5.5. ★ The limiting distribution for the results in some hypothetical measurement is given by the triangular function shown in Figure 5.18, where the value of $f(0)$ is

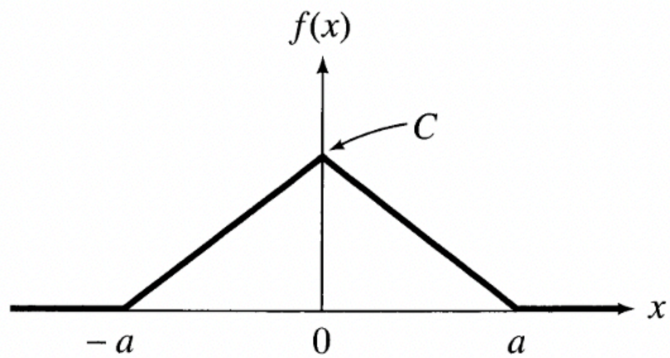

Figure 5.18. A triangular distribution; for Problem 5.5.

called C. **(a)** What is the probability of a measurement outside the range between $x = -a$ and $x = a$? **(b)** What is the probability of a measurement with $x > 0$? **(c)** Use the normalization condition (5.13) to find C in terms of a. **(d)** Sketch this function for the cases that $a = 1$ and $a = 2$.

5.7. ★ For the exponential distribution of Problem 5.6, what is the probability for a result $t > \tau$? What for $t > 2\tau$? (Notice that these probabilities also give the fraction of the original atoms that live longer than τ and 2τ.)

5.9. ★★ The limiting distribution for the results in some hypothetical measurement has the form

$$f(x) = \begin{cases} C & \text{for } |x| < a \\ 0 & \text{otherwise.} \end{cases}$$

(a) Use the normalization condition (5.13) to find C in terms of a. **(b)** Sketch this function. Describe in words the distribution of results governed by this distribution. **(c)** Use Equations (5.15) and (5.16) to calculate the mean and standard deviation that would be found after many measurements.

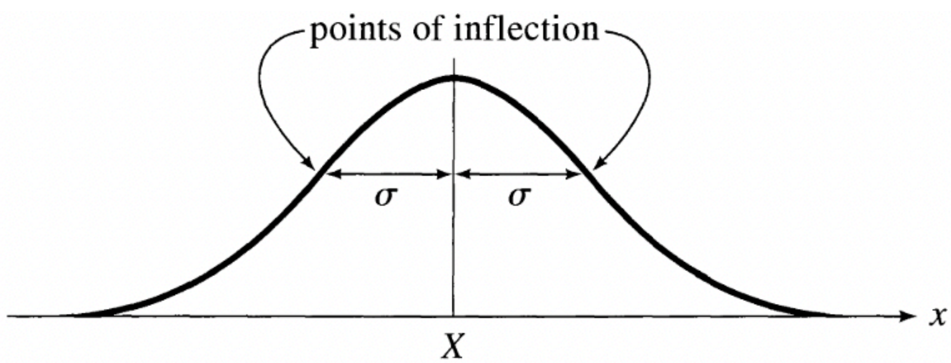

Figure 5.20. The points $X \pm \sigma$ are the points of inflection of the Gauss curve; for Problem 5.13.

5.13. ★ One way to define the width σ of the Gauss distribution is that the points $X \pm \sigma$ are the two points of inflection (Figure 5.20), where the curvature changes sign; that is, where the second derivative is zero. Prove this claim.

5.21. ★★ The Giraffe Club of Casterbridge is a club for young adults (18–24 years) who are unusually tall. There are 2,000 women between the ages of 18 and 24 in Casterbridge, and the heights of women in this age range are distributed normally with a mean of $5'5\frac{1}{2}''$ and standard deviation of $2\frac{1}{2}''$. **(a)** If the Giraffe Club initially sets its minimum height for women at $5'10''$, approximately how many women are eligible to join? **(b)** A year or so later, the club decides to double its female membership by lowering the minimum height requirement; what would you recommend that the club set as its new minimum height for women (to the nearest half inch)?

5.23. ★★★ We have seen that for the normal distribution, the standard deviation gives the 68% confidence range. This result is not necessarily true for other distributions, as the following problem illustrates: Consider the exponential distribution of Problem 5.6, $f(t) = (1/\tau)e^{-t/\tau}$ (for $t \geqslant 0$; $f(t) = 0$ for $t < 0$). The parameter τ is the mean value of t (that is, after many measurements, $\bar{t} = \tau$). **(a)** Use the integral (5.16) to prove that τ is also the standard deviation, $\sigma_t = \tau$. (This result is a noteworthy property of this distribution—that the mean value is equal to the standard deviation.) **(b)** By doing the necessary integral, find the probability that any one value would fall in the range $\bar{t} \pm \sigma_t$.

5.25. ★ A student measures a time t eight times with the following results (in tenths of a second):

Value, t_k:	75	76	77	78	79	80
Occurrences, n_k:	2	3	0	0	2	1

(a) Assuming these measurements are normally distributed, what should be your best estimates for the true value and the standard deviation? **(b)** Based on these estimates, what is the probability that a ninth measurement would be 81 or more? (Because the measurements are rounded to the nearest integer, this probability is that for a value $t \geqslant 80.5$.)

5.27. ★★ **(a)** Based on the data of Problem 5.24, find the mean, the standard deviation, and the uncertainty of your value for the SD [the last using Equation (5.46)]. **(b)** Recalculate the probability asked for in part (b) of Problem 5.24 assuming the true value of σ is $\sigma_x - \delta\sigma_x$, and, once again, assuming σ is really $\sigma_x + \delta\sigma_x$. Comment on the difference in your two answers here.

5.31. ★★ Listed here are 40 measurements $t_1, \ldots, t_{40}$ of the time for a stone to fall from a window to the ground (all in hundredths of a second).

63	58	74	78	70	74	75	82	68	69
76	62	72	88	65	81	79	77	66	76
86	72	79	77	60	70	65	69	73	77
72	79	65	66	70	74	84	76	80	69

(a) Compute the standard deviation σ_t for the 40 measurements. **(b)** Compute the means $\bar{t}_1, \ldots, \bar{t}_{10}$ of the four measurements in each of the 10 columns. You can think of the data as resulting from 10 experiments, in each of which you found the *mean of four timings.* Given the result of part (a), what would you expect for the standard deviation of the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$? What is it? **(c)** Plot histograms for the 40 individual measurements $t_1, \ldots, t_{40}$ and for the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$. [Use the same scales and bin sizes for both plots so they can be compared easily. Bin boundaries can be chosen in various ways; perhaps the simplest is to put one boundary at the mean of all 40 measurements (72.90) and to use bins whose width is the standard deviation of the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$.]