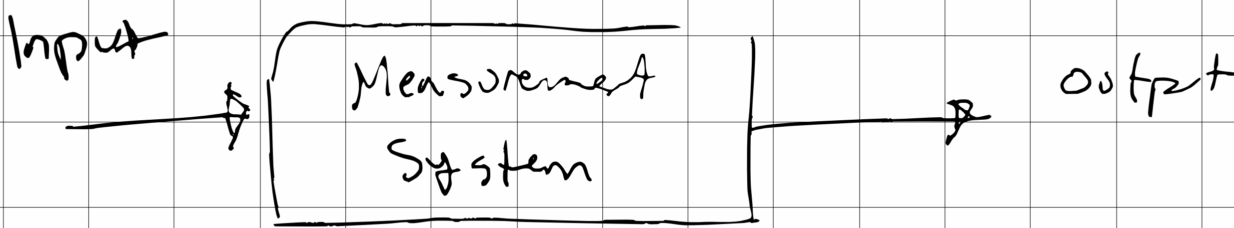
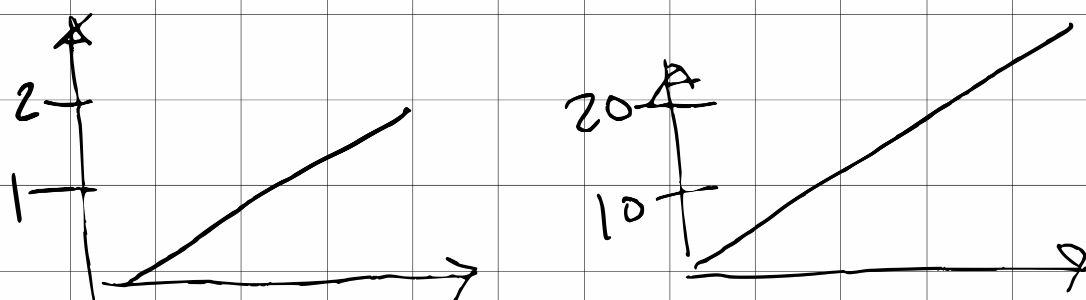
ME 133 lecture 9 2/1/2022

Last time:



→ amplitude linearity

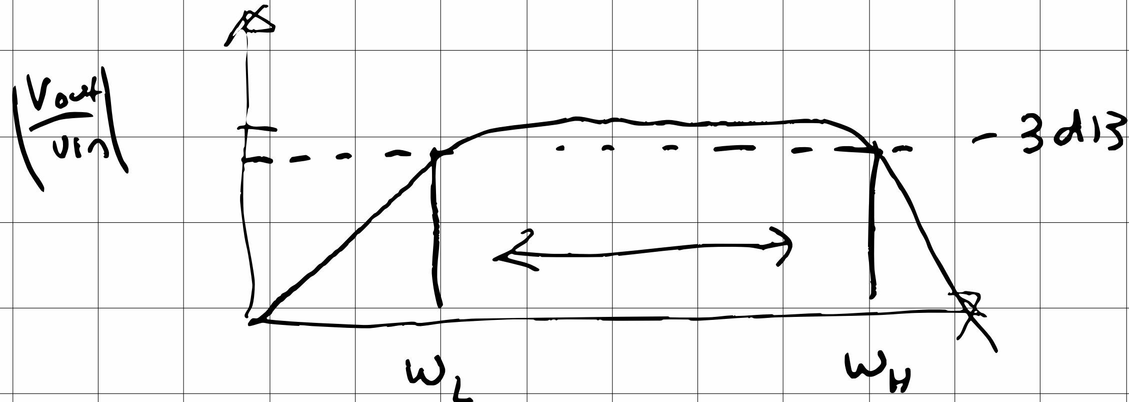


$$V_{out} = \alpha V_{in}$$

→ Review of Fourier Analysis

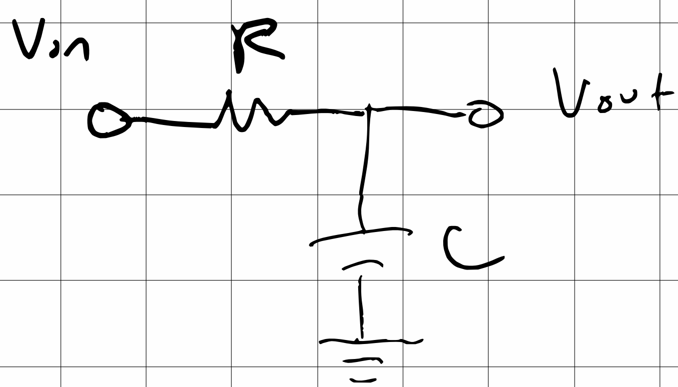
→ Bandwidth & Freq. Response

- $-3\ dB$ → "half power point"



Today:
- Bandwidth example
- phase linearity
- Dynamic Characteristics
- Zero-order system
- first-order system

Example: low pass filter



$$V_{out} = \left(\frac{Z_C}{Z_R + Z_C} \right) V_{in}$$

$$Z_C = \frac{1}{j\omega C} \qquad \frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$$

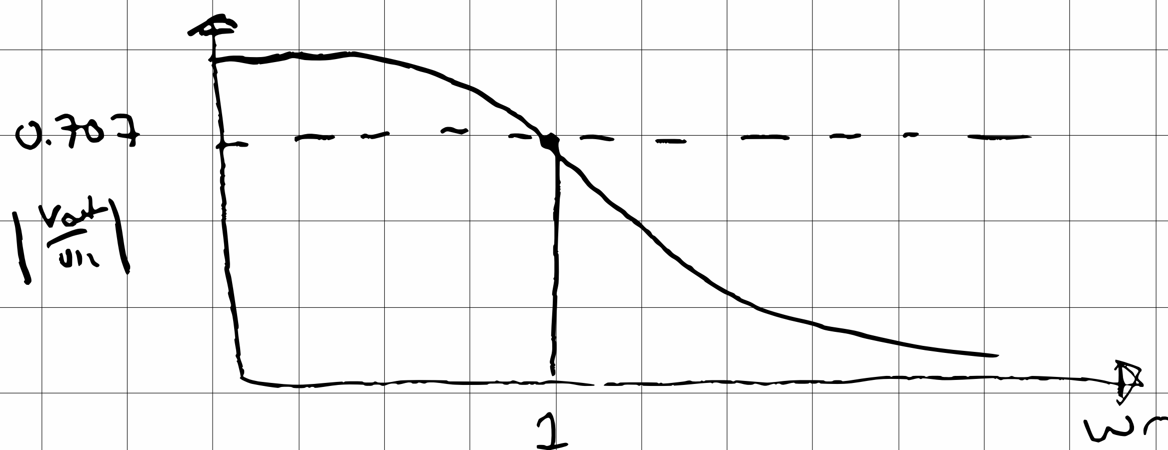
$$Z_R = R$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

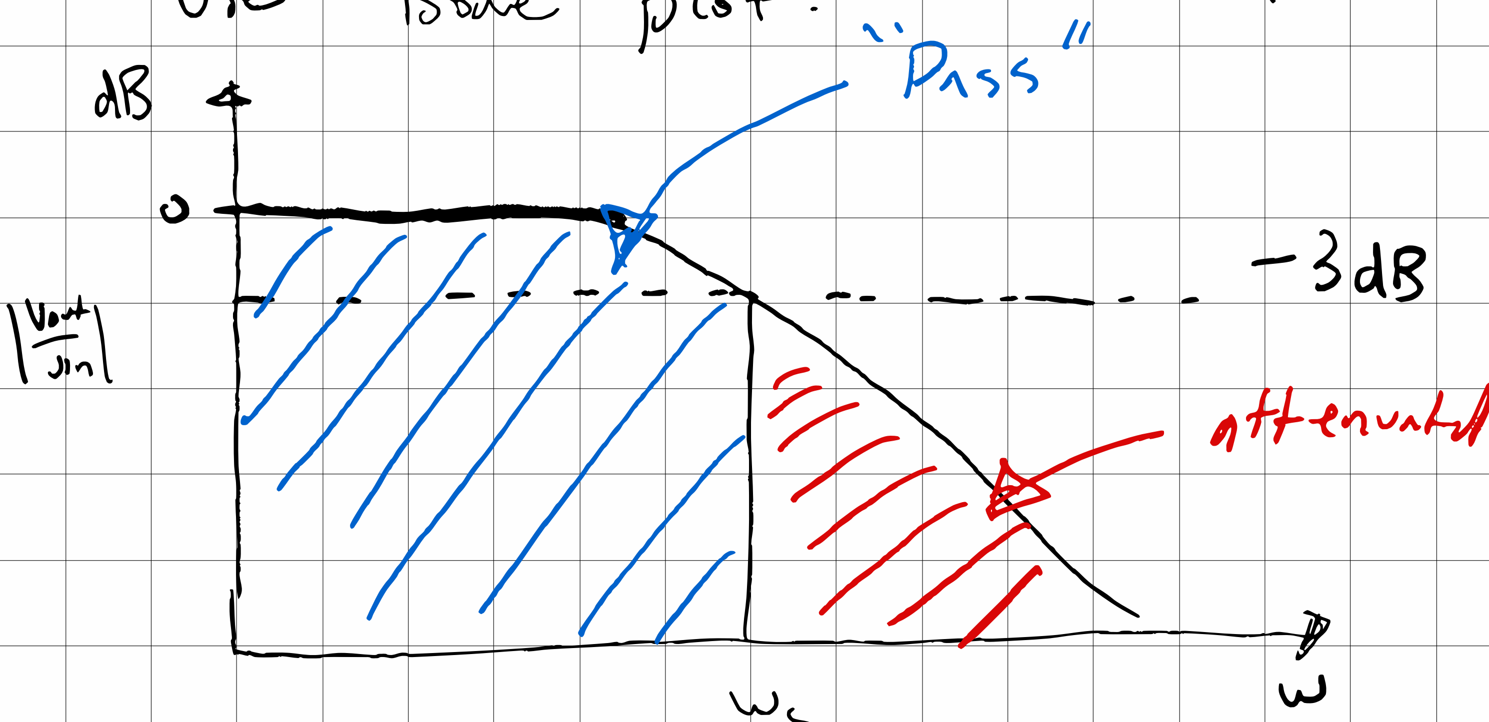
let $\omega_c = \frac{1}{RC}$ (cutoff freq)

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\omega/\omega_c = \omega_r \qquad \left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + \omega_r^2}}$$

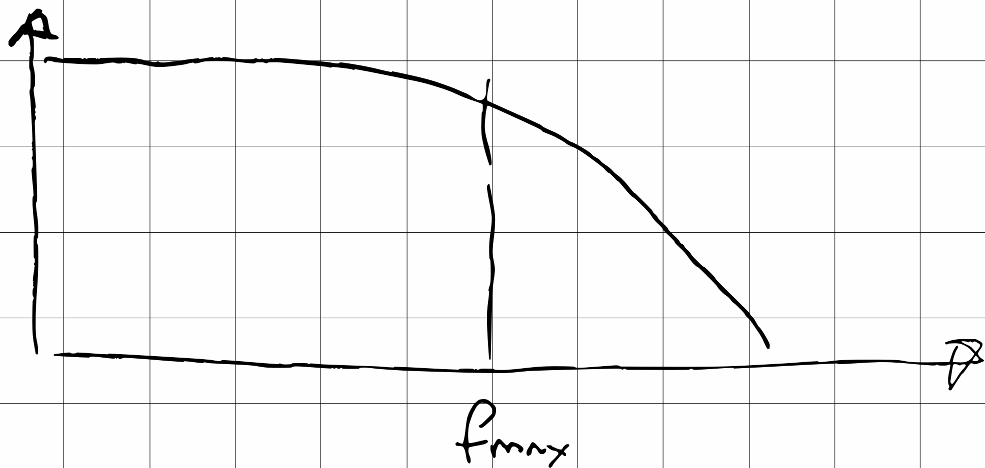


Use Bode plot:



Bandwidth applies to all systems

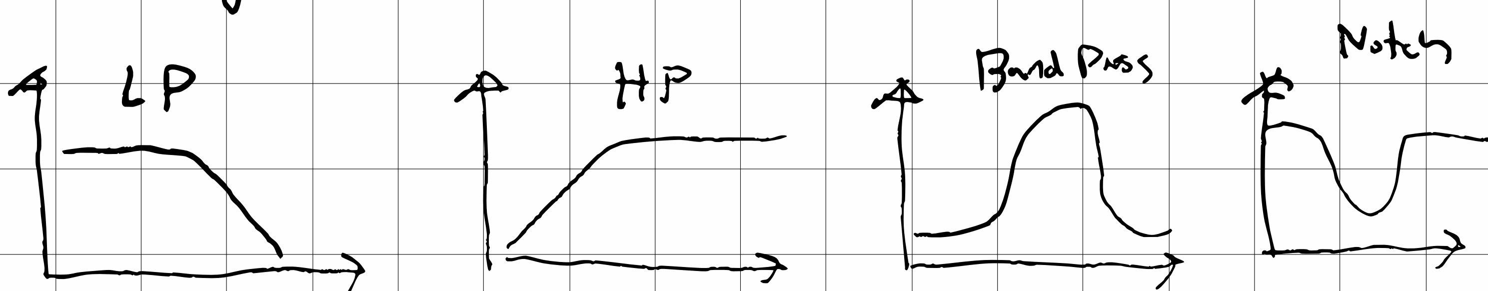
- Actuators have a Bandwidth



track sine waves up to f_{max}

- Sensors have bandwidth

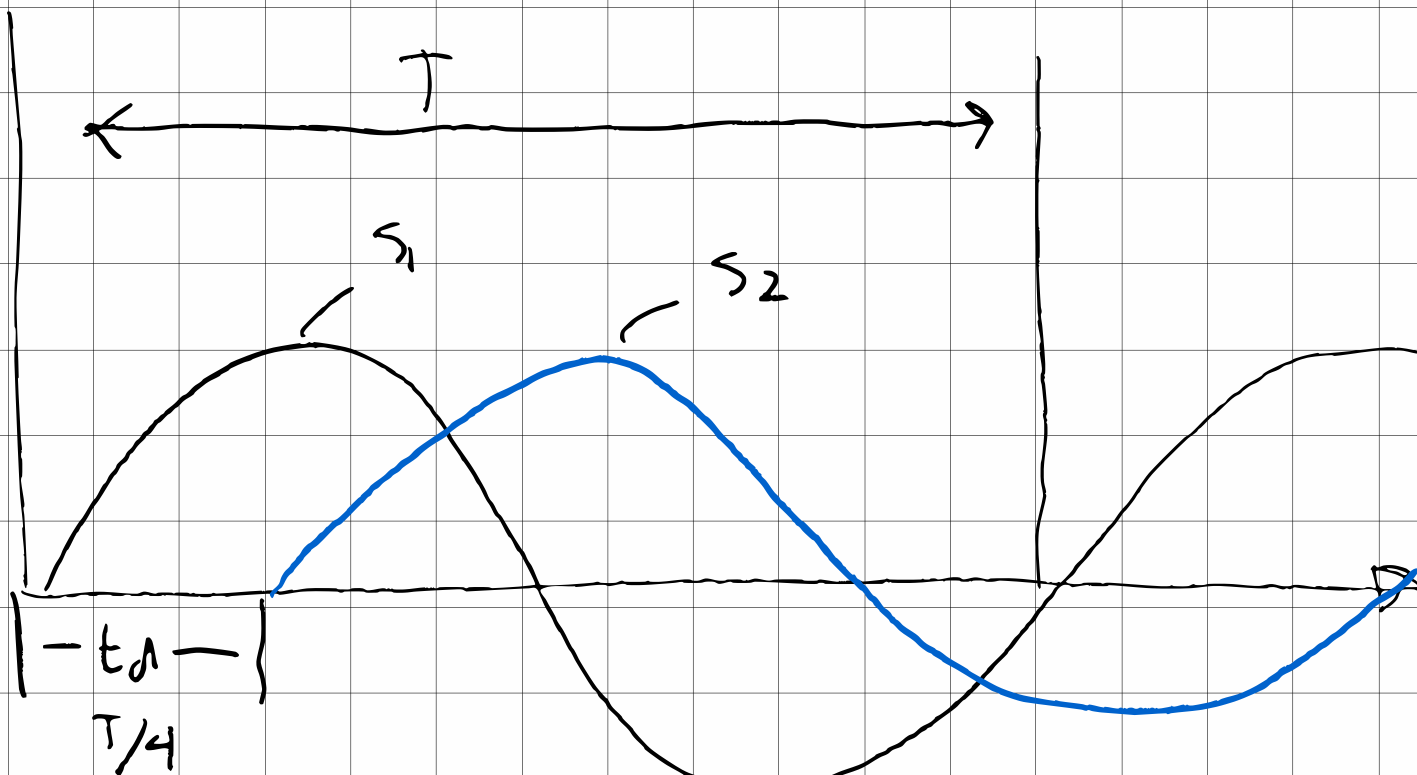
- Bandwidth is the most important design consideration for filters!



- systems w/ non linear effects have amplitude dependent Freq. Res.

Phase Linearity

→ how well a system preserves the phase relationship between freq. components



$s_1 = \sin(\omega t)$

$s_2 = \sin(\omega t + \phi)$

$$\phi = ?$$

$$= 360° \cdot \frac{t_d}{T}$$

$$= 360° \cdot \frac{T/4}{T}$$

$$= 90°$$

$T = \frac{1}{f}$, $\boxed{\phi = 2\pi f \cdot td}$

→ phase shift is freq. dependent.

→ we want equal time shift (td) for all frequencies

$\therefore$ $\underline{\phi = K \cdot f}$ phase linearity

phase shift (ϕ) is proportional to freq. → equal td of each component.

When a system doesn't have phase & amplitude linearity the output is <u>distorted</u> !

Dynamic Characteristics of a system

General an ODE:

$$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = \sum_{m=0}^{m} B_m \frac{d^m X_{in}}{dt^m}$$

$\left.\begin{matrix} A_n \\ B_m \end{matrix}\right\}$ constant!

$\longrightarrow$ represent physical properties

$$a_2 \ddot{x} + a_1 \dot{x} + a_0 x = b_1 \dot{u} + b_0 u$$

$$m \ddot{x} + b \dot{x} + K x = u$$

$$RC \dot{V}_o + V_o = V_{in}$$

ODE can model all types of physical systems!

$\rightarrow$ one of the most important tools for engineers.

Zero - Order System

$N = M = 0$, <u>Static</u> system

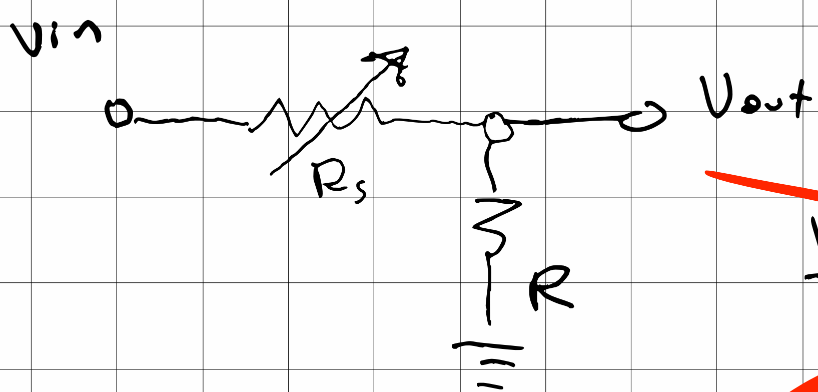
$$A_0 X_{out} = B_0 X_{in}$$

$$X_{out} = \underbrace{\frac{B_0}{A_0}}\, X_{in} = \underbrace{K}\, X_{in}$$

K: gain or sensitivity

→ Zero-order system have no distortion

Recall our voltage divider sensor circuit :



Gain depends on
Force -> Rs function

$$\cancel{\frac{V_{out}}{V_{in}} = \left(\frac{R}{R_s + R}\right)}$$

$$\cancel{\text{gain} = f(R_s)}$$

Same idea for potentiometer

First Order System

$N = 1, \; M = 0$

$$A_1 \frac{d X_{out}}{dt} + A_0 X_{out} = B_0 X_{in}$$

$$A_1 \dot{x} + A_0 x = B_0 u$$

$$\rightarrow \dot{x} = \frac{-A_0}{A_1} x + \frac{B_0}{A_1} u$$

$$\boxed{\dot{x} = Ax + Bu}$$

state space form

all higher order ODEs can be written a a system of first order ODEs.

$$A_1 \dot{x}_{out} + A_0 x_{out} = B_0 x_{in}$$

$$\underbrace{\frac{A_1}{A_0}} \dot{x}_{out} + x_{out} = \underbrace{\frac{B_0}{A_0}}_{K} x_{in}$$

$K = \frac{B_0}{A_0}$ static gain
or
sensitivity

time constant $\tau = \frac{A_1}{A_0}$

$$\tau \dot{x}_{out} + x_{out} = K x_{in}$$

'standard form'

$$\tau \dot{x}_{out} + x_{out} = K X_{in}$$

Step Response :

$$X_{in} = \begin{cases} 0 & t < 0 \\ A_{in} & t \geq 0 \end{cases}$$

Solution : Assume $X_{out} = Ce^{st}$

$X(0) = 0$

write characteristic equation :

$$\tau s + 1 = 0 \qquad s = -\frac{1}{\tau}$$

homogenous solution : $X_{out_h} = C e^{-t/\tau}$

then, the particular solution : $(\dot{x} = 0)$

$$X_{out_p} = K A_{in}$$

General Solution:

$$X_{out}(t) = X_{out_h} + X_{out_p}$$

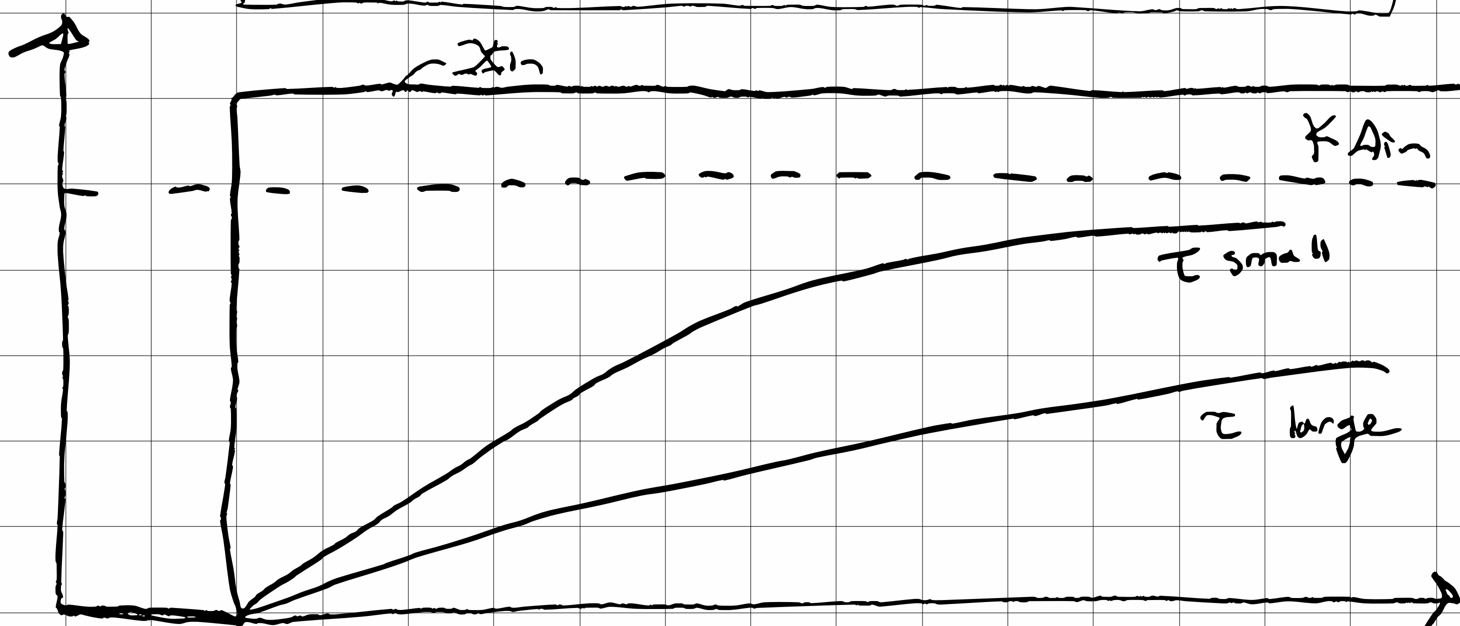
$$= Ce^{-t/\tau} + KA_{in}$$

Use our I.C.

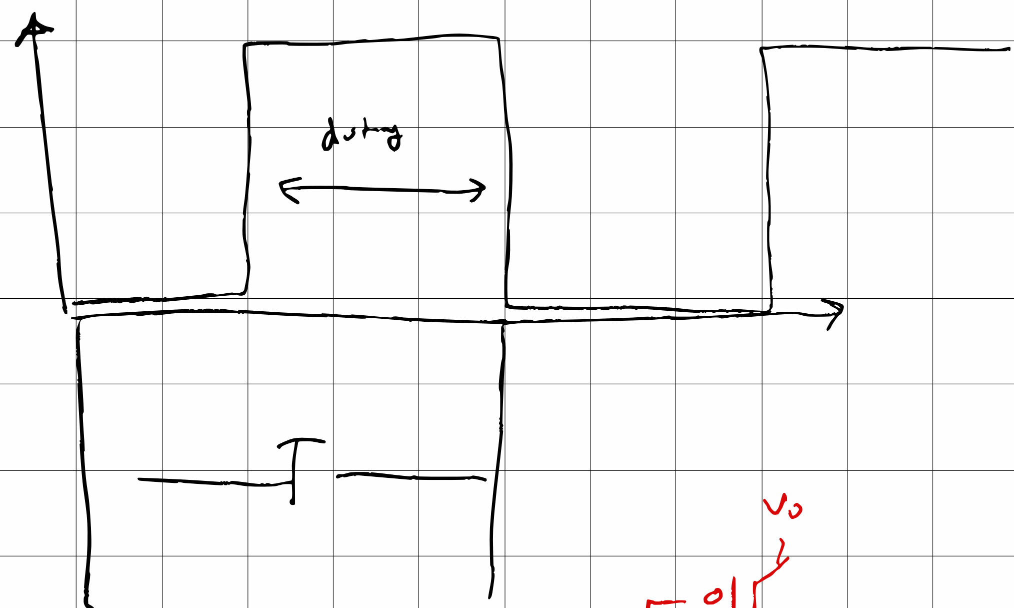
$$0 = C + KA_{in}$$

$$C = -KA_{in}$$

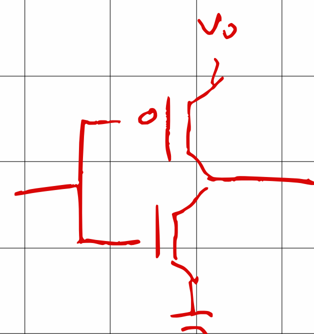
$$\therefore \boxed{X_{out} = KA_{in}\left(1 - e^{-t/\tau}\right)}$$

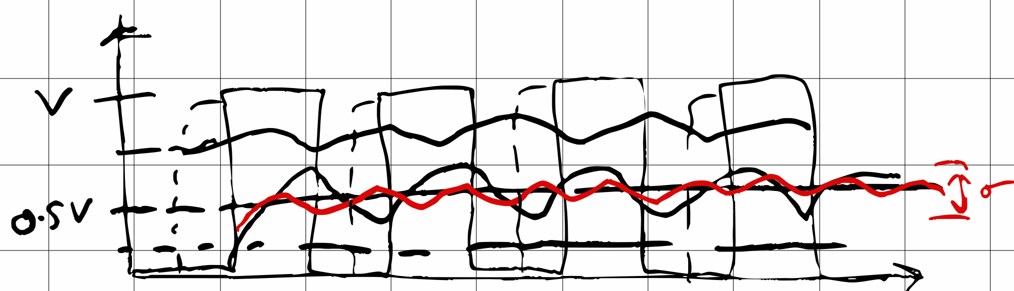


PWM



→ DAC





$$X_{out}(t=\tau) = KA_m \underbrace{\left(1 - e^{-1}\right)}_{0.6321\ldots}$$

after 1 time constant the signal reaches 63% of S.S.

when $t = 4\tau$

$$X_{out}(t=4\tau) = KA_m \underbrace{\left(1 - e^{-4}\right)}_{0.982}$$

$$= KA_m \; 98\%$$

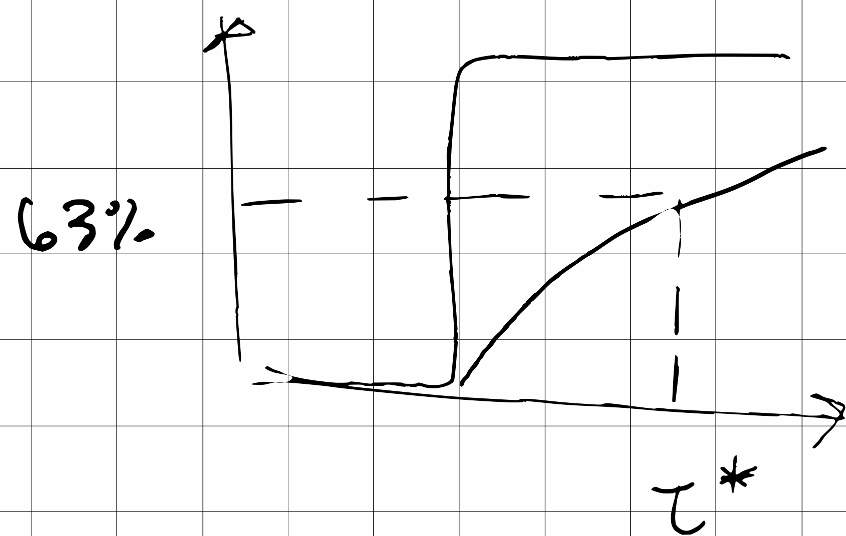
First-order system reaches S.S. after $t = 4\tau$

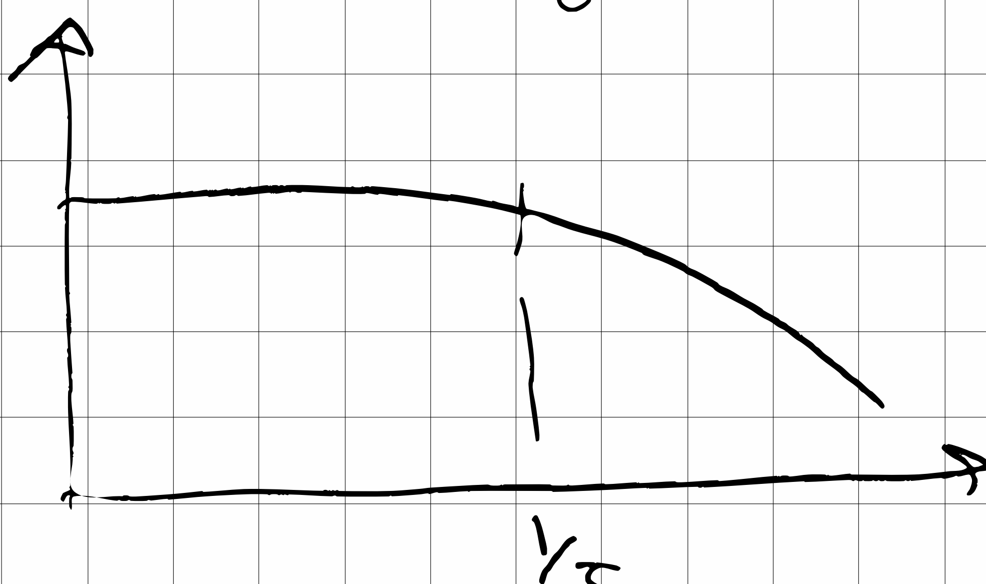
System ID of First Order

How to estimate τ ?

$\rightarrow$ Step response!

Method one:





Method 2: $1 - \frac{x_{out}}{KA_{in}} = e^{-t/\tau}$

$$\underbrace{\ln\left(1 - \frac{x_{out}}{KA_{in}}\right)}_{z} = -\frac{t}{\tau}$$

$$z = -\frac{t}{\tau}$$

$$\frac{z}{t} = -\frac{1}{\tau}$$