ME170b Lecture 2 1/20/23

Experimental Techniques

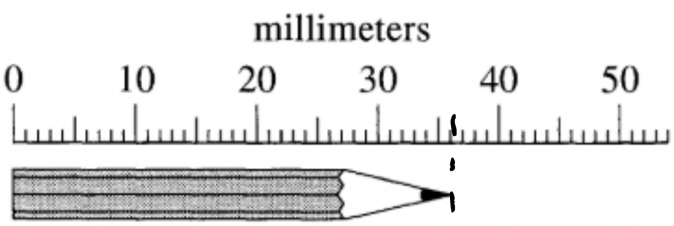
Last time:

- Hypotheses
- lab report
- uncertainties

Today: CH. 2

- standard form
- discrepancy
- fractional uncertainty
- graphical methods
- difference & multiplication of measurements

How should uncertainties in an experiment be reported?



x

measured value $x = x_{best} \pm \delta x$

experimenter's best estimate

uncertainty

highest & lowest probable value

* In general, "95% confidence interval (CI)"

CH. 4

How many significant figures for uncertainty?

$$\boxed{g = 9.82 \pm 0.02 \ m/s^2}$$

what's wrong with this estimate:

$$(\text{measured } g) = 9.82 \pm 0.02385 \text{ m/s}^2.$$

no good!

δx is an estimate → should not be stated with too much precision.

$\boxed{\delta x \text{ have almost always one significant figure.}}$

These are ‘rules of thumb’ — should use best judgment

If our estimate $\delta x = 0.14$

one sigfig: $\delta x = 0.1$

That's nearly 50% difference.

if leading digit is 1 (or 2) it okay to have 2 sig fig.

Significant figures of the estimate is determined <u>after</u> uncertainty

what's wrong with this estimate:

measured speed $=$ 6051.78 ± 30 m/s $\rightarrow$ 6050 ± 30

[redacted]

The last sig fig of best estimate should be same order of magnitude as the uncertainty.

One caveat

any measurement that will used in a calculation should retain at least one more sig. fig. than finally justified.

Scientific Notations

$$\text{measured charge} = 1.61 \times 10^{-19} \pm 5 \times 10^{-21} \text{ coulombs.}$$

$$(1.61 \pm 0.05) \times 10^{-19} \text{ coulombs}$$

Simpler & clearer to put best estimate & uncertainty in same form.

Quick Check 2.2. Rewrite each of the following measurements in its most appropriate form:

(a) $v = 8.123456 \pm 0.03\cancel{12}$ m/s

(b) $x = 3.1234 \times 10^4 \pm 2$ m

(c) $m = 5.6789 \times 10^{-7} \pm 3 \times 10^{-9}$ kg.

a. $v = 8.12 \pm 0.03$ m/s

b. 31234 ± 2 m or $3.1234 \pm 0.0002 \times 10^{4}$ m

c. $5.68 \pm 0.03 \times 10^{-7}$ kg.

Discrepancy

The difference between two measured values of the same quantity

A: $15 \pm 1\ \Omega$

B: $25 \pm 2\ \Omega$

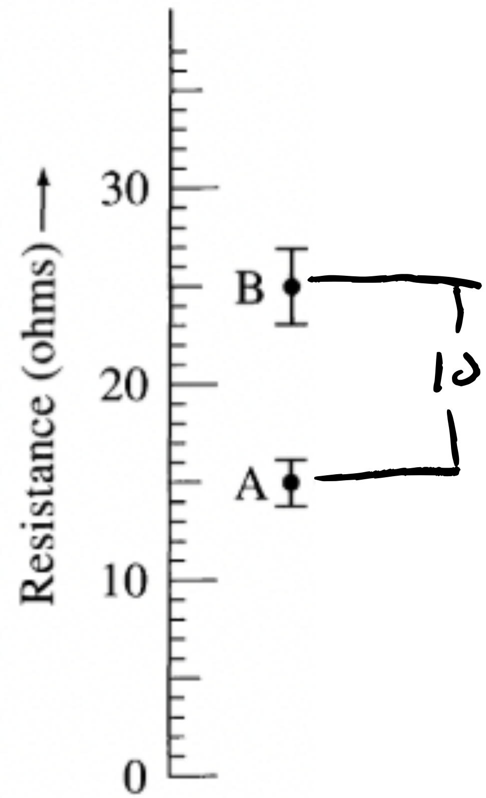
discrepany $= 25 - 15 = 10\ \Omega$

Is this a "significant" difference?

Discrepancy

Student A: 15 ± 1 ohms

Student B: 25 ± 2 ohms,



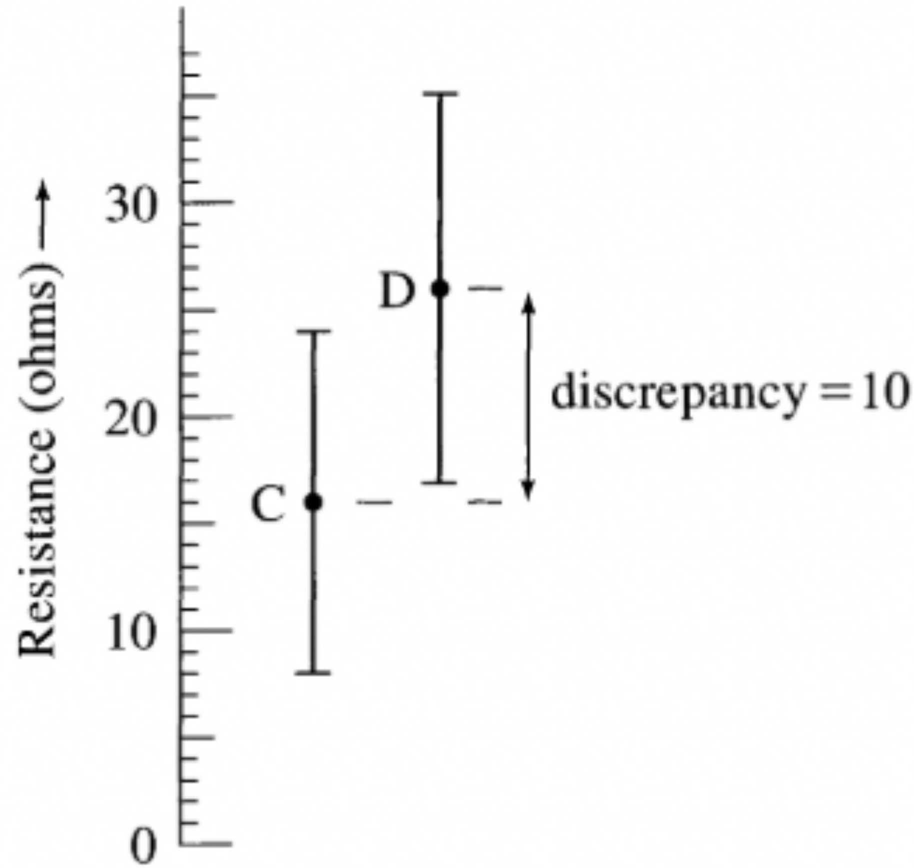
Is it *significant*?
Why or why not?

Yes, it is "significant", because the discrepancy is incompatible with both measurement.

Discrepancy

Student C: 16 ± 8 ohms

Student D: 26 ± 9 ohms.



Is it *significant*?
Why or why not?

No, because the true value is compatible with both measurements

True Error

the difference between a measured value & the **true value**

→ true value can **almost never** be known

ex: accepted value of universal gas constant

$$R = 8.31451 \pm 0.00007 \text{ J/mol}\cdot\text{K}$$

Can you think of something that we know true value?

$$\pi = C/D$$

Experiments without drawing a conclusion (testing a hypothesis) has no merit.

- A single measurement for an experiment by itself is not interesting.
- Experiments should compare two or more numbers.

Ex. (a) measurement compared with theoretical value. H: The measurement = theory

(b.) several measurements & how they relate to eachother through a physical law.

Example: Experiment for measuring a quantity with a known value

H: the measurement will be equal to the known value.

Procedure:

1. measure the quantity
2. estimate the uncertainty
3. compare with accepted value

A: $v = 329 \pm 5$ m/s

$v^* = 331$ m/s

How to present results?

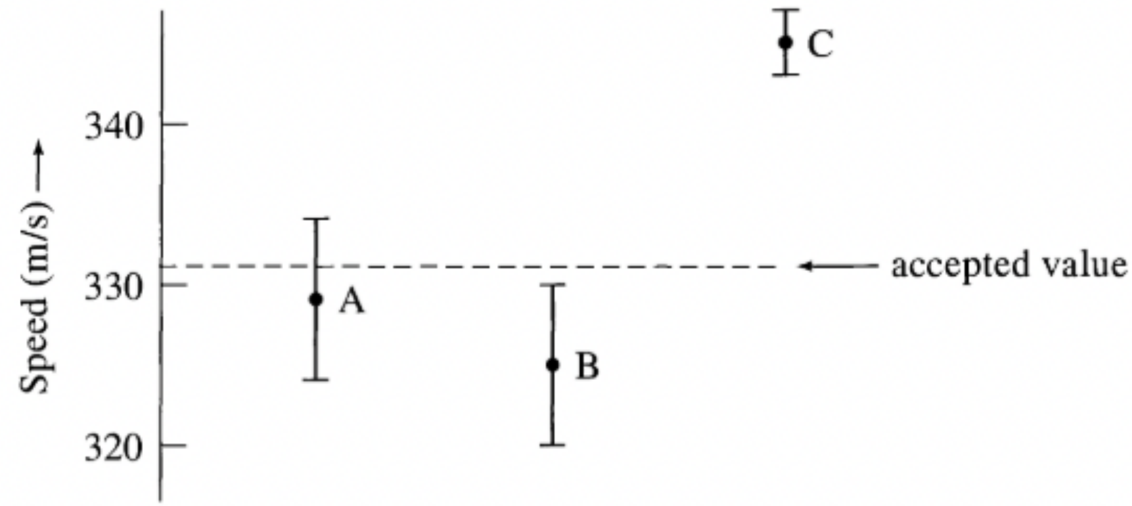
Example: Experiment for measuring a quantity with a known value

H: the measurement with be equal to the known value

A's measured speed $= 329 \pm 5$ m/s,

B's measured speed $= 325 \pm 5$ m/s,

C's measured speed $= 345 \pm 2$ m/s



accepted

A: "The expected value lies within our margin error, (see Fig. 1), therefore the data supports our hypothesis."

C: "The discrepancy is 14 m/s, approx. 7 times the uncertainty in our measurement – reject H."

Example: An experiment <u>comparing two measurements</u>

H: the measurements will be equal.

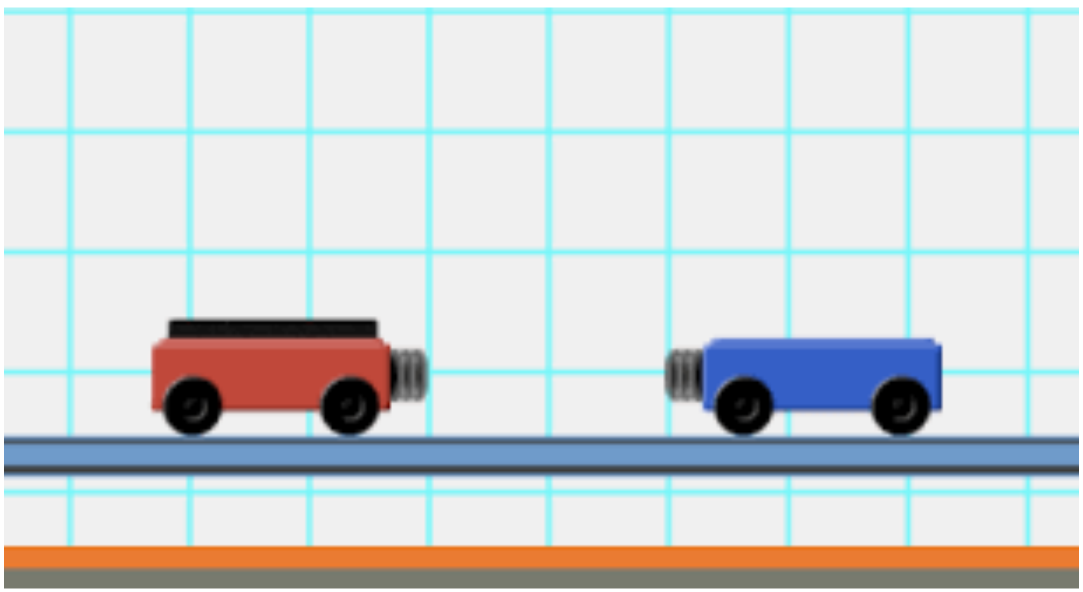
Conservation of momentum: total momentum of an isolated system is constant.

Example: An experiment comparing two measurements

H: the measurements with be equal

Conservation of momentum states total momentum of isolated system is constant

We can use an experiment with two measurements to confirm theory.



$p = mv$

$p_1 = p_2 \leftarrow H$

before after

measure mass

measure velocity before

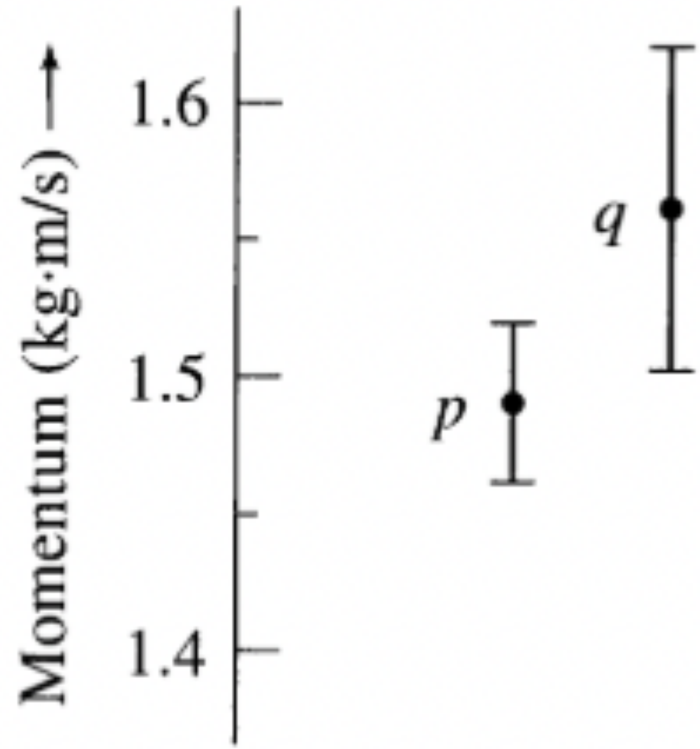
measure velocity after

Example: An experiment comparing two measurements

H: the measurements with be equal

initial momentum $p = 1.49 \pm 0.03$ kg·m/s

final momentum $q = 1.56 \pm 0.06$ kg·m/s.



Does data support H?

The measurement overlap! - confirming H

→ "this data is consistent w/ theory"

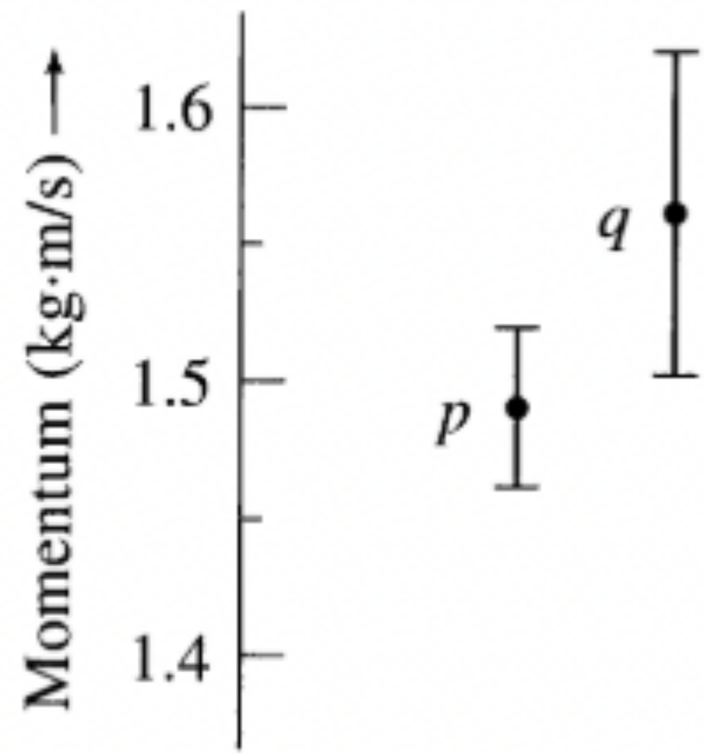
Example: An experiment comparing two measurements

H: the measurements with be equal

initial momentum $p = 1.49 \pm 0.03$ kg·m/s

final momentum $q = 1.56 \pm 0.06$ kg·m/s.

Does the data support our hypothesis? Why/why not?



Example: An experiment <u>comparing two measurements</u>

Another way is to subtract p - q — the result should be zero!

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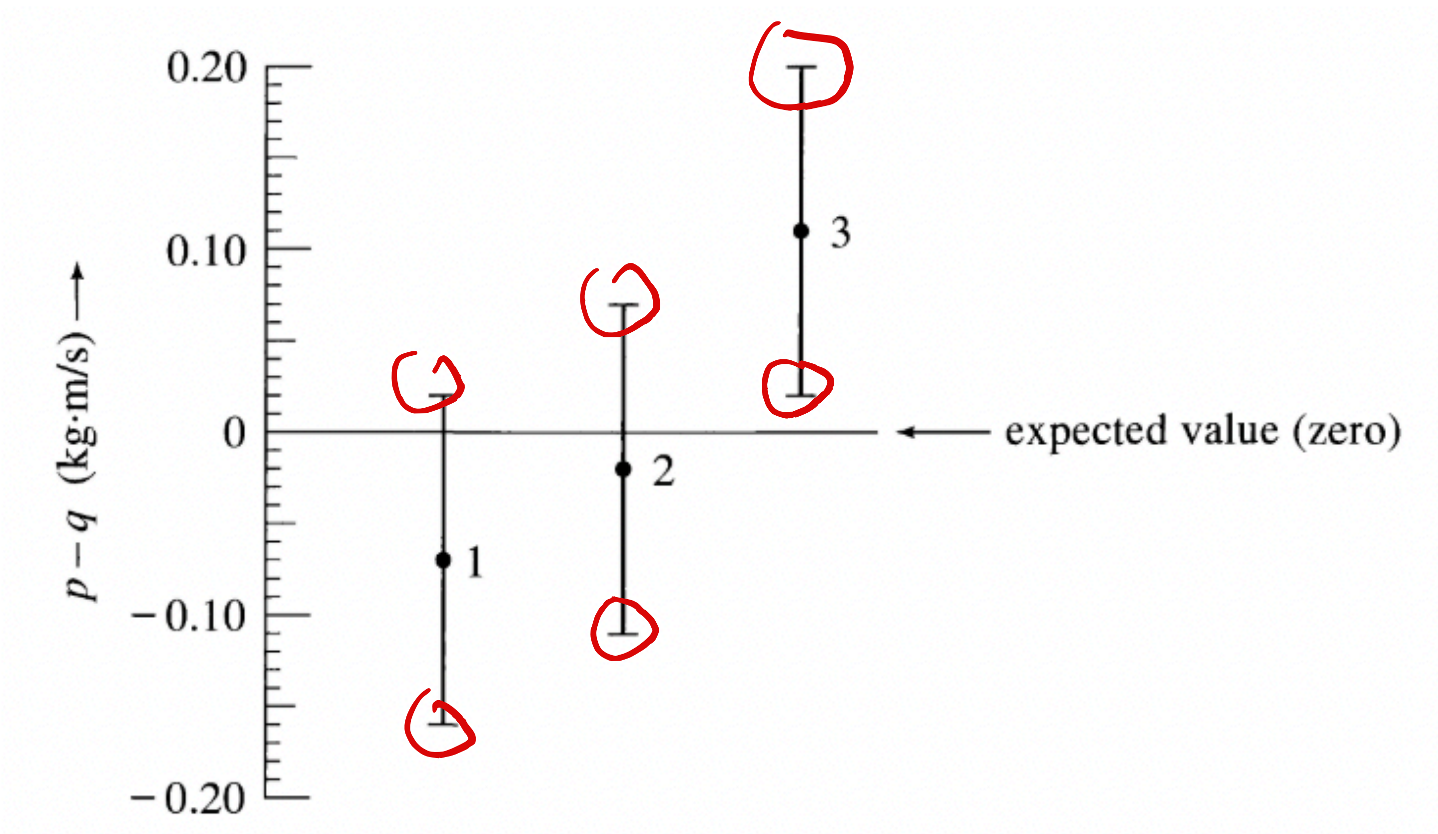

Uncertainty in Difference (provisional rule)

Uncertainty in Difference

$$\underbrace{x \;; \; y}_{\text{best estimates}} \quad ; \quad \underbrace{\delta x \quad \delta y}_{\text{uncertainty}}$$

$$q = x - y$$

$$\delta q \approx \delta x + \delta y$$

Single measurements are not reliable!

What is the best way to display repeated measurements?

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Table 2.1. Measured momenta (kg·m/s).

Trial number	Initial momentum p (all ± 0.03)	Final momentum q (all ± 0.06)
1	1.49	1.56
2	3.10	3.12
3	2.16	2.05
etc.		

Difference

± 0.09

Quick Check 2.3. In an experiment to measure the latent heat of ice, a student adds a chunk of ice to water in a styrofoam cup and observes the change in temperature as the ice melts. To determine the mass of ice added, she weighs the cup of water before and after she adds the ice and then takes the difference. If her two measurements were

$$\text{(mass of cup \& water)} = m_1 = 203 \pm 2 \text{ grams}$$

and

$$\text{(mass of cup, water, \& ice)} = m_2 = 246 \pm 3 \text{ grams,}$$

find her answer for the mass of ice, $m_2 - m_1$, with its uncertainty, as given by the provisional rule (2.18).

$\boxed{43 \pm 5\ g}$

Graphical Methods

Physical Laws typically imply a relationship between quantities

Ex. Hookes Law

$$F = Kx \Rightarrow x = F/K$$

> hang mass → measure displacement

H: the data will obey Hookes Law

Graphical Methods

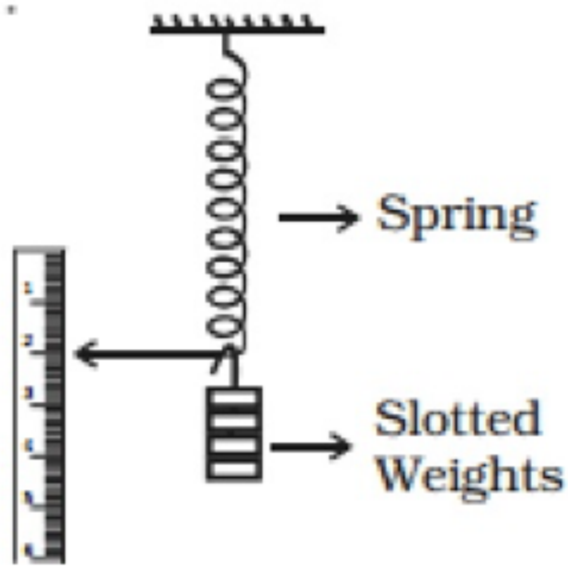


Fig. Experimental setup to verify Hooke's law

$$F = Kx \Rightarrow x = F/K$$

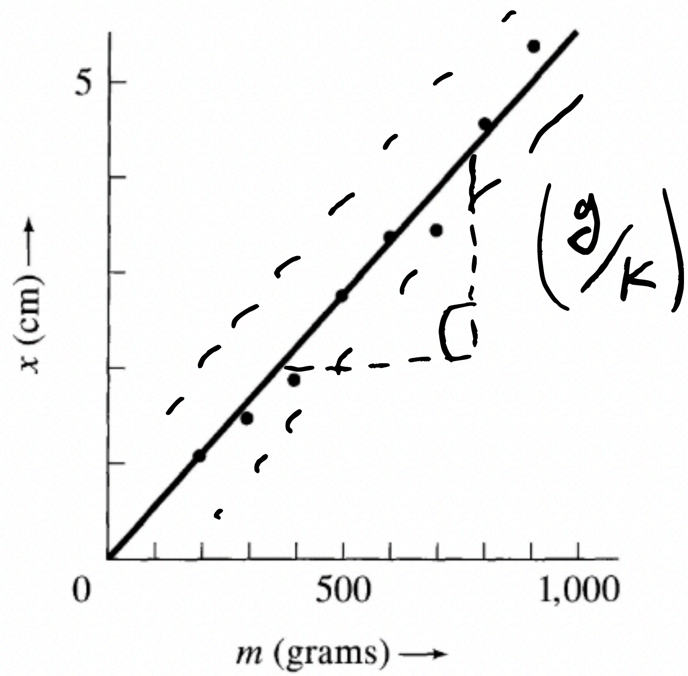
$$F = mg$$

$$x = mg/K = \left(\frac{g}{K}\right)m$$

Graphical Methods

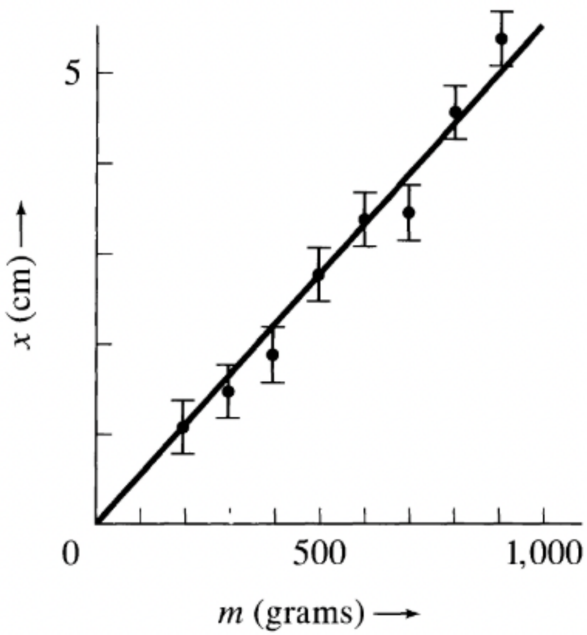
Table 2.3. Load and extension.

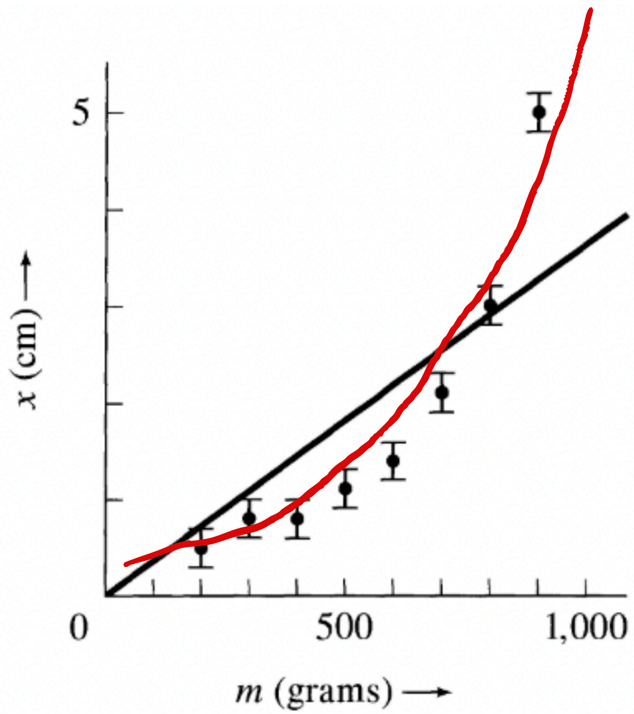
Load m (grams) (δm negligible)	200	300	400	500	600	700	800	900
Extension x (cm) (all ± 0.3)	1.1	1.5	1.9	2.8	3.4	3.5	4.6	5.4



Graphical Methods

Better to include error bars

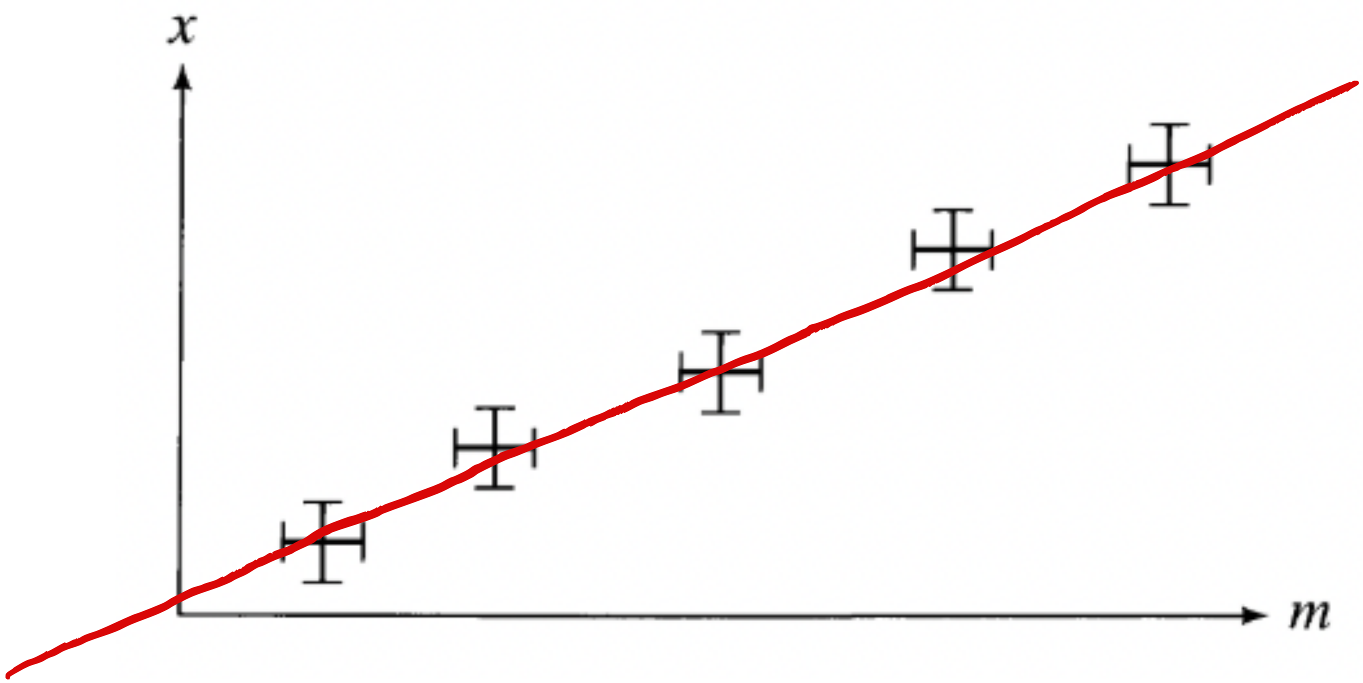




Left plot does [redacted] support hypothesis (k = constant)

Graphical Methods

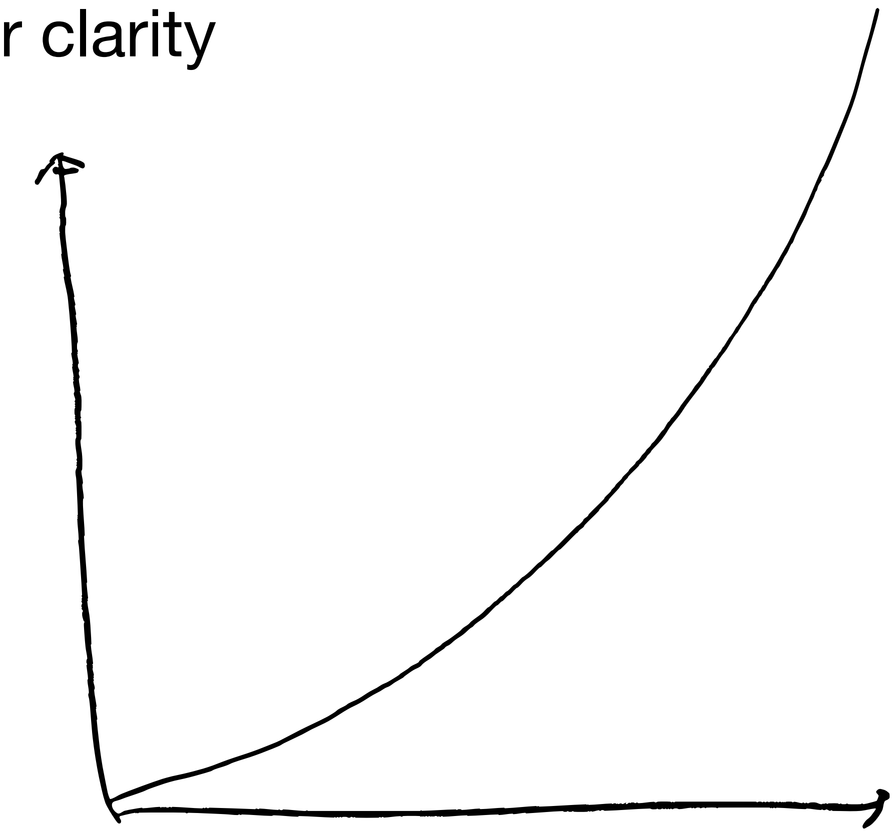
What if mass also has uncertainty?



Graphical Methods

Different ways to show data for clarity

$$y = Ax^2$$



Graphical Methods

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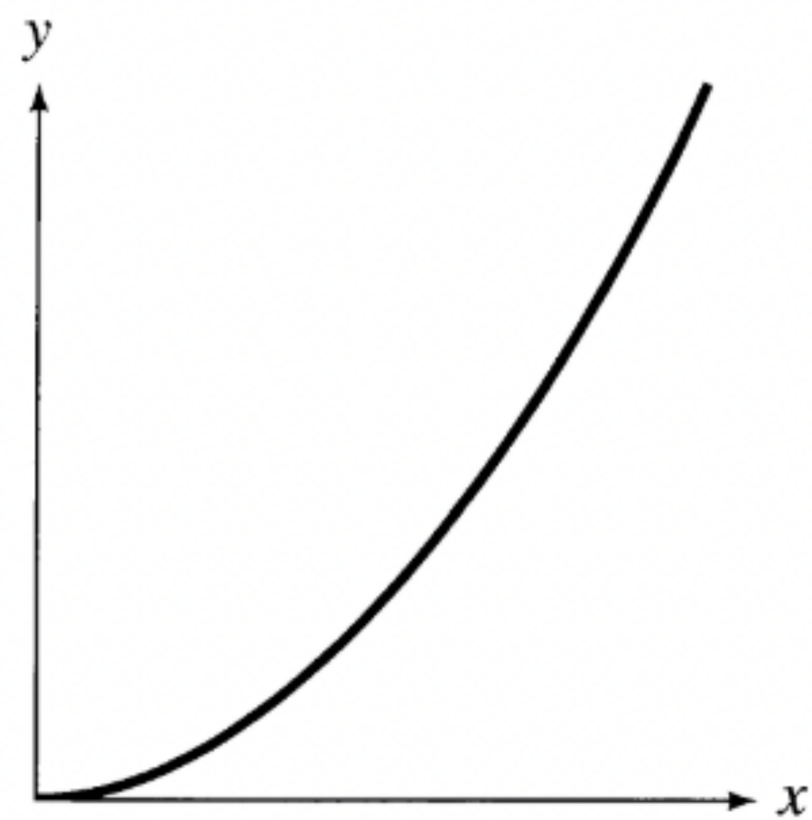
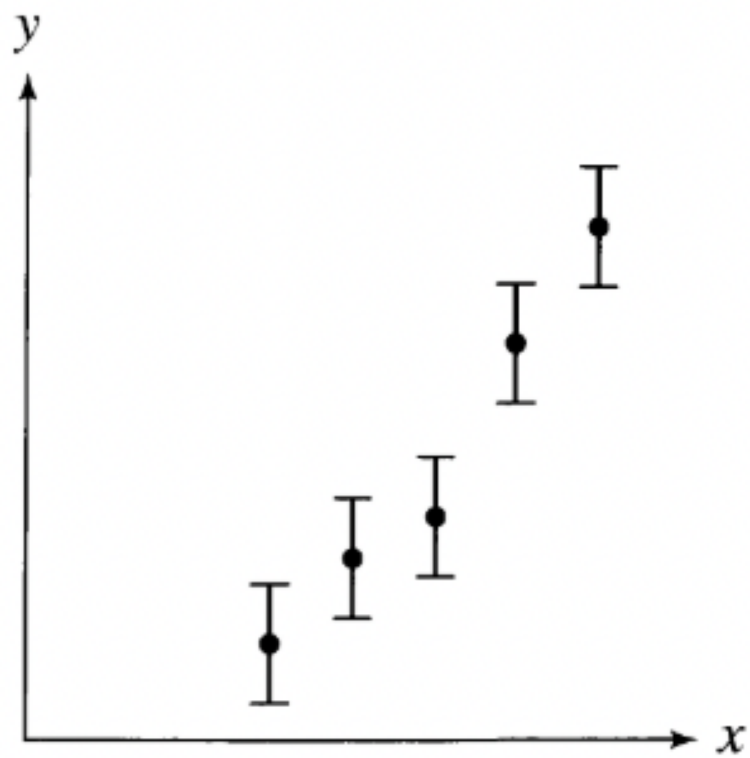


Graphical Methods

Different ways to show data for clarity



x^2

Does follow

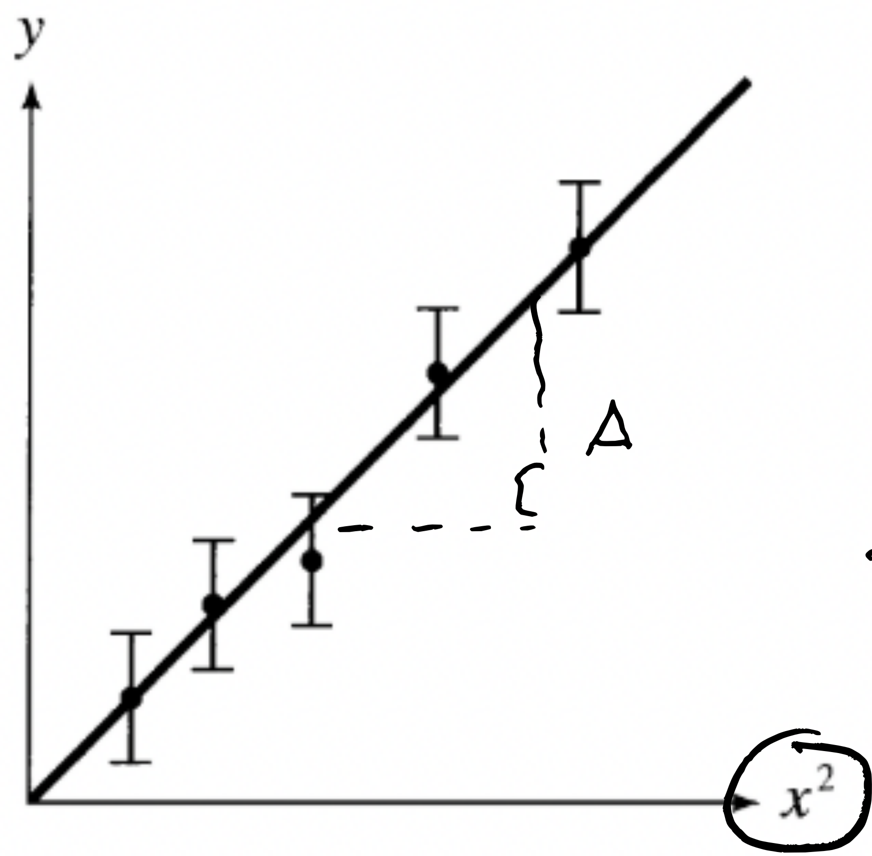
$y = Ax^2$?

→ its hard to

determined

Graphical Methods

Different ways to show data for clarity



We can plot y vs. x^2 which has linear relationship

$$y = Ae^{Bx}$$

$$\rightarrow \ln(y) = \ln(A) + Bx$$

Fractional uncertainty

$$x = x_{best} \pm \underline{\underline{\delta x}}$$

→ indicating the relibability on precision of the measurement.

$$FV = \frac{\delta x}{|x_{best}|}$$

relative uncertainty

Fractional uncertainty allows a more intuitive sense of uncertainty

$$l = 50 \pm 1 \quad cm$$

$$\frac{\delta l}{|l_{best}|} = \frac{1\,cm}{50\,cm} = 0.02$$

$$l = 50\,cm \pm 2\%$$

FU indicates the quality of a measurement

10% or more $\rightarrow$ rough estimate

Quick Check 2.4. Convert the errors in the following measurements of the velocities of two carts on a track into fractional errors and percent errors: **(a)** $v = 55 \pm 2$ cm/s; **(b)** $u = -20 \pm 2$ cm/s. **(c)** A cart's kinetic energy is measured as $K = 4.58$ J $\pm$ 2%; rewrite this finding in terms of its absolute uncertainty. (Because the uncertainties should be given to one significant figure, you ought to be able to do the calculations in your head.)

a. $v = 55 \pm 3\%$ cm/s

b. $u = -20 \pm 10\%$ cm/s

c. $K = 4.58 \pm 0.09$ J

Fractional Uncertainty and Significant Figures

Fractional Uncertainty and Significant Figures

Table 2.4. Approximate correspondence between significant figures and fractional uncertainties.

Number of significant figures	Corresponding fractional uncertainty is	
	between	or roughly
1	10% and 100%	50%
2	1% and 10%	5%
3	0.1% and 1%	0.5%

Multiplying measurements

$$x = x_{best} \pm \delta x$$

$$x = x_{best}\left(1 \pm \frac{\delta x}{|x_{best}|}\right)$$

$p = mv$

$$m = m_{best}\left(1 \pm \frac{\delta m}{|m_{best}|}\right)$$

$$v = v_{best}\left(1 \pm \frac{\delta v}{|v_{best}|}\right)$$

Multiplying measurements

$$p = p_{best}\left(1 + \frac{\delta p}{|p_{best}|}\right)$$ we want!

$$p_{best} = m_{best} v_{best}$$

largest value of p:

$$p_{large} = m_{best} v_{best} \underbrace{\left(1 + \frac{\delta m}{|m_{best}|}\right)\left(1 + \frac{\delta v}{|v_{best}|}\right)}$$

$$\left(1 + \frac{\delta m}{|m_{best}|}\right)\left(1 + \frac{\delta v}{|v_{best}|}\right) = 1 + \frac{\delta m}{|m_{best}|} + \frac{\delta v}{|v_{best}|} + \cancel{\frac{\delta m \, \delta v}{|m_{best}| \, |v_{best}|}}$$

Multiplying measurements

$$P = m_{best} v_{best} \left(1 \pm \left[\frac{\delta m}{|m_{best}|} + \frac{\delta v}{|v_{best}|} \right] \right)$$

For multiplication:

$$\frac{\delta q}{|q_{best}|} \approx \frac{\delta x}{|x_{best}|} + \frac{\delta y}{|y_{best}|}$$

$$q = x \cdot y$$

Multiplying measurements

Quick Check 2.5. To find the area of a rectangular plate, a student measures its sides as $l = 9.1 \pm 0.1$ cm and $b = 3.3 \pm 0.1$ cm. Express these uncertainties as percent uncertainties and then find the student's answer for the area $A = lb$ with its uncertainty. (Find the latter as a percent uncertainty first and then convert to an absolute uncertainty. Do all error calculations in your head.)