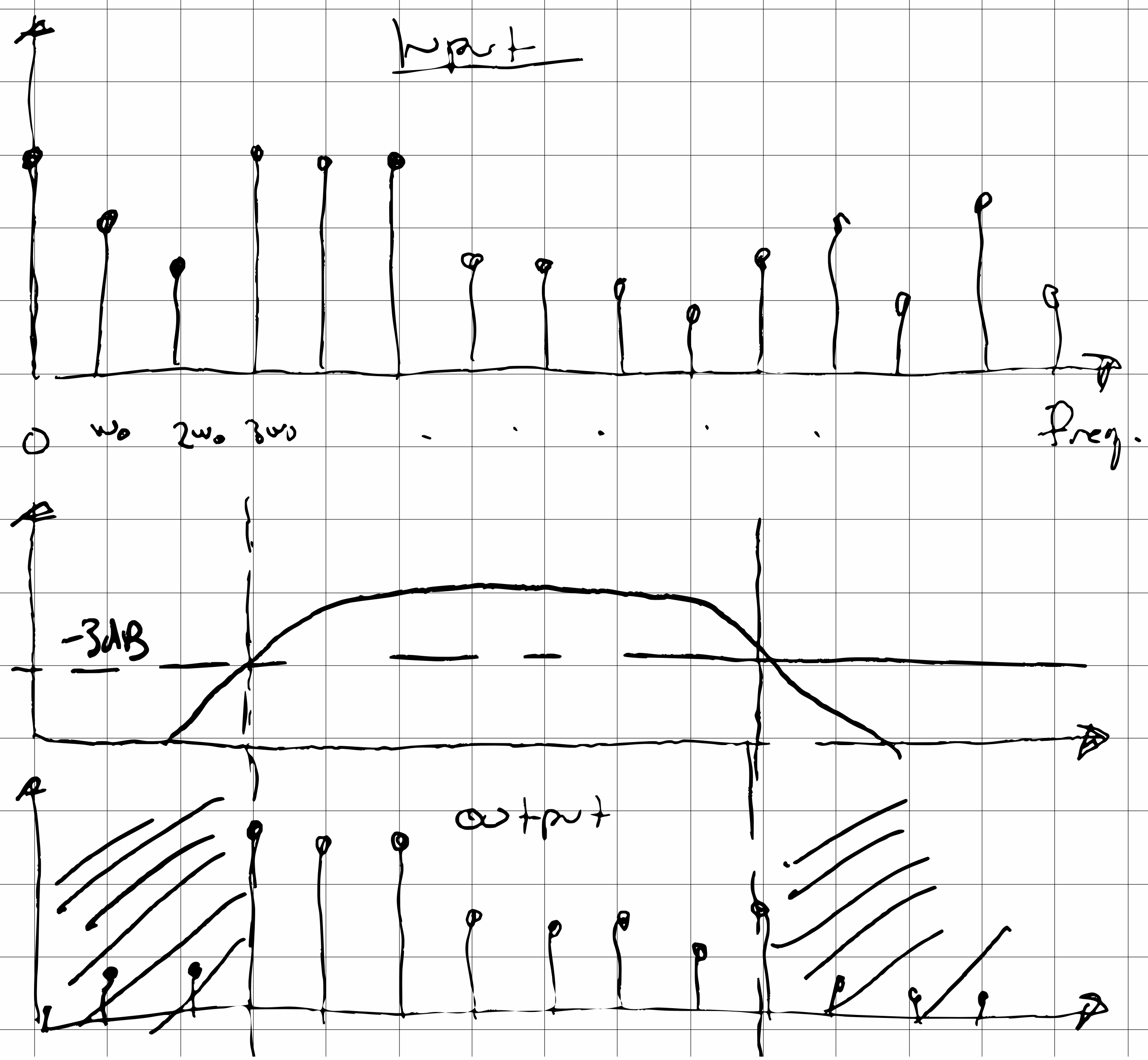
$$\frac{A_{out}}{A_{in}} = \sqrt{\frac{P_{out}}{P_{in}}} = \frac{1}{\sqrt{2}} \approx 0.707$$

$$dB = 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) \approx \underline{\underline{-3\,dB}}$$

ω_L & ω_H $\rightarrow$ cutoff freq.
or corner freq.

<u>large bandwidth</u> $\rightarrow$ <u>high fidelity</u>

If you know the frequency response of your system (measurement device, sensor, actuator, filter) and you know the freq. content of the input, then you can predict the out freq. content.



- How to determine bandwidth of a system?

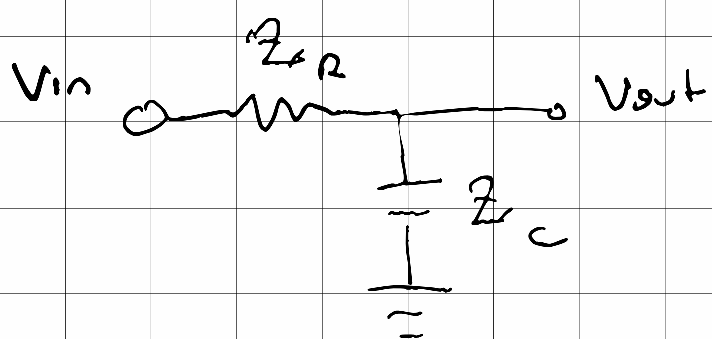
→ Field "System Identification"

→ send pure sine waves as input for all frequency.

→ frequency sweep!

→ Gaussian noise

Example: lowpass filter.



$$V_{out} = \left(\frac{Z_C}{Z_R + Z_C} \right) V_{in}$$

$$Z_C = \frac{1}{j\omega C} \quad \Big\rangle \quad \frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$$

$$Z_R = R$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

let $\omega_c = \frac{1}{RC}$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}} \quad \leftarrow \text{next time}$$