ME133 Lecture 9 — make up lecture 2/8/23

Last time:

> Measurement systems

> Amplitude Linearity

> Bandwidth and Frequency Response

Today:

> Phase linearity

> Intro to system dynamics

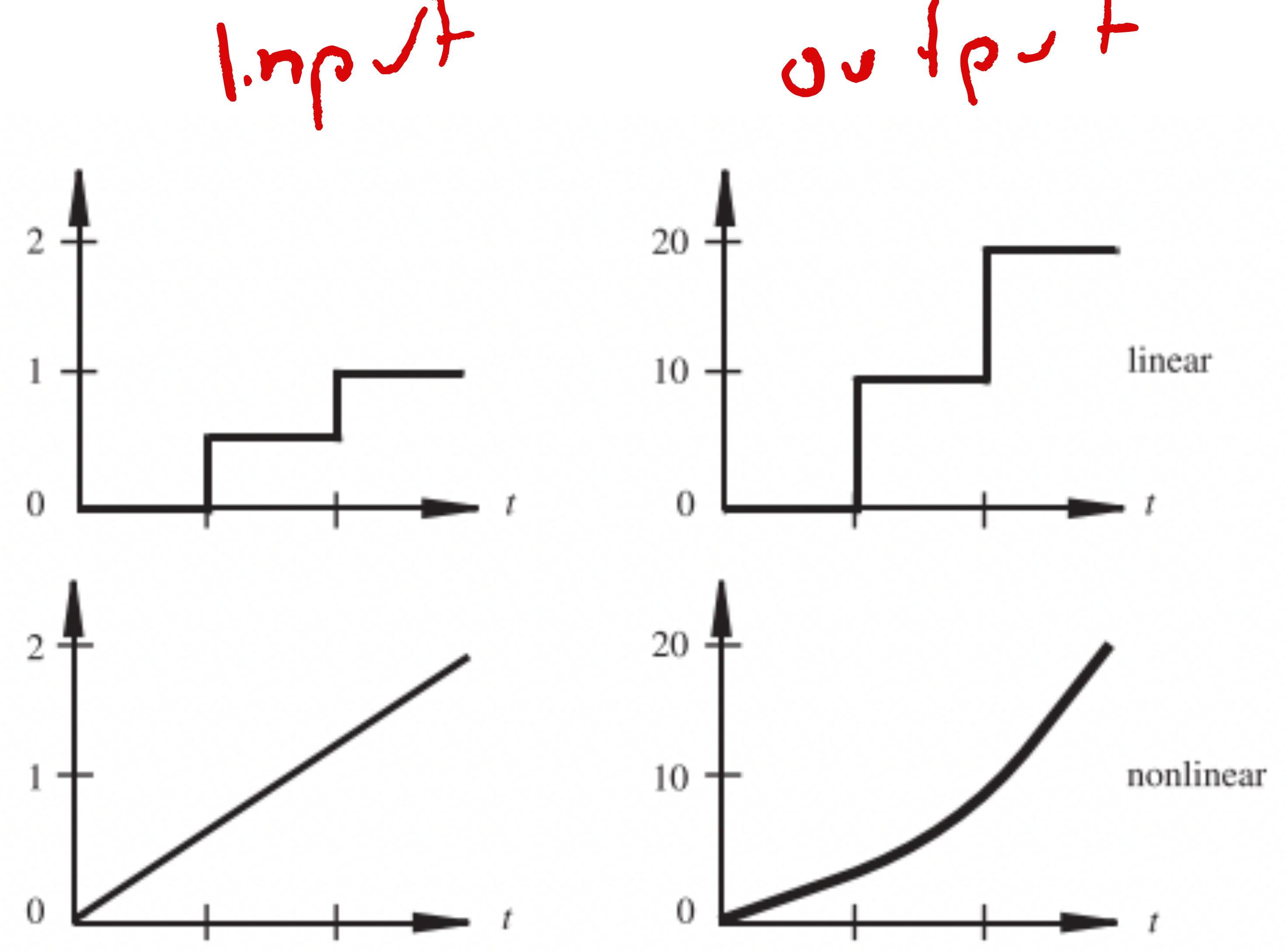
> First Order Systems

Three most important considerations for a measurement system

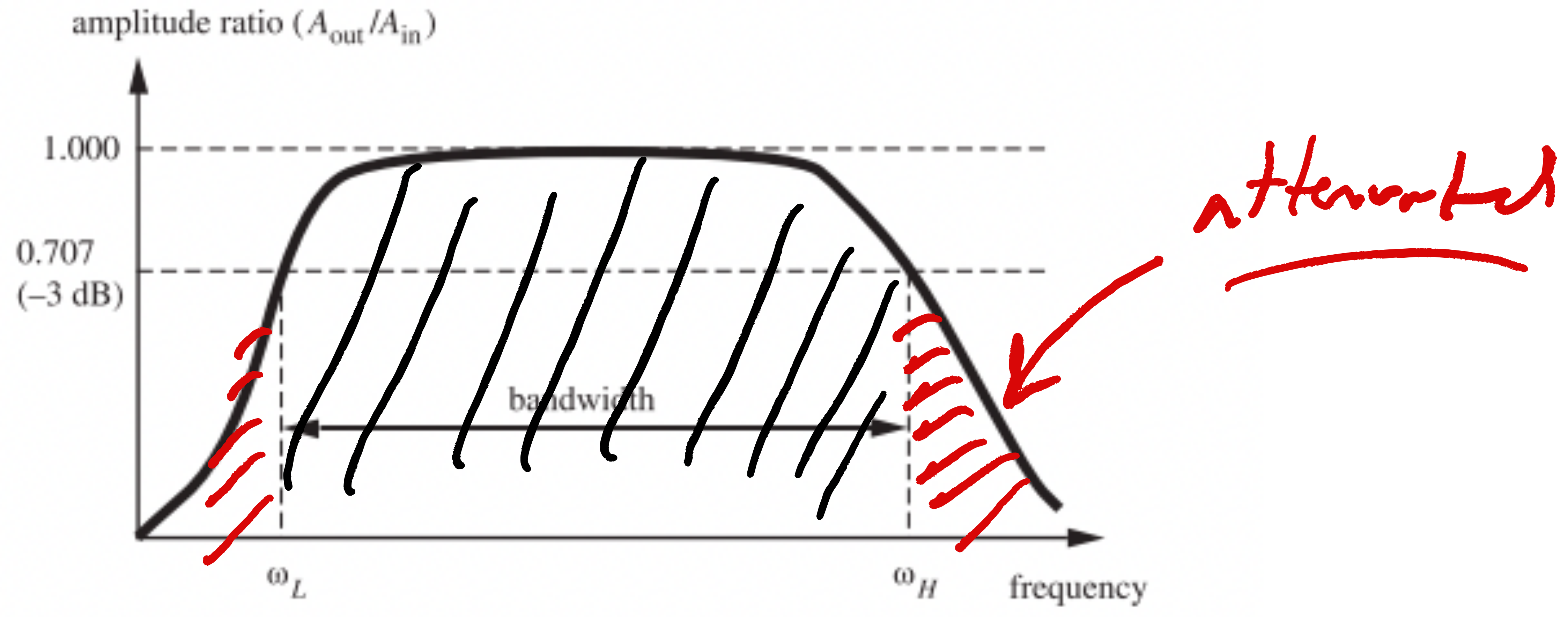
1. Amplitude linearity

2. Adequate bandwidth

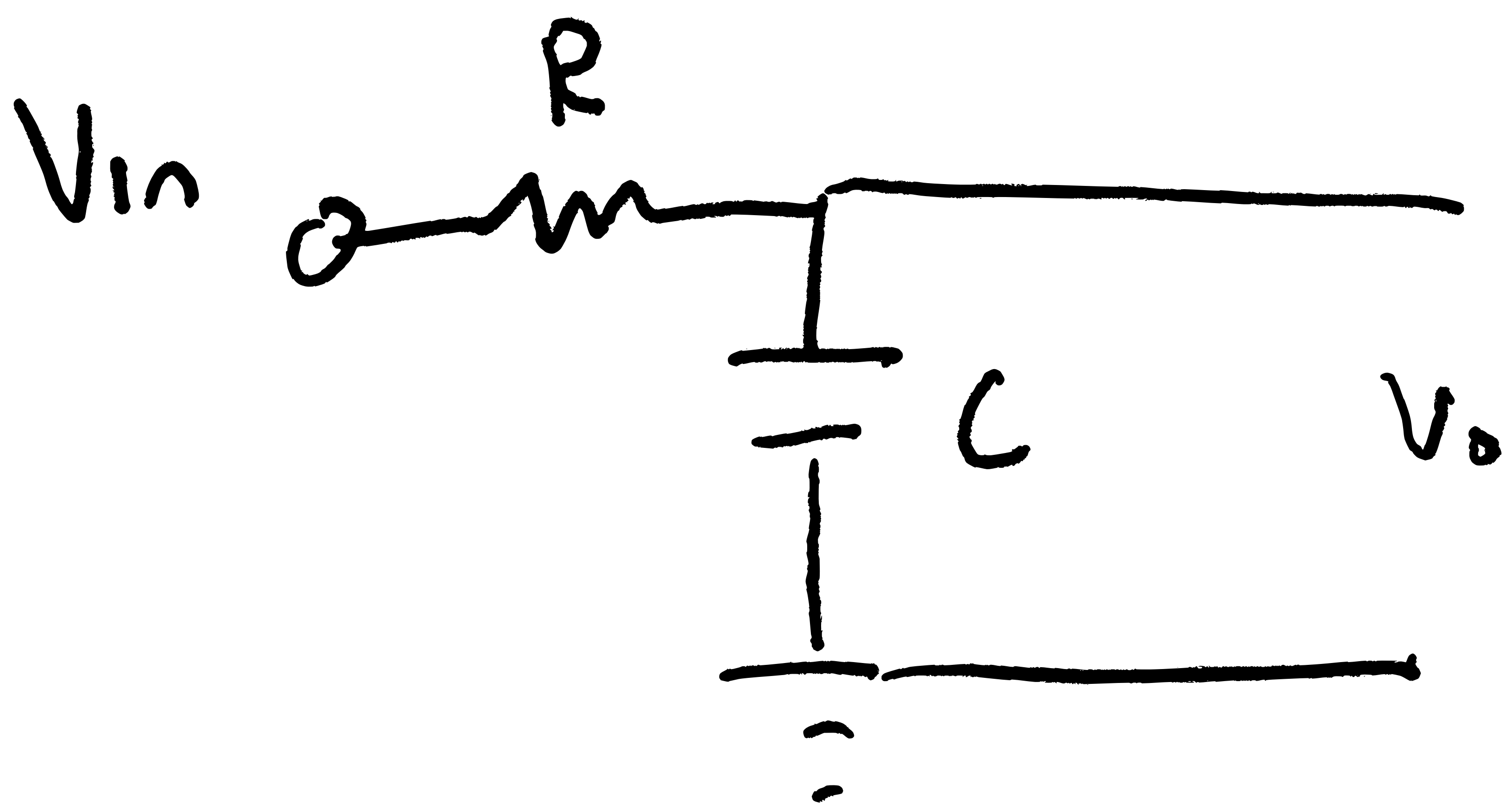
3. Phase linearity



½ power point



Example: Bandwidth of a lowpass filter



$$V_{out} = \left(\frac{Z_c}{Z_c + Z_R} \right) V_{in}$$

RC-filters

$$Z_c = \frac{1}{j\omega C}$$

$$Z_R = R$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega \underbrace{RC} + 1}$$

Example: Bandwidth of a lowpass filter

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

let $\omega_c = \frac{1}{RC}$ (cutoff freq.

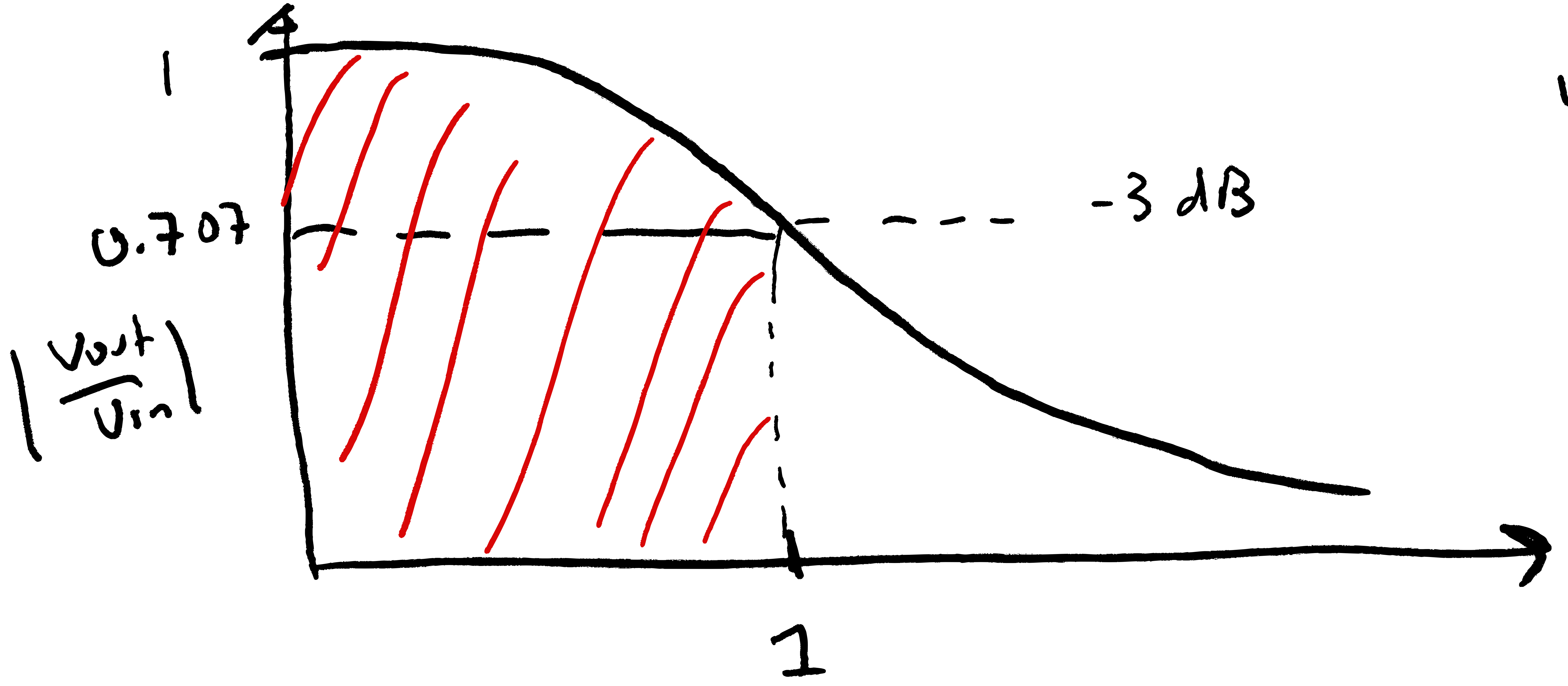
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\frac{\omega}{\omega_c} = \omega_r$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \omega_r^2}}$$

Example: Bandwidth of a lowpass filter

$$\frac{\omega}{\omega_c} = \omega_r \qquad \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+\omega_r^2}} = \frac{1}{\sqrt{2}} = \underline{\underline{0.707}}$$

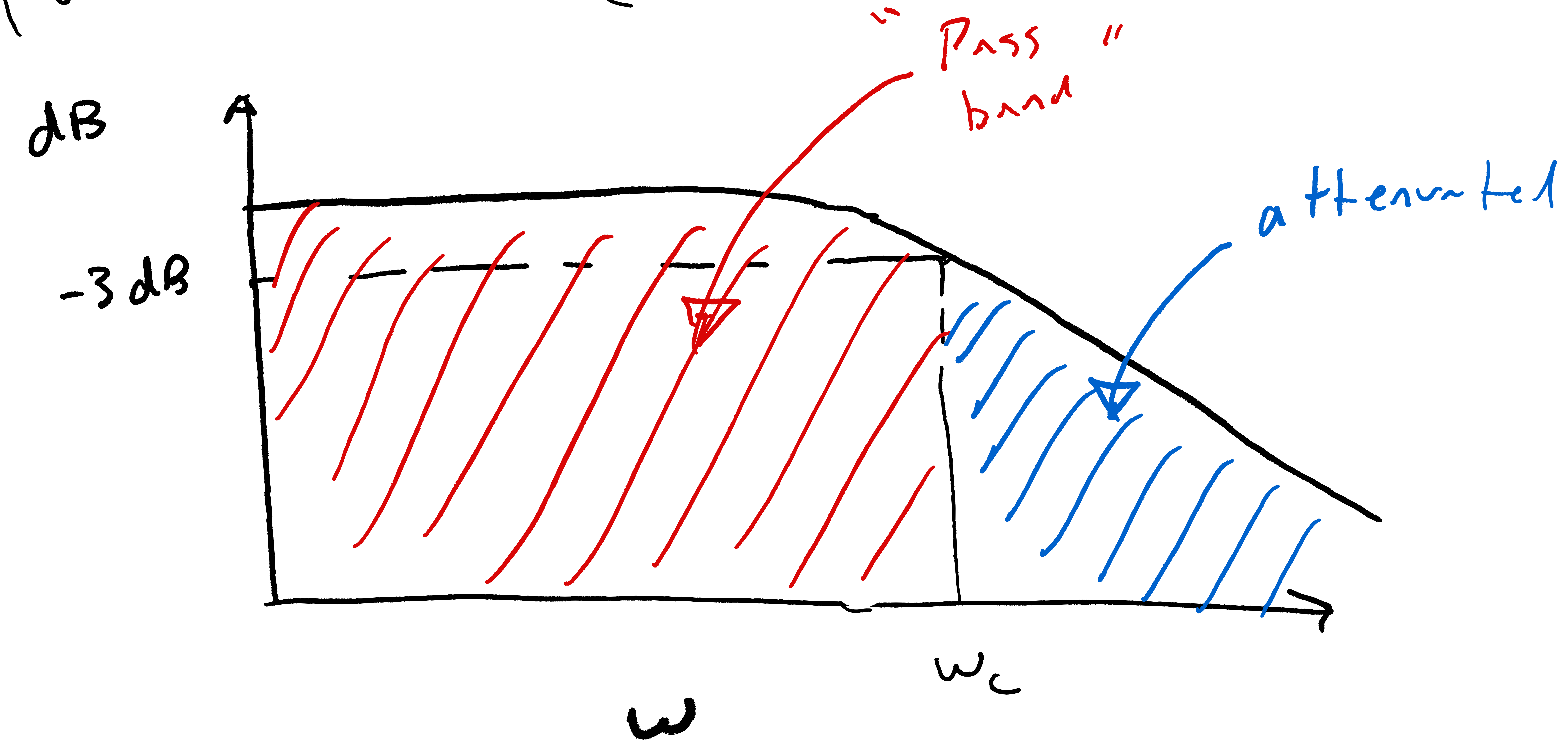


$$\omega_r = 1 = \underline{\omega = \omega_c}$$

$$dB = -3\,dB$$

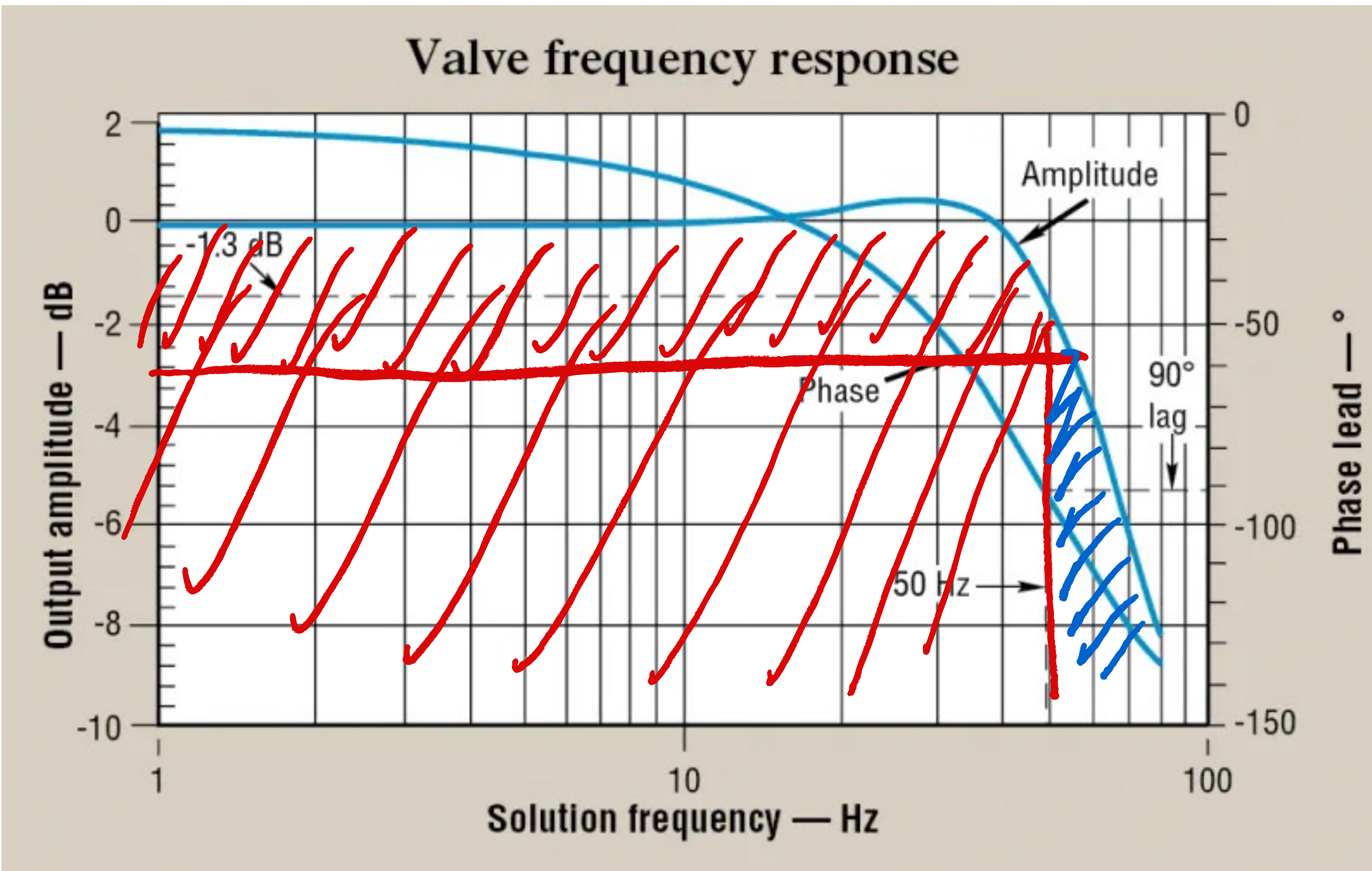
Example: Bandwidth of a lowpass filter

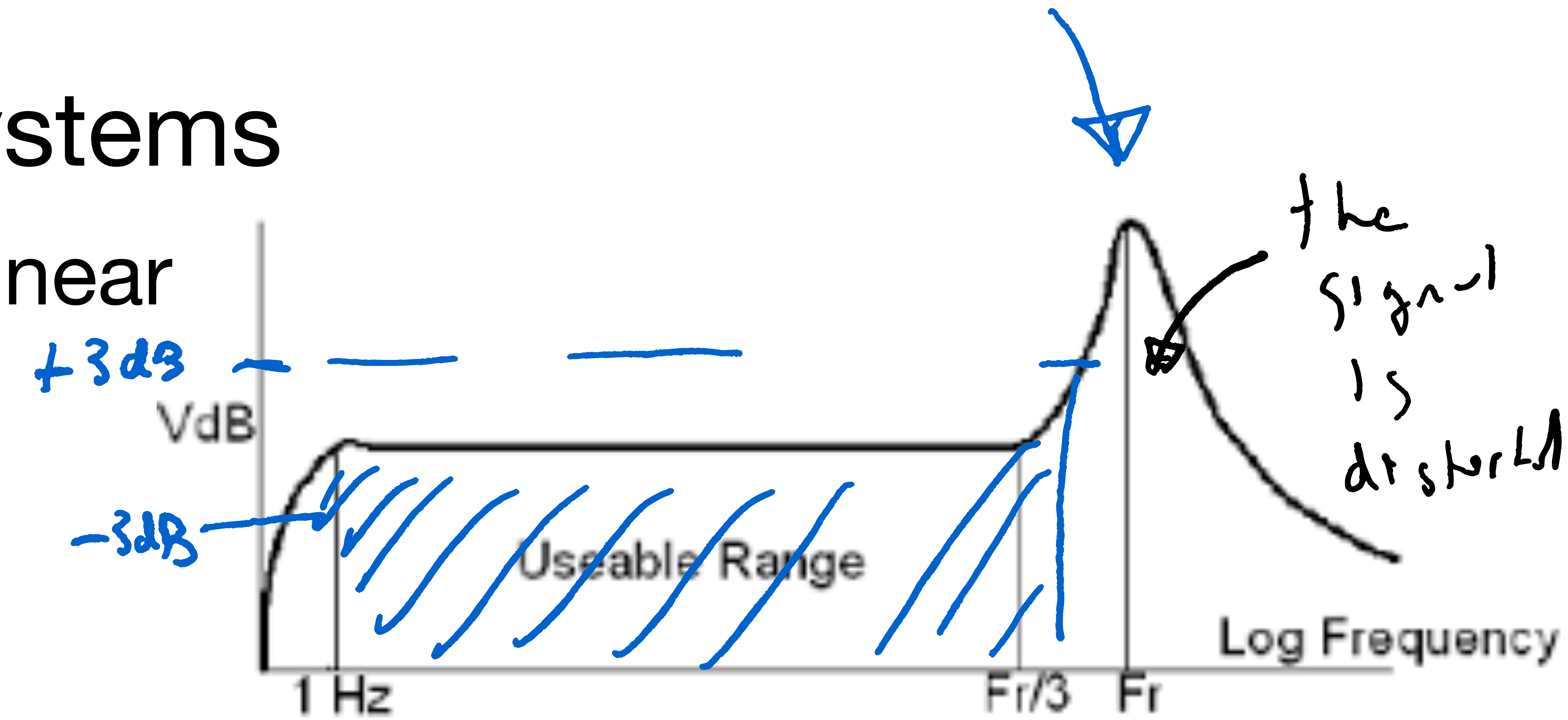
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$



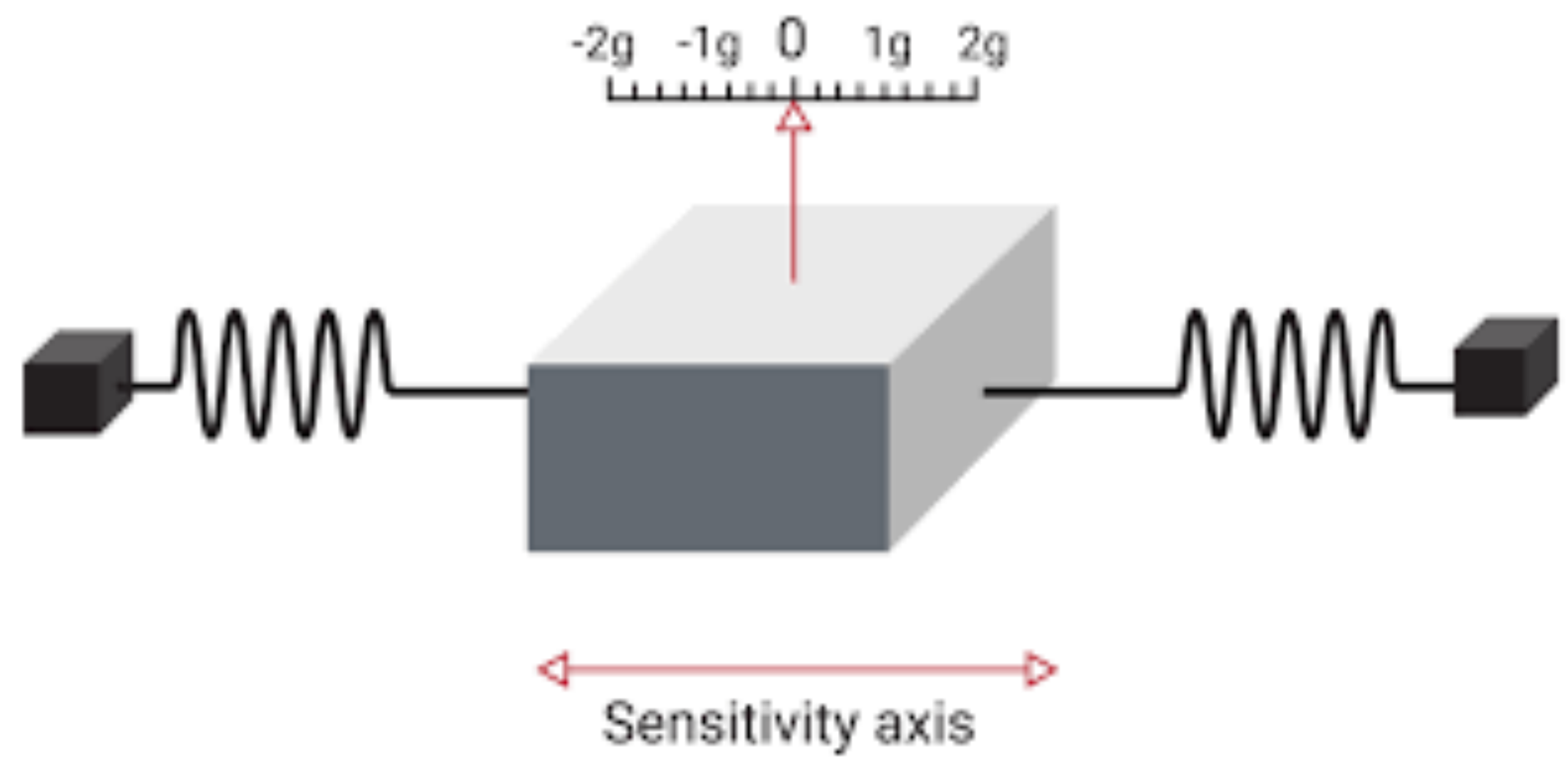
Bandwidth applies to all (linear) systems

many systems can be approximated as linear

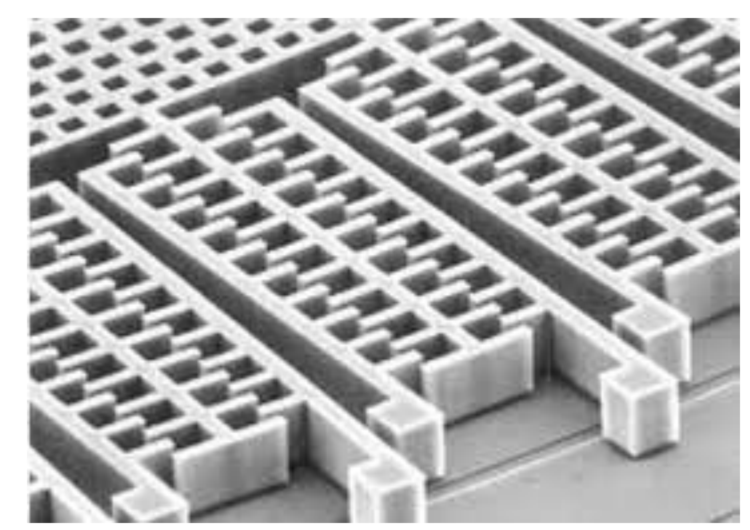




Accelerometer Frequency Response



actual device

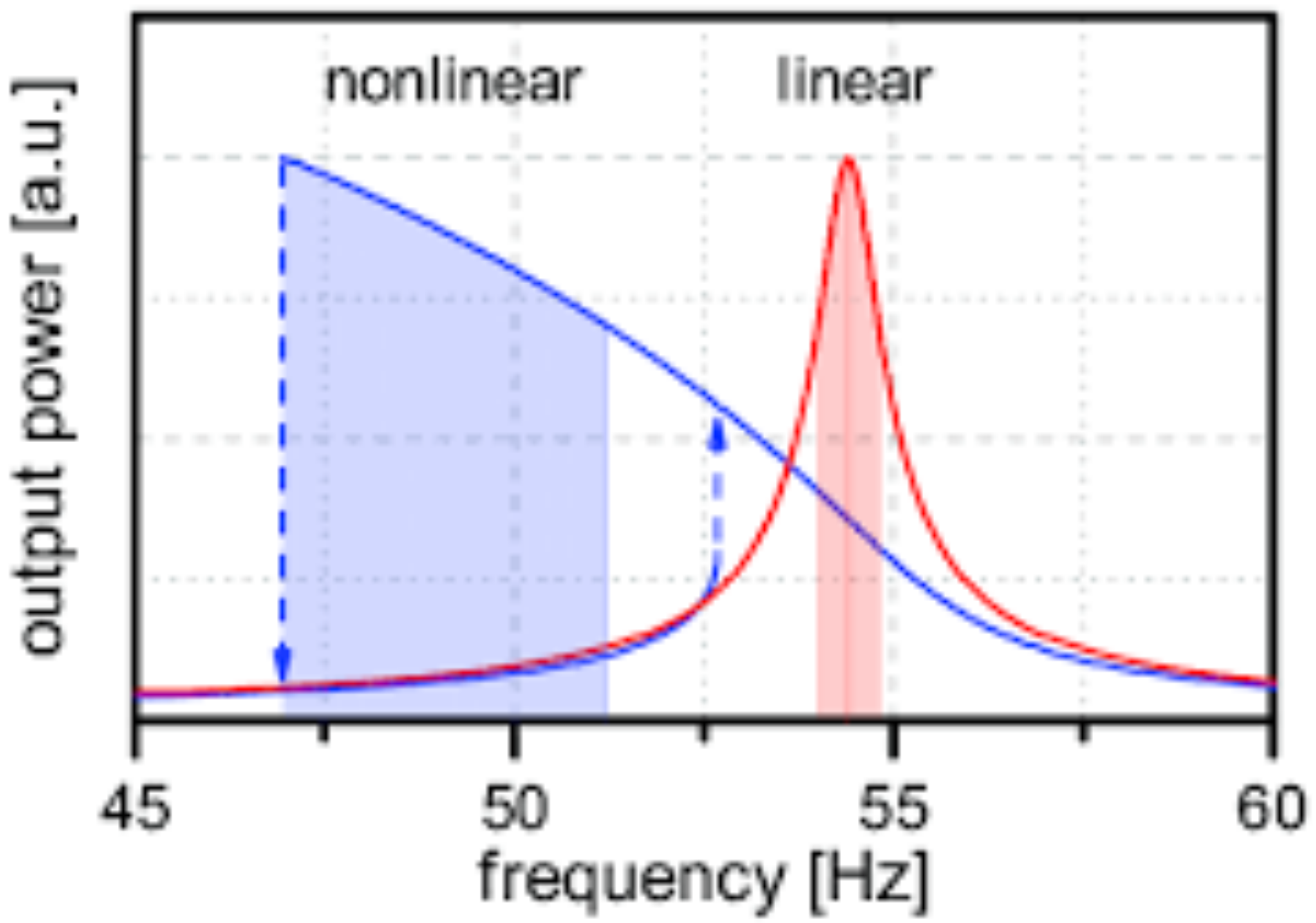


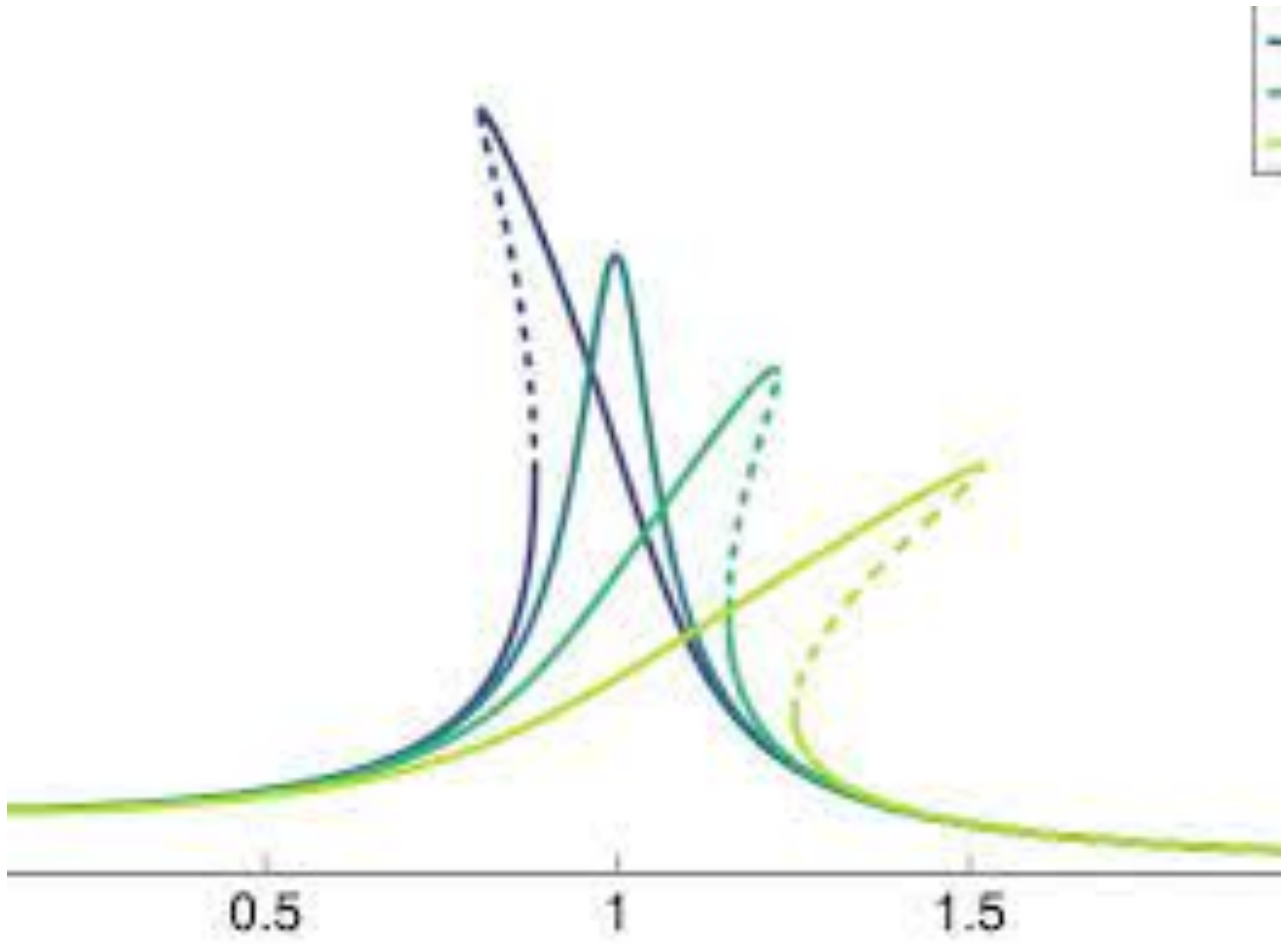
MEMS

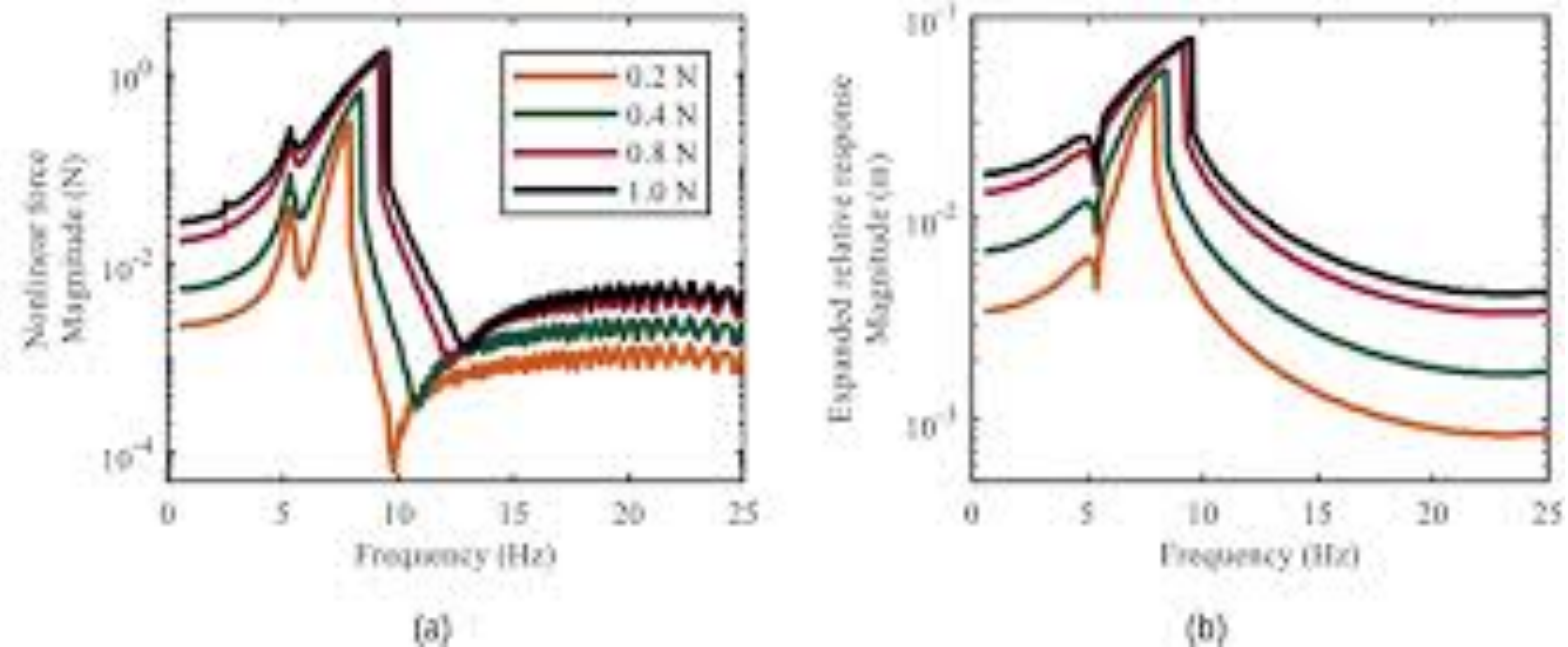
→ micro electromechanical system

nonlinear systems have ‘funky’ frequency response

often it is input amplitude dependent







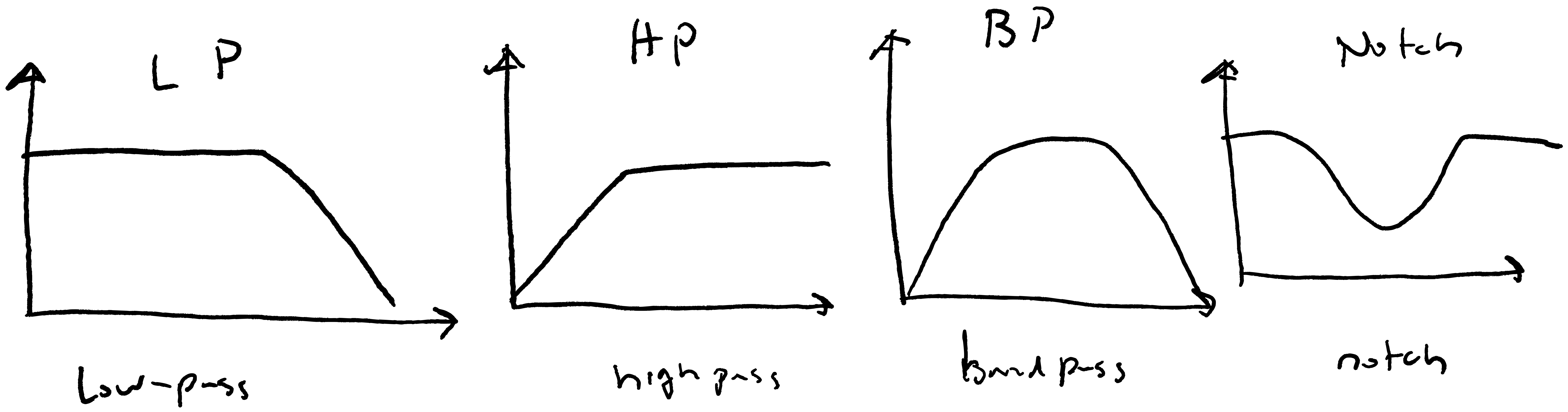
we won’t discuss these

Bandwidth is critical design consideration for all components

some components we can control the bandwidth

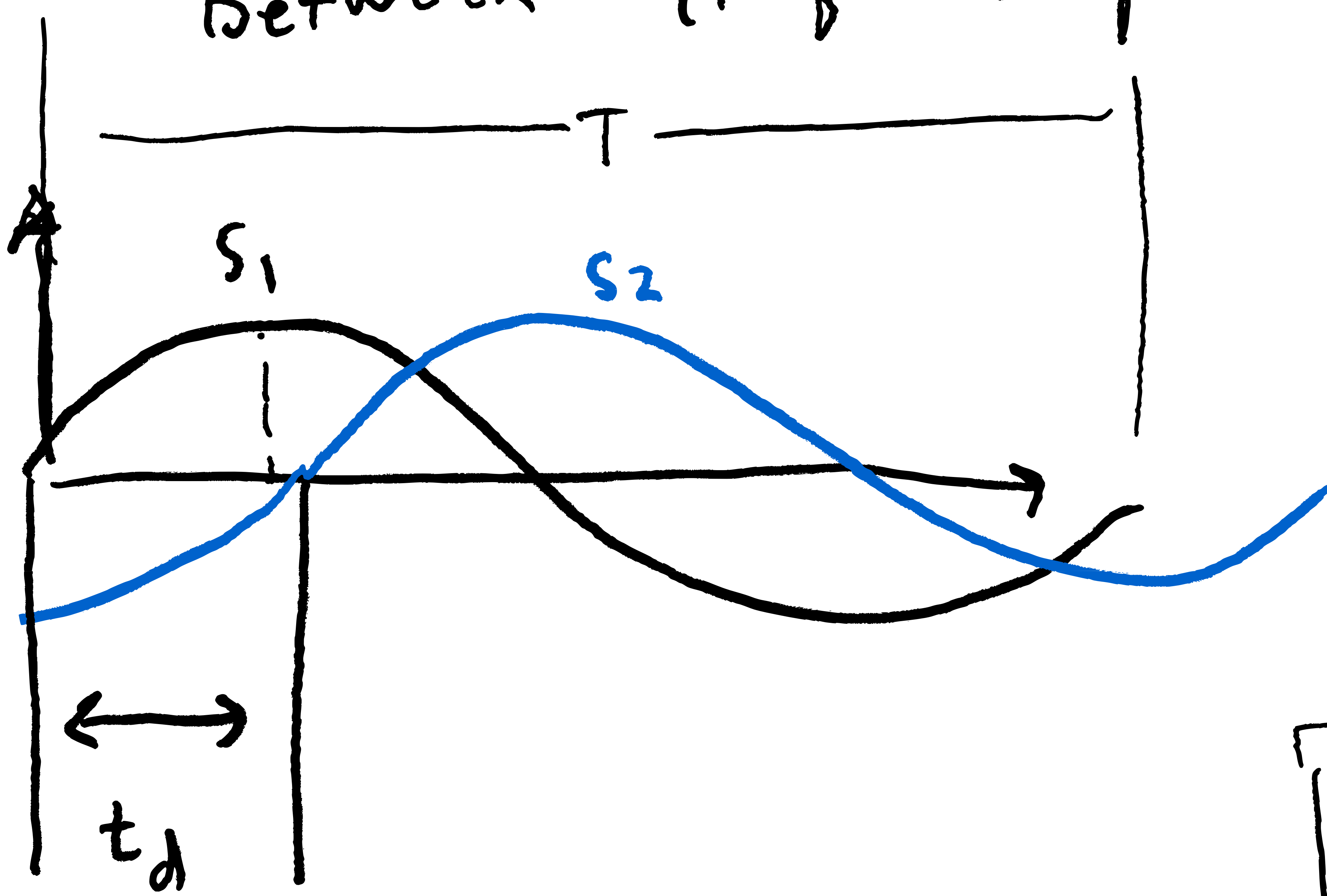
others we cannot

Filters — we design the bandwidth!



Phase linearity

→ how well a system preserves the phase relationship between freq. components



$T/4$

$s_1 = \sin(\omega t)$

$s_2 = \sin(\omega t + \phi)$

phase shift

$\phi = ?$

$= 360° \cdot \frac{t_d}{T}$

$= 360° \cdot \frac{T/4}{T}$

$= \underline{\underline{90°}}$

key idea: $T = \frac{1}{f}$

$$\boxed{\phi = 2\pi f \cdot t_d}$$

→ phase shift is freq. dependent.

→ we want equal time shifts (t_d) for all freq.

$$\boxed{\therefore \underbrace{\phi = K \cdot f}\ \text{phase linearity}}$$

phase shift (ϕ) is proportional to freq. → equal t_d for freq.

Phase linearity: Summary

When a system (measurement system) doesn't have phase & amplitude linearity the output signal is distorted!

Dynamic Characteristics of a System

What does it mean for something to be 'dynamic'? time dependence.

$$\boxed{\dot{x} = f(x, t)} \leftarrow \text{dynamics}$$

We will focus on linear systems:

$$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = \sum_{m=0}^{m} B_m \frac{d^m X_{in}}{dt^n}$$

A_n, B_n $\rightarrow$ constant!

$\rightarrow$ represent physical properties of the system

Dynamic Characteristics of a System

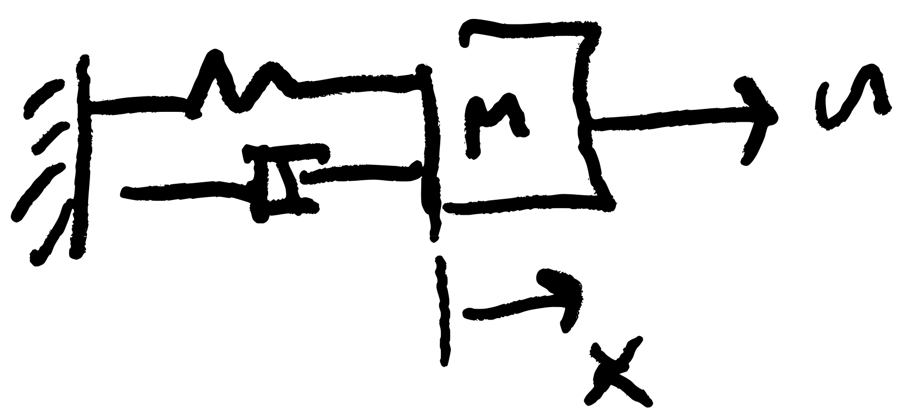
$$a_2 \ddot{x} + a_1 \dot{x} + a_0 x = b_1 \dot{u} + b_0 u$$

$$m \ddot{x} + b \dot{x} + K x = u$$

Forcing function

u = input

x = output



$$RC \dot{V}_o + V_o = V_{in}$$

low-pass

ODE can model all types of physical systems.

one of the most important tools for engineers!

Dynamic Characteristics of a System

Zero Order System

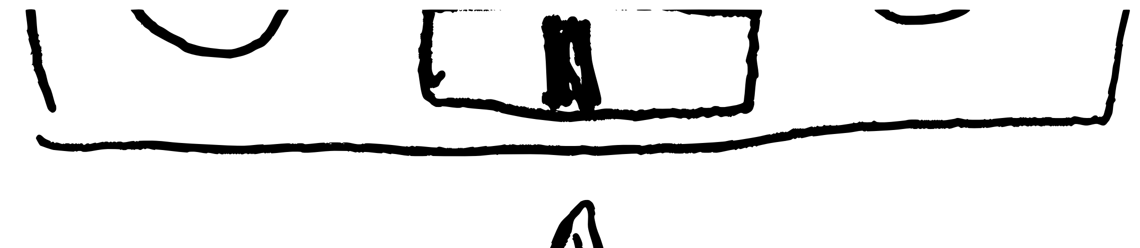
$$A_0 X_{out} = B_0 X_{in}$$

$$\Rightarrow X_{out} = \frac{B_0}{A_0} X_{in} = K X_{in}$$

K: gain or sensitivity

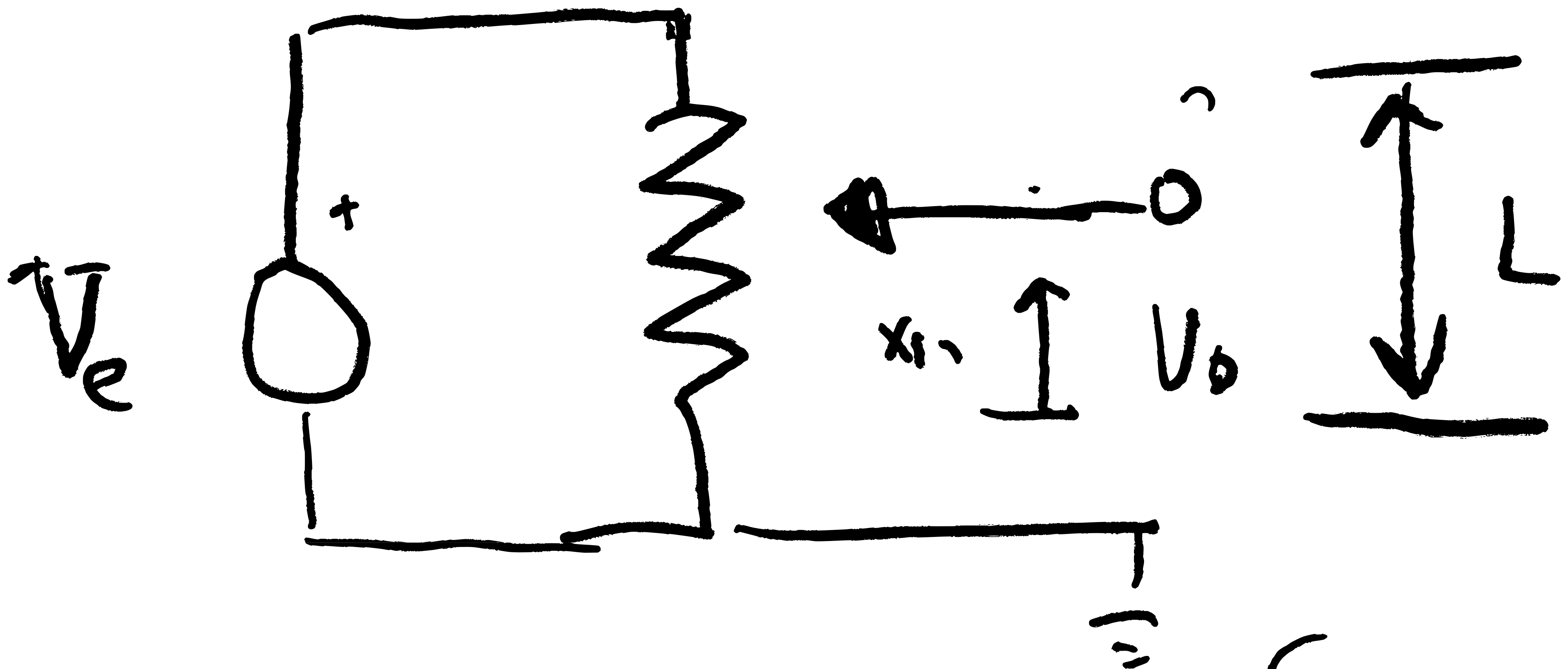
→ zero order systems have <u>no dynamics</u>

zero order systems have zero distortion.



Example: Potentiometer

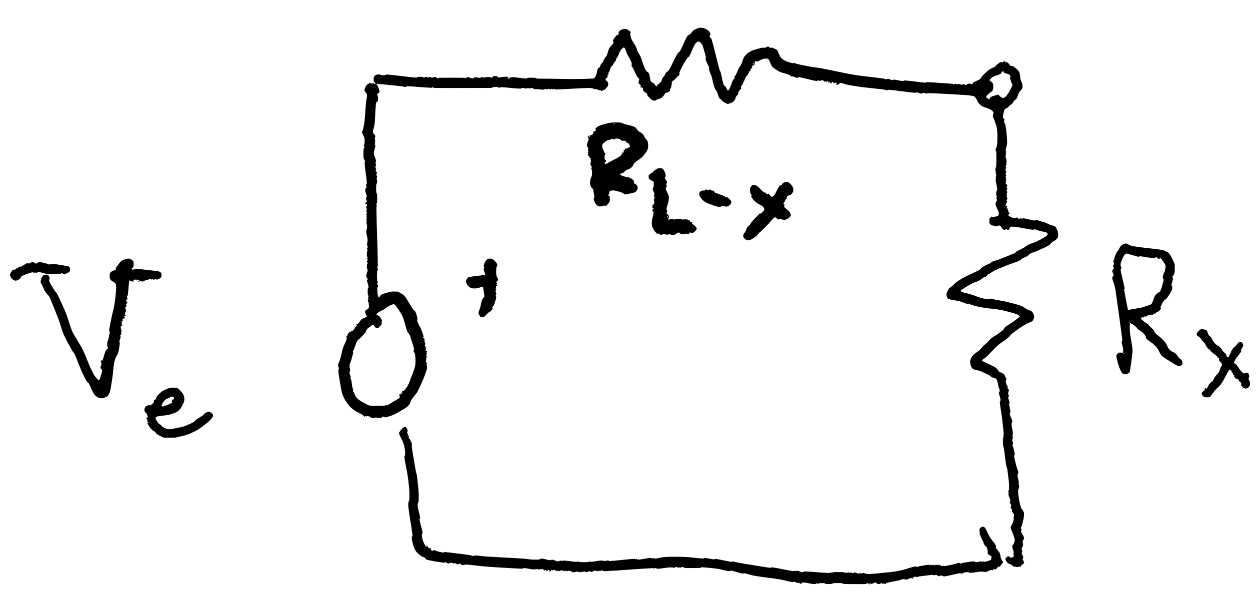
slide to changes which deck is active



$$V_{out} = \frac{R_x}{R_p} V_e$$

$$\begin{cases} R_{L-x} = R_p - R_x \\ R_x = \left(\frac{x_{in}}{L}\right) \cdot R_p \end{cases}$$

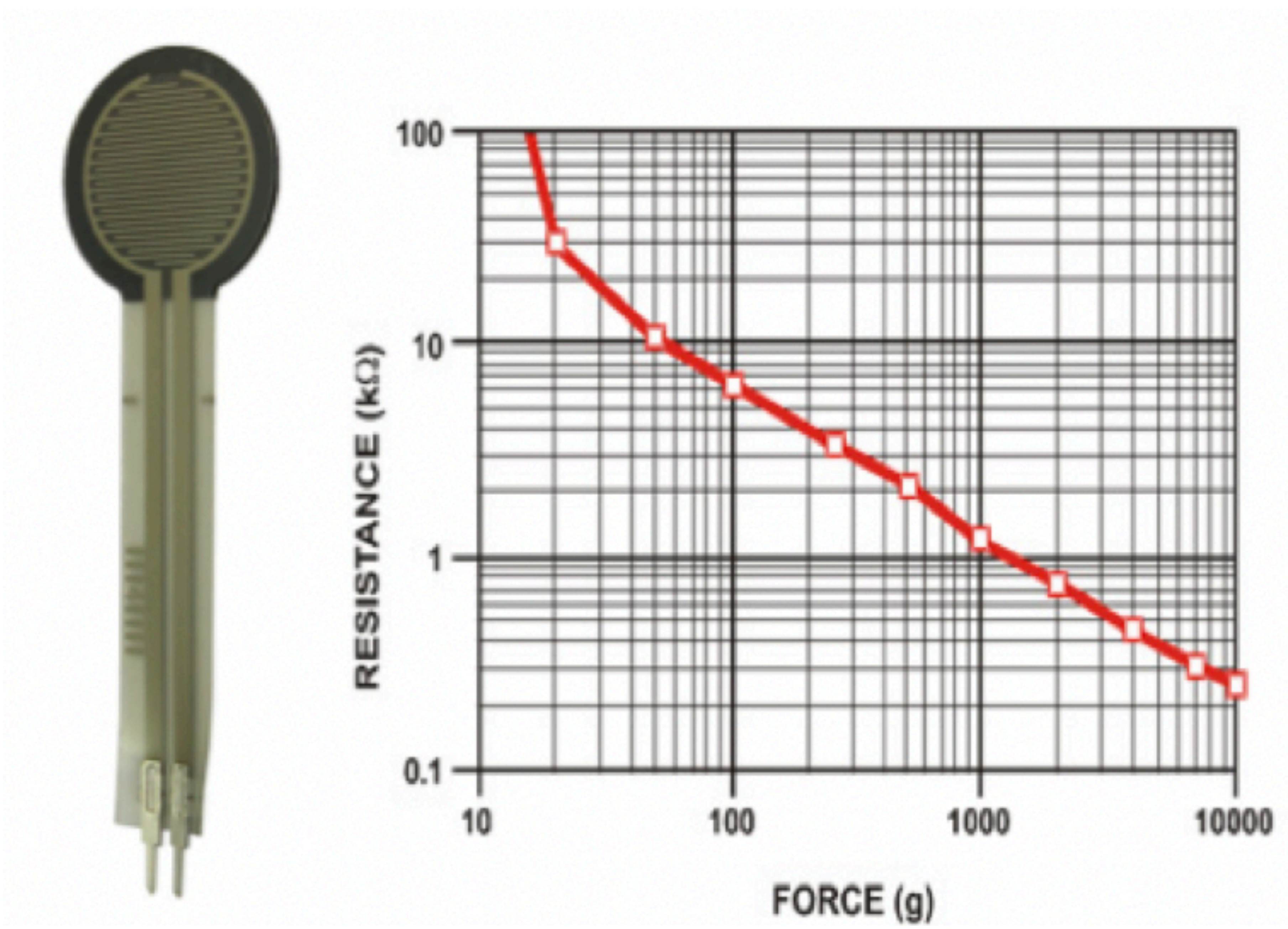
$$V_{out} = \underbrace{\left(\frac{V_e}{L}\right)}_{K \,:\, \text{gain}} \cdot X_{in}$$

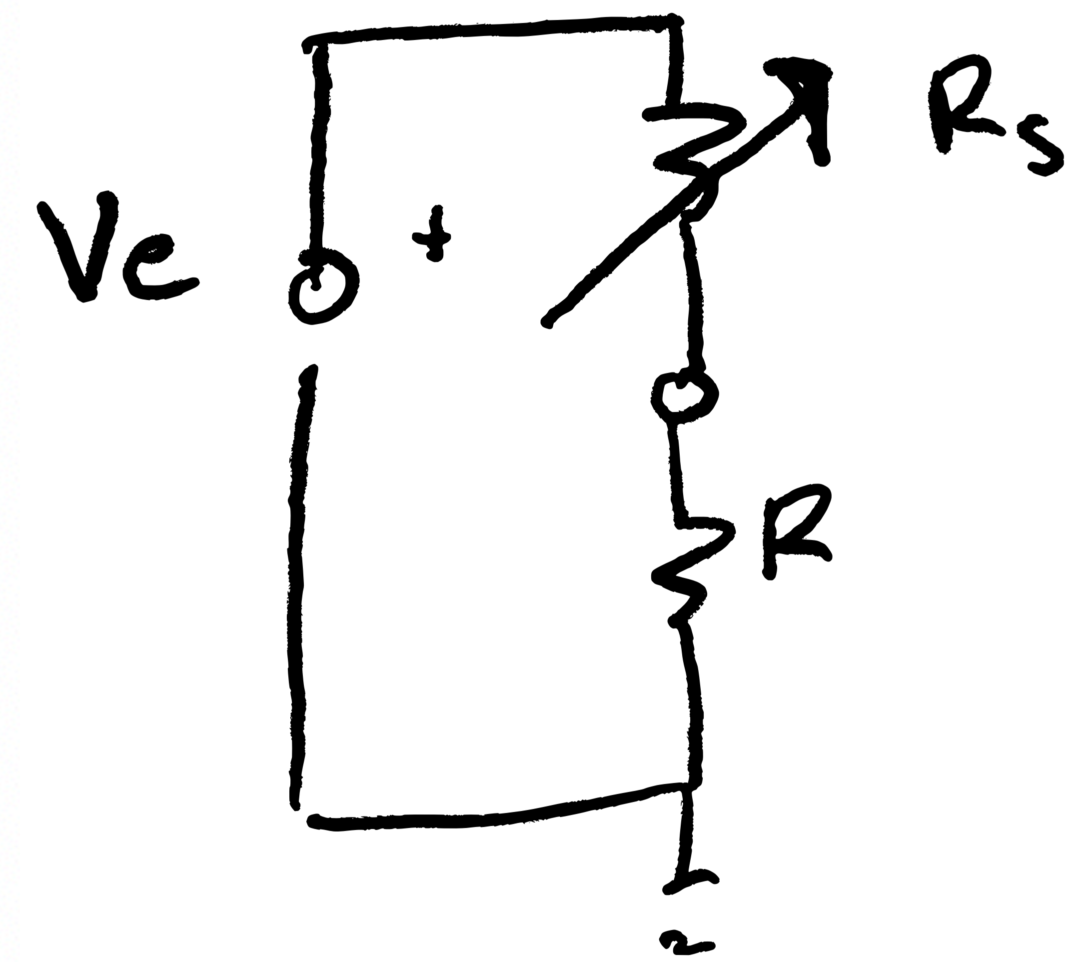


R_p : total resistance

R_x : the resistance of bottom leg of pot.

Example: FSR





$$V_o = \left(\frac{R}{R_s + R}\right) V_e$$

$$V_o = \frac{R}{f(\text{force}) + R} V_e$$

$$V_o = \frac{R}{f(F_{in}) + R} V_e$$

$R_s = f(\text{force})$

we know this function

$$V_o = \underbrace{\tilde{f}(F_{in}) \cdot V_e}_{\text{gain}}$$

Gain is nonlinear, but it is static

→ doesn't w/ time.

First order systems

$N = 1, \quad M = 0$

$$A_1 \frac{dX_{out}}{dt} + A_0 X_{out} = B_0 X_{in}$$

$$A_1 \dot{x} + A_0 x = B_0 u$$

Scalars

$$\dot{x} = \overbrace{-\frac{A_0}{A_1}}^{\text{Scalars}} x + \overbrace{\frac{B_0}{A_1}}^{\text{Scalars}} u$$

$$\boxed{\dot{x} = A x + B u}$$

state space form

all higher order ODEs can be written as a system of first order ODEs.

$$A_1 \dot{X}_{out} + A_0 X_{out} = B_0 X_{in}$$

$$\underbrace{\frac{A_1}{A_0}}_{\text{time constant } \tau} \dot{X}_{out} + X_{out} = \underbrace{\frac{B_0}{A_0}}_{\text{static gain } K} X_{in}$$

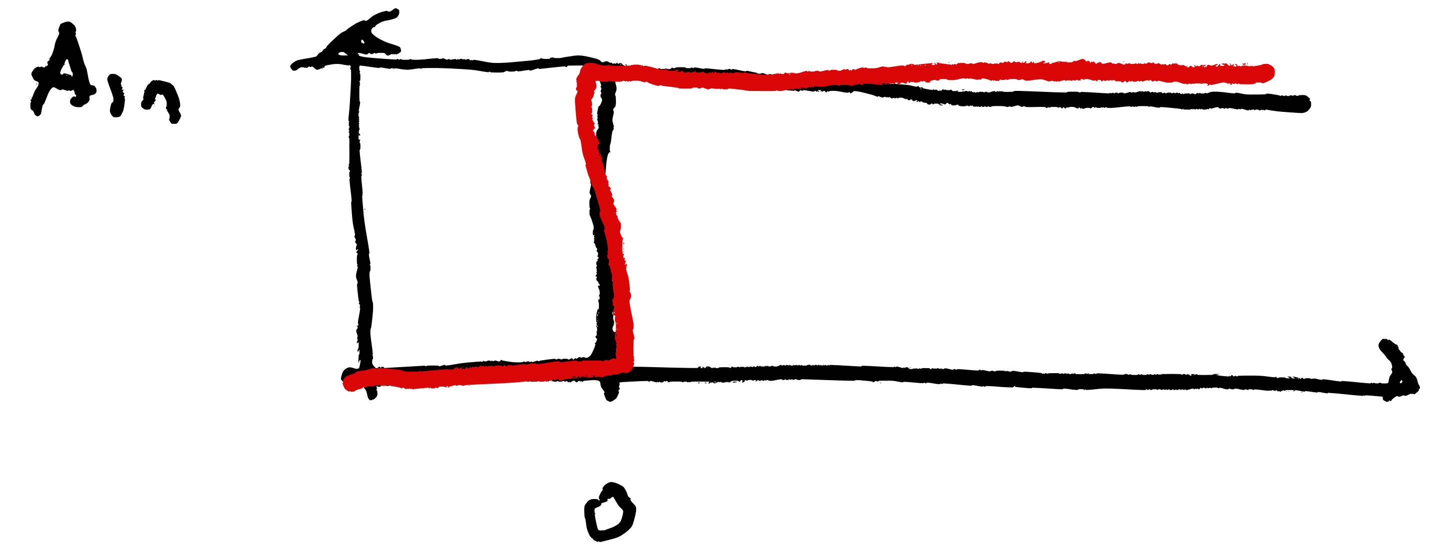
time constant τ

static gain K

$$\tau \dot{x}_{out} + x_{out} = K x_{in}$$

standard form

First Order Step Response



$$X_{in} = \begin{cases} 0 & t < 0 \\ A_{in} & t \geq 0 \end{cases}$$

Solution:

means $x(t)$ for all t.

Assume $x_{out} = Ce^{st}$

$x_{out}(0) = 0$

write the characteristic eq:

$\tau s + 1 = 0$, $s = -1/\tau$

homogeneous solution: $X_{out} = Ce^{-t/\tau}$

Particular Solution ($\dot{x} = 0$)

$X_{out_p} = KA_{in}$

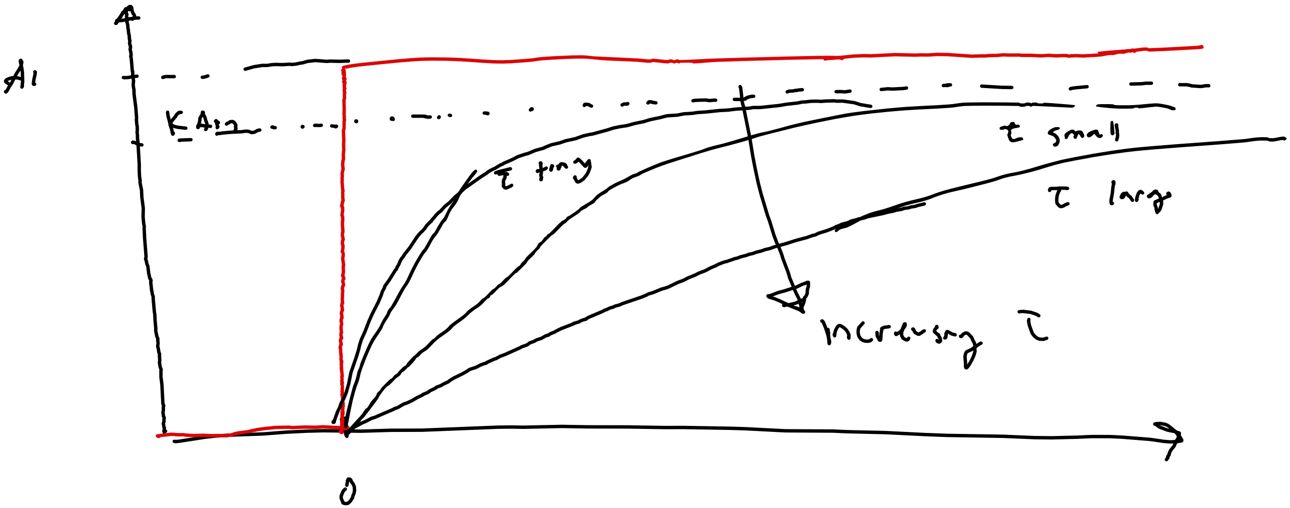
$x_{out} = Ce^{-t/\tau} + KA_{in}$

use our I.C.

$0 = C + KA_{in}$

$C = -KA_{in}$

$$\boxed{X_{out} = KA_{in}\left(1 - e^{-t/\tau}\right)}$$


A1
K A1n
τ tiny
τ small
τ large
increasing τ
0

First Order Step Response

$$X_{out}(t) = KA_{in}\left(1 - e^{-t/\tau}\right)$$

$$X_{out}(t = \tau) = KA_{in}\underbrace{\left(1 - e^{-1}\right)}_{0.6321\ldots}$$

after 1 time constant the output signal reaches 63% of steady state

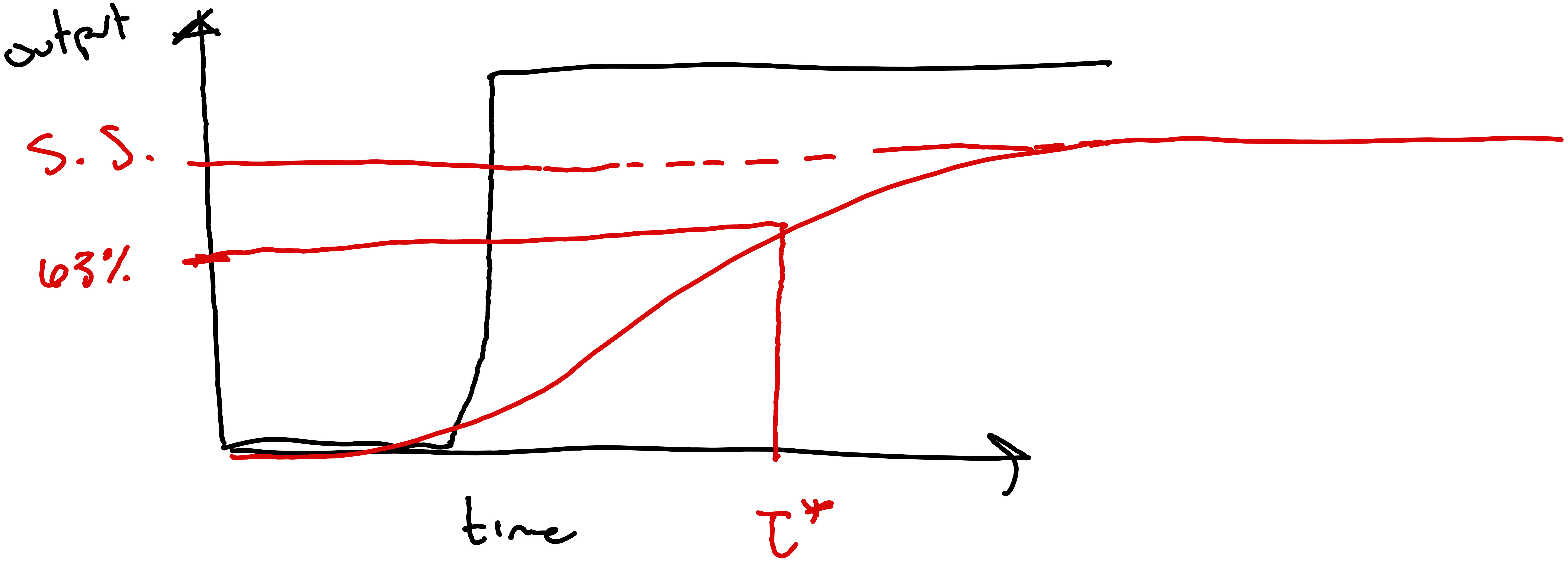
$$X_{out}(t = 4\tau) = KA_{in}\underbrace{\left(1 - e^{-4}\right)}_{0.982}$$

after 4 time constants System reaches steady state

Sytem Identification

How to estimate τ ?

$\longrightarrow$ step response



Sytem Identification

$$\frac{X_{out} - KA_m}{KA_m} = -e^{-t/\tau}$$

$$1 - \frac{X_{out}}{KA_m} = e^{-t/\tau}$$

$$\ln\left(1 - \frac{X_{out}}{KA_m}\right) = -t/\tau$$

$$Z = -t/\tau$$ ← natural log of output

$$\frac{dZ}{dt} = -1/\tau$$