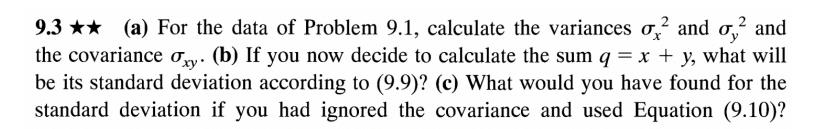
9.1. ★ Calculate the covariance for the following four measurements of two quantities x and y.

x: 20 23 23 22

y: 30 32 35 31



9.5. $\star\star$ Imagine a series of N measurements of two fixed lengths x and y that were made to find the value of some function q(x, y). Suppose each pair is measured with a different tape; that is, the pair (x_1, y_1) is measured with one tape, (x_2, y_2) is measured with a second tape, and so on. (a) Assuming the main source of errors is that some of the tapes have shrunk and some stretched (uniformly, in either case), show that the covariance σ_{xy} is bound to be positive. (b) Show further, under the same conditions, that $\sigma_{xy} = \sigma_x \sigma_y$; that is, σ_{xy} is as large as permitted by the Schwarz inequality (9.11).

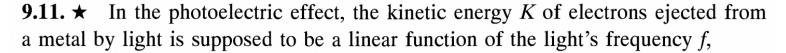
[Hint: Assume that the *i*th tape has shrunk by a factor λ_i , that is, present length = (design length)/ λ_i , so that a length that is really X will be measured as $x_i = \lambda_i X$. The moral of this problem is that there are situations in which the covariance is certainly not negligible.]

9.9. \star Calculate the correlation coefficient r for the following six pairs of measurements:

$$x = 1 \quad 2 \quad 3 \quad 5 \quad 6 \quad 7$$

$$y = 5 \quad 6 \quad 6 \quad 8 \quad 8 \quad 9$$

Do the calculations yourself, but if your calculator has a built-in function to compute r, make sure you know how it works, and use it to check your value.



$$K = hf - \emptyset, \tag{9.20}$$

where h and \emptyset are constants. To check this linearity, a student measures K for N different values of f and calculates the correlation coefficient r for her results. (a) If she makes five measurements (N = 5) and finds r = 0.7, does she have significant support for the linear relation (9.20)? (b) What if N = 20 and r = 0.5?

