ME170b Lecture 3 1/27/23

Experimental Techniques

Last time:
- > Standard Form
- > Discrepancy
- > Fractional Uncertainty
- > Graphical Methods
- > Difference and multiplication

Today:
- > Uncertainty in measurements Review
- > Square root rule
- > Revisit sum/difference prod/quotient
- > Independence
- > uncertainty in functions
- > General Forumula

Propagation of Uncertainties

We already discussed basic propagation

Uncertainty in a Difference
(Provisional Rule)

If two quantities x and y are measured with uncertainties δx and δy, and if the measured values x and y are used to calculate the difference $q = x - y$, the *uncertainty in* q is the *sum of the uncertainties in* x *and* y:

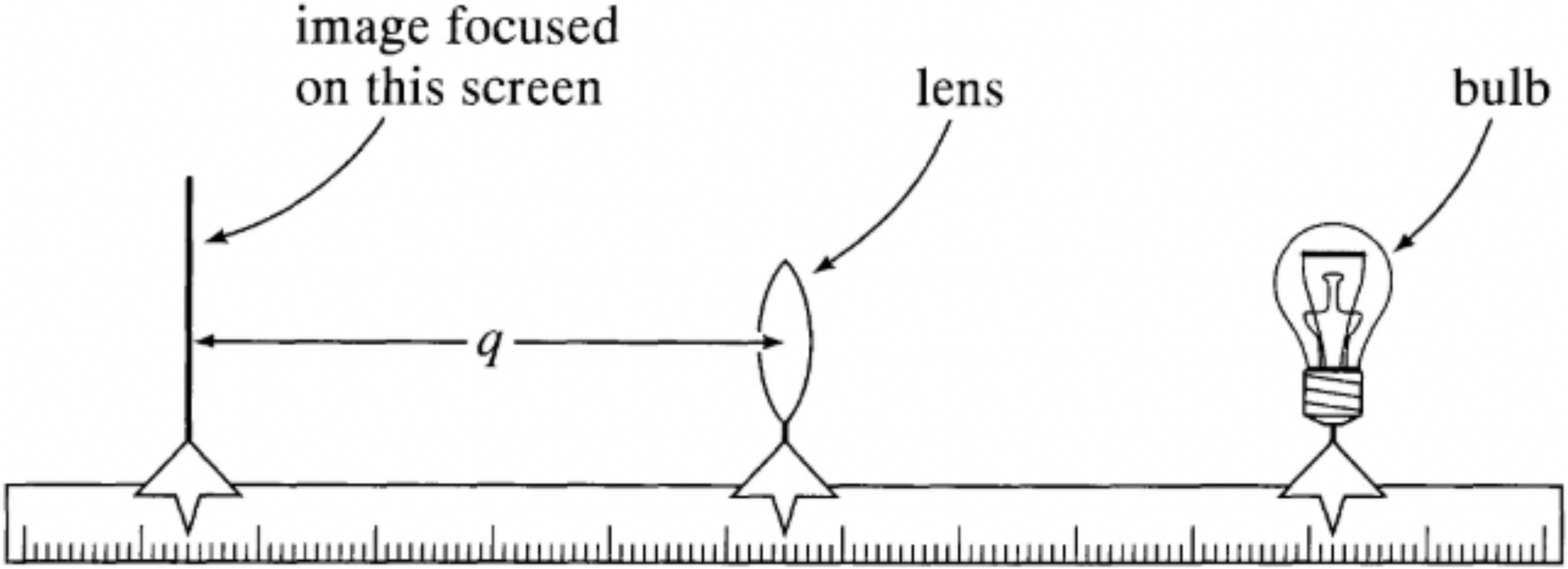
$$\delta q \approx \delta x + \delta y. \tag{2.18}$$

Uncertainty in a Product
(Provisional Rule)

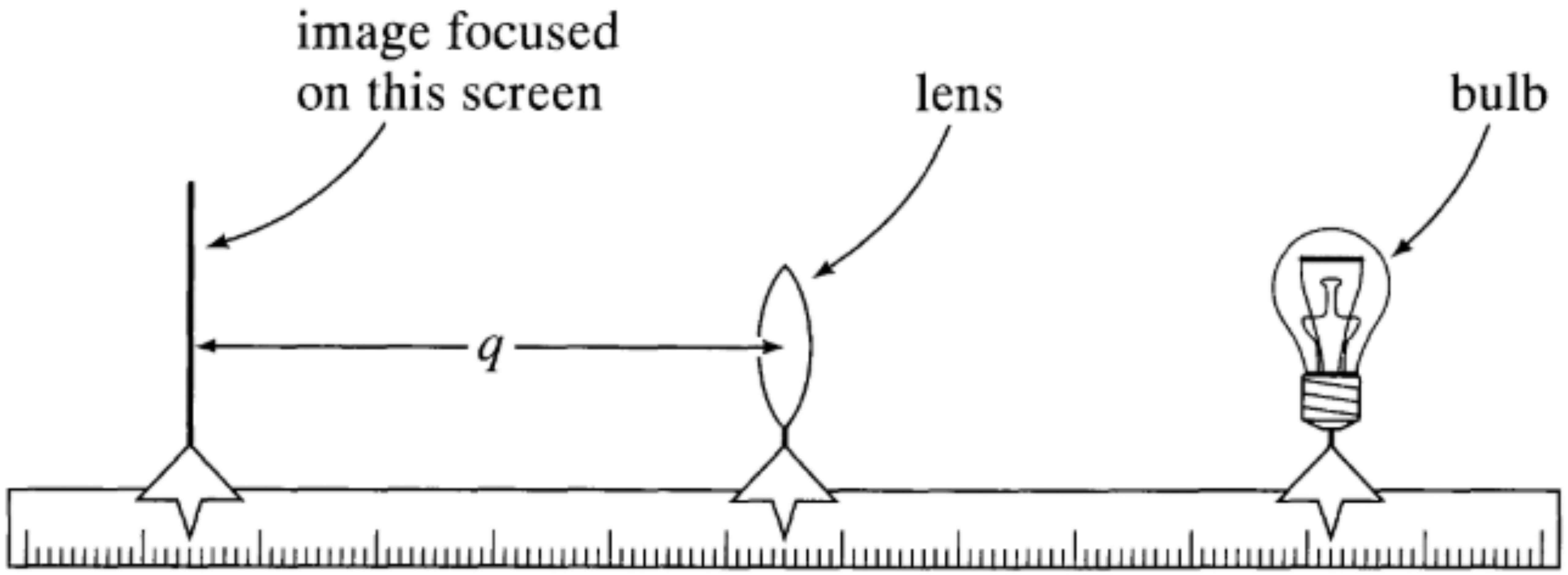
If two quantities x and y have been measured with small fractional uncertainties $\delta x/|x_{\text{best}}|$ and $\delta y/|y_{\text{best}}|$, and if the measured values of x and y are used to calculate the product $q = xy$, then the *fractional uncertainty in* q *is the sum of the fractional uncertainties in* x *and* y,

$$\frac{\delta q}{|q_{\text{best}}|} \approx \frac{\delta x}{|x_{\text{best}}|} + \frac{\delta y}{|y_{\text{best}}|}. \tag{2.28}$$

First, review uncertainty in direct measurements



Recall how to find uncertainty in direct measurements



What are some challenges in determining uncertainty?

- where is center of lens
- could be a range of distance with image focused
- challenge is neither point is clearly defined
  - problem of definition

What're are the techniques we can use?

- repeated measures
- digital devices only specify sig. figs.

$$x = x_{best} \pm \delta x$$

Counting experiments and uncertainty

A demographer want to know the average births at a given hospital

<u>H: The average births at Hospital Y is equal to the average births in city X.</u>

Counting experiments - the square root rule

Quick Check 3.1. **(a)** To check the activity of a radioactive sample, an inspector places the sample in a liquid scintillation counter to count the number of decays in a two-minute interval and obtains 33 counts. What should he report as the number of decays produced by the sample in two minutes? **(b)** Suppose, instead, he had monitored the same sample for 50 minutes and obtained 907 counts. What would be his answer for the number of decays in 50 minutes? **(c)** Find the percent uncertainties in these two measurements, and comment on the usefulness of counting for a longer period as in part (b).

Review for uncertainty propagation for Difference/Addition

Key Idea: you can use the highest and lowest probable values to estimate new uncertainty

Review for uncertainty propagation for Difference/Addition

Uncertainty in Sums and Differences
(Provisional Rule)

If several quantities $x, \ldots, w$ are measured with uncertainties $\delta x, \ldots, \delta w$, and the measured values used to compute

$$q = x + \cdots + z - (u + \cdots + w),$$

then the uncertainty in the computed value of q is the sum,

$$\delta q \approx \delta x + \cdots + \delta z + \delta u + \cdots + \delta w, \tag{3.4}$$

of all the original uncertainties.

Same idea applies to products/quotients, but fractional form is used

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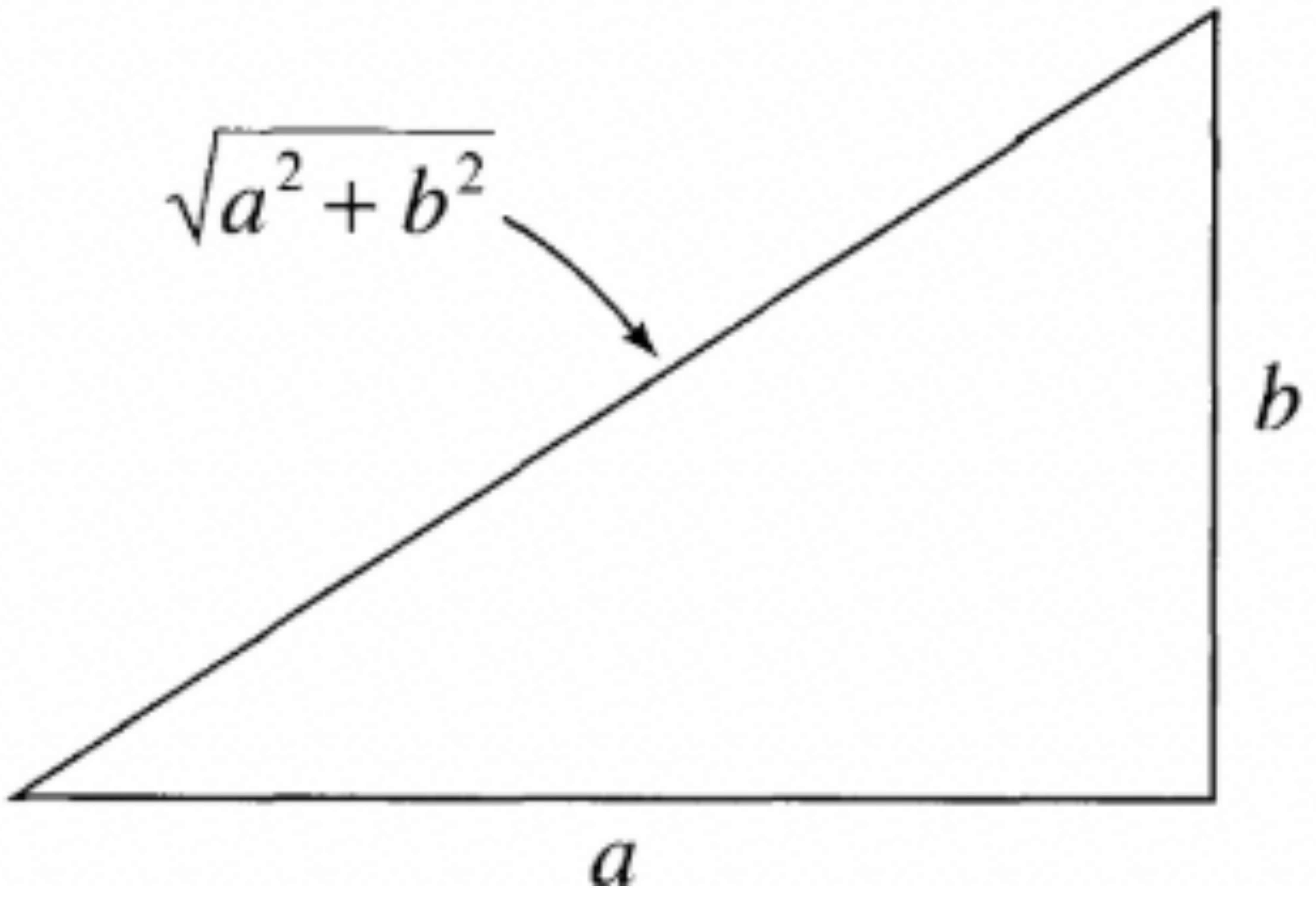
Special Cases: Multiplication with a constant

Special Cases: Powers

Independent Uncertainties in Sums

Let’s explore why the original formulation is conservative

What should be done?



$$\sqrt{a^2 + b^2} < a + b$$

Summary for new uncertainty estimates

Uncertainty in Sums and Differences

Suppose that $x, \ldots, w$ are measured with uncertainties $\delta x, \ldots, \delta w$ and the measured values used to compute

$$q = x + \cdots + z - (u + \cdots + w).$$

If the uncertainties in $x, \ldots, w$ are known to be *independent and random,* then the uncertainty in q is the quadratic sum

$$\delta q = \sqrt{(\delta x)^2 + \cdots + (\delta z)^2 + (\delta u)^2 + \cdots + (\delta w)^2}$$

of the original uncertainties. In any case, δq is never larger than their ordinary sum,

$$\delta q \leqslant \delta x + \cdots + \delta z + \delta u + \cdots + \delta w.$$

Uncertainties in Products and Quotients

Suppose that $x, \ldots, w$ are measured with uncertainties $\delta x, \ldots, \delta w$, and the measured values are used to compute

$$q = \frac{x \times \cdots \times z}{u \times \cdots \times w}.$$

If the uncertainties in $x, \ldots, w$ are *independent and random,* then the fractional uncertainty in q is the sum in quadrature of the original fractional uncertainties,

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}.$$

In any case, it is never larger than their ordinary sum,

$$\frac{\delta q}{|q|} \leqslant \frac{\delta x}{|x|} + \cdots + \frac{\delta z}{|z|} + \frac{\delta u}{|u|} + \cdots + \frac{\delta w}{|w|}.$$

Summary for new uncertainty estimates

Quick Check 3.6. Suppose you measure three numbers as follows:

$$x = 200 \pm 2, \quad y = 50 \pm 2, \quad z = 20 \pm 1,$$

where the three uncertainties are independent and random. What would you give for the values of $q = x + y - z$ and $r = xy/z$ with their uncertainties?

Arbitrary Functions of One Variable

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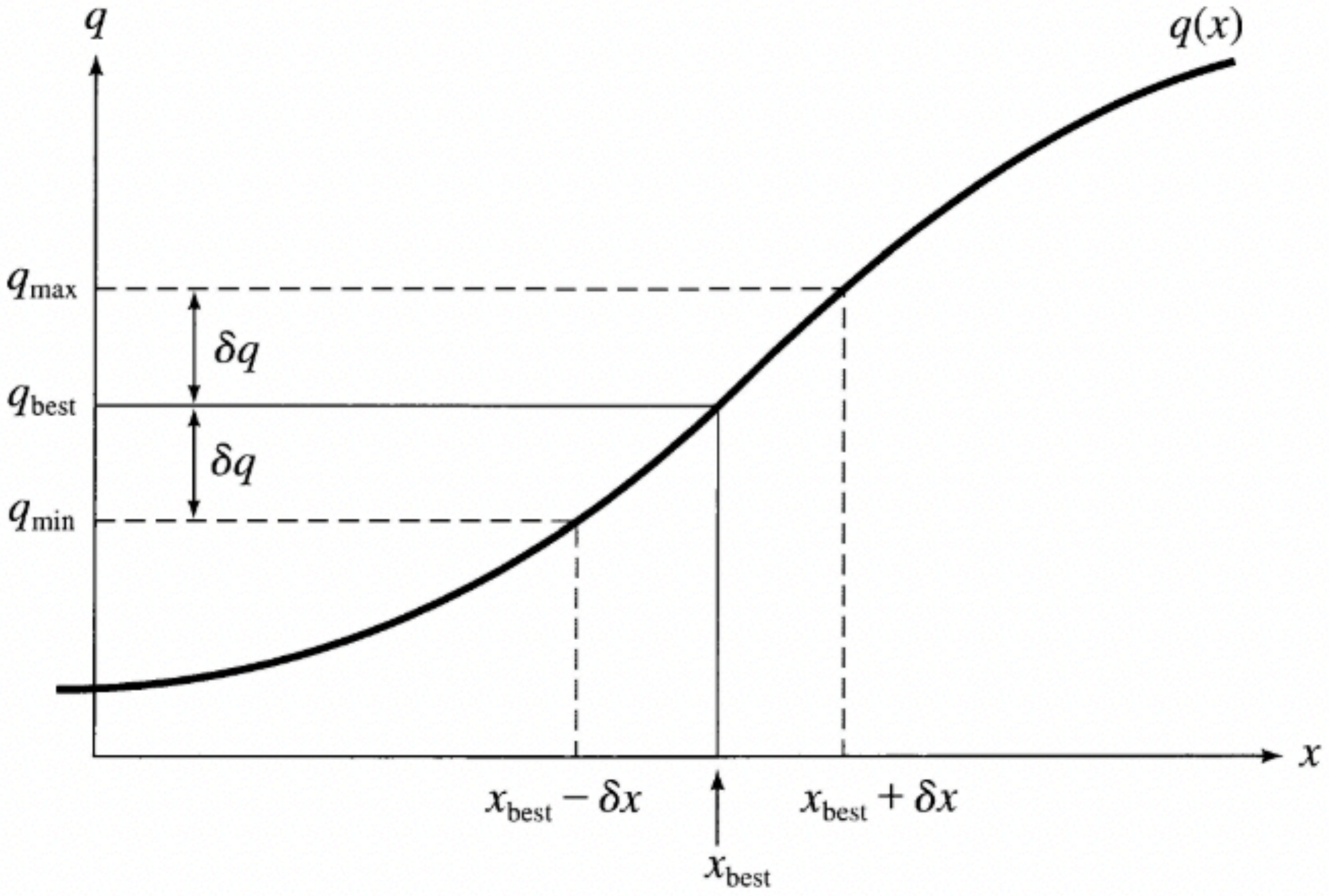
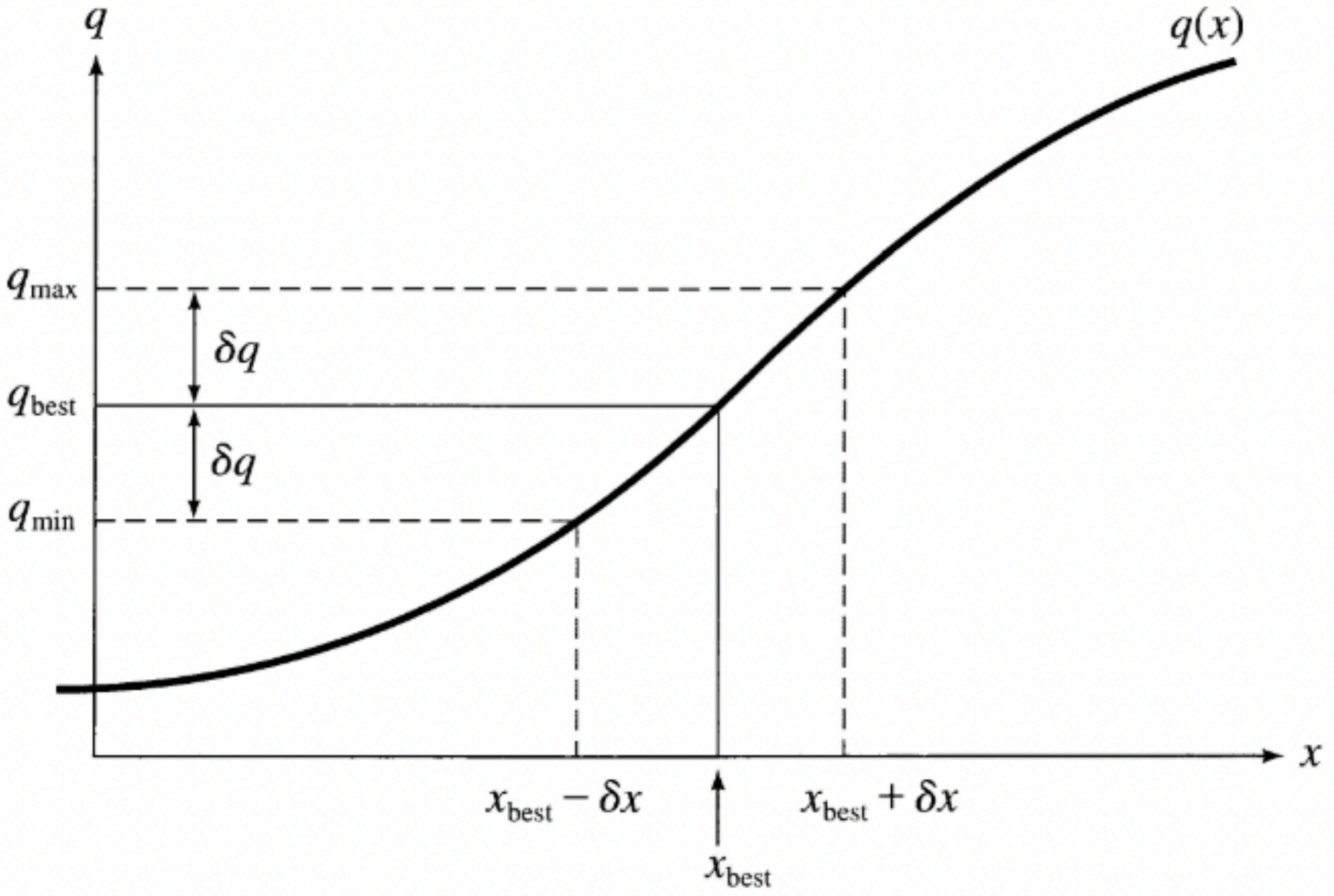
How should we approach this?

Arbitrary Functions of One Variable



Arbitrary Functions of One Variable



Arbitrary Functions of One Variable

Uncertainty in Any Function of One Variable

If x is measured with uncertainty δx and is used to calculate the function $q(x)$, then the uncertainty δq is

$$\delta q = \left| \frac{dq}{dx} \right| \delta x.$$

Arbitrary Functions of One Variable

Quick Check 3.7. Suppose you measure x as 3.0 ± 0.1 and then calculate $q = e^x$. What is your answer, with its uncertainty?

Special Case — power

Special Case — power

Uncertainty in a Power

If x is measured with uncertainty δx and is used to calculate the power $q = x^n$ (where n is a fixed, known number), then the fractional uncertainty in q is $|n|$ times that in x,

$$\frac{\delta q}{|q|} = |n| \frac{\delta x}{|x|}.$$

Special Case — power

Putting Everything Together: Propagation Step-by-Step

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$$q = x(y - z \sin u)$$

Quick Check 3.9. Suppose you measure three numbers as follows:

$$x = 200 \pm 2, \quad y = 50 \pm 2, \quad z = 40 \pm 2,$$

where the three uncertainties are independent and random. Use step-by-step propagation to find the quantity $q = x/(y - z)$ with its uncertainty. [First find the uncertainty in the difference $y - z$ and then the quotient $x/(y - z)$.]

General Formula for Error Propagation

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General Formula for Error Propagation

Uncertainty in a Function of Several Variables

Suppose that $x, \ldots, z$ are measured with uncertainties $\delta x, \ldots, \delta z$ and the measured values are used to compute the function $q(x, \ldots, z)$. If the uncertainties in $x, \ldots, z$ are independent and random, then the uncertainty in q is

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\,\delta x\right)^2 + \cdots + \left(\frac{\partial q}{\partial z}\,\delta z\right)^2}.$$

In any case, it is never larger than the ordinary sum

$$\delta q \leqslant \left|\frac{\partial q}{\partial x}\right| \delta x + \cdots + \left|\frac{\partial q}{\partial z}\right| \delta z.$$

General Formula for Error Propagation