9.1. ★ Calculate the covariance for the following four measurements of two quantities x and y.

x:	20	23	23	22
y:	30	32	35	31

9.3 ★★ **(a)** For the data of Problem 9.1, calculate the variances $\sigma_x{}^2$ and $\sigma_y{}^2$ and the covariance σ_{xy}. **(b)** If you now decide to calculate the sum $q = x + y$, what will be its standard deviation according to (9.9)? **(c)** What would you have found for the standard deviation if you had ignored the covariance and used Equation (9.10)?

9.5. ★★ Imagine a series of N measurements of two fixed lengths x and y that were made to find the value of some function $q(x, y)$. Suppose each pair is measured with a different tape; that is, the pair (x_1, y_1) is measured with one tape, (x_2, y_2) is measured with a second tape, and so on. **(a)** Assuming the main source of errors is that some of the tapes have shrunk and some stretched (uniformly, in either case), show that the covariance σ_{xy} is bound to be positive. **(b)** Show further, under the same conditions, that $\sigma_{xy} = \sigma_x \sigma_y$; that is, σ_{xy} is as large as permitted by the Schwarz inequality (9.11).

[Hint: Assume that the ith tape has shrunk by a factor λ_i, that is, present length = (design length)/λ_i, so that a length that is really X will be measured as $x_i = \lambda_i X$. The moral of this problem is that there are situations in which the covariance is certainly not negligible.]

9.9. ★ Calculate the correlation coefficient r for the following six pairs of measurements:

$$x = 1 \quad 2 \quad 3 \quad 5 \quad 6 \quad 7$$

$$y = 5 \quad 6 \quad 6 \quad 8 \quad 8 \quad 9$$

Do the calculations yourself, but if your calculator has a built-in function to compute r, make sure you know how it works, and use it to check your value.

9.11. ★ In the photoelectric effect, the kinetic energy K of electrons ejected from a metal by light is supposed to be a linear function of the light's frequency f,

$$K = hf - \phi, \tag{9.20}$$

where h and ϕ are constants. To check this linearity, a student measures K for N different values of f and calculates the correlation coefficient r for her results. **(a)** If she makes five measurements ($N = 5$) and finds $r = 0.7$, does she have significant support for the linear relation (9.20)? **(b)** What if $N = 20$ and $r = 0.5$?

9.15. ★★ Draw a scatter plot for the six data pairs of Problem 9.9 and the least-squares line that best fits these points. Find their correlation coefficient r. Based on the probabilities listed in Appendix C, would you say these data show a significant linear correlation? Highly significant?