ME170b Lecture 4

2/3/23

Experimental Techniques

Last time:

General Formula for error propogation

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \cdots + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

$$\delta q \leq \left|\frac{\partial q}{\partial x}\right|\delta x + \cdots + \left|\frac{\partial q}{\partial z}\right|\delta z$$

Today:

Statistics CH. 4

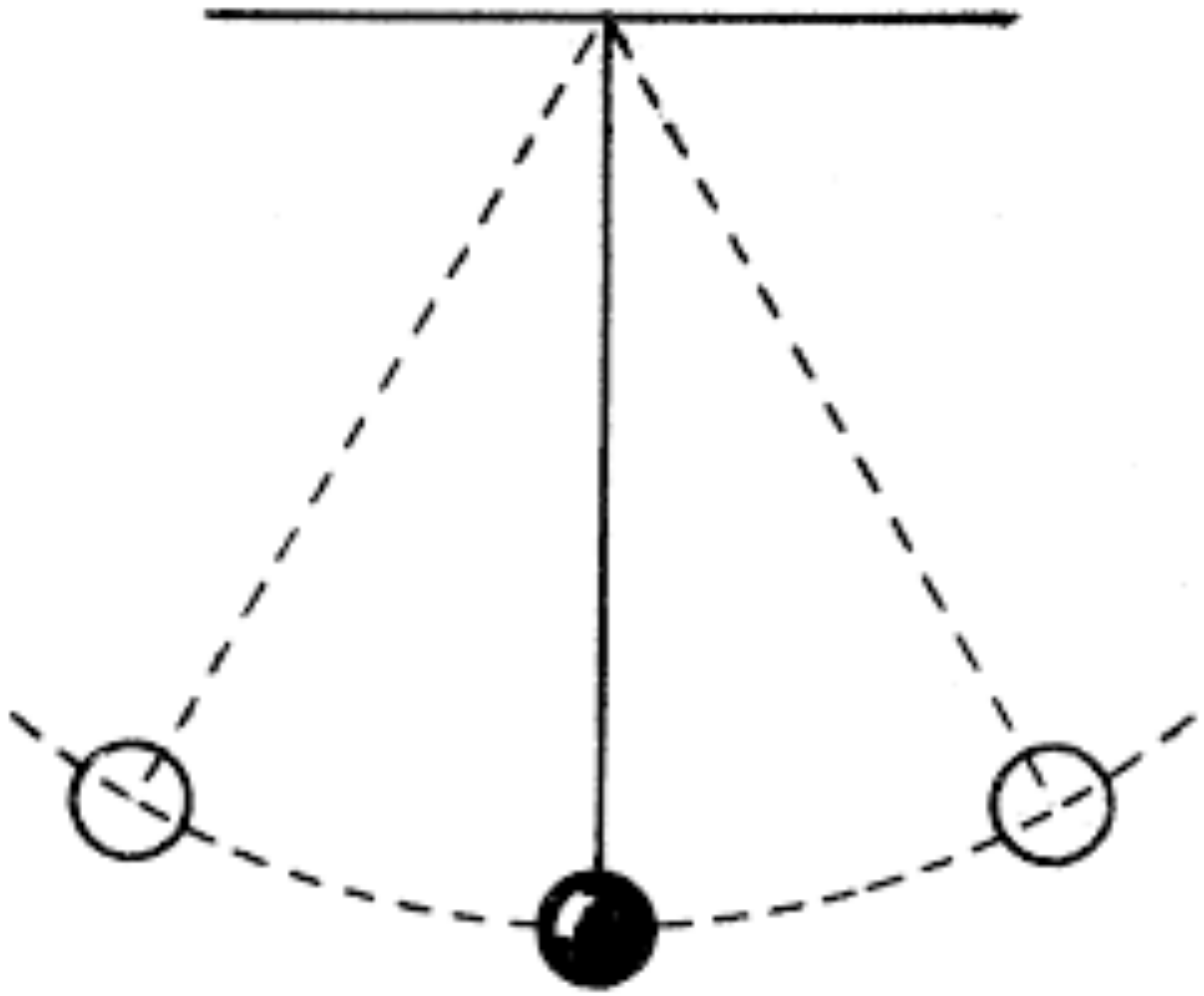
Histogram CH. 5

first.

Checking Understanding: Propagation of Uncertainties

Suppose we want to confirm the value of g, the acceleration due to gravity

$H: g_{measure} = 9.81$

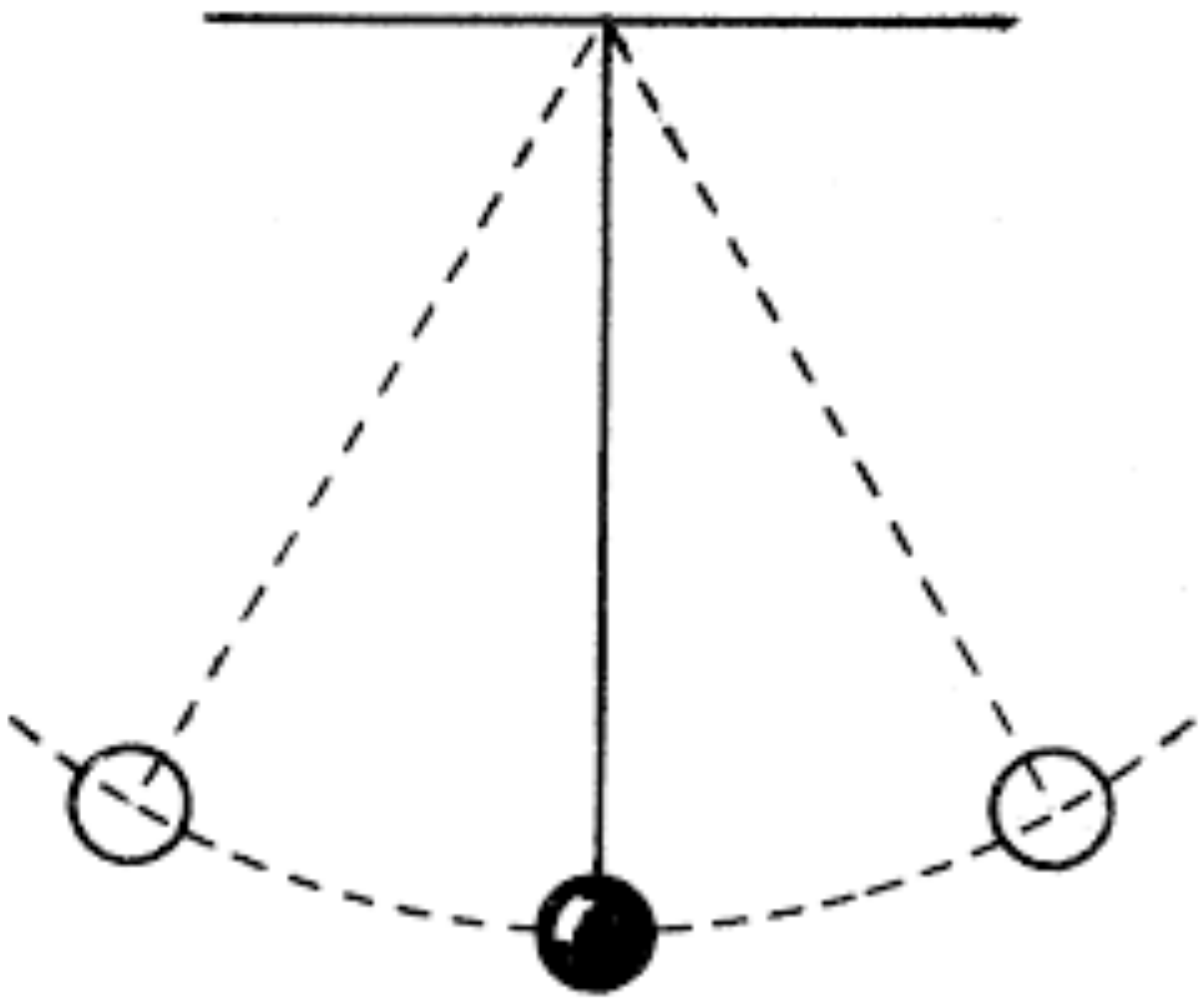


Experiment: Use a pendulum, measure length l, measure the period T, estimate g:

$$T = 2\pi \sqrt{l/g}$$

$$\boxed{g = 4\pi^2 \, l/T^2} \quad g(l, T)$$

Checking Understanding: Propagation of Uncertainties



How can we track error propagation?

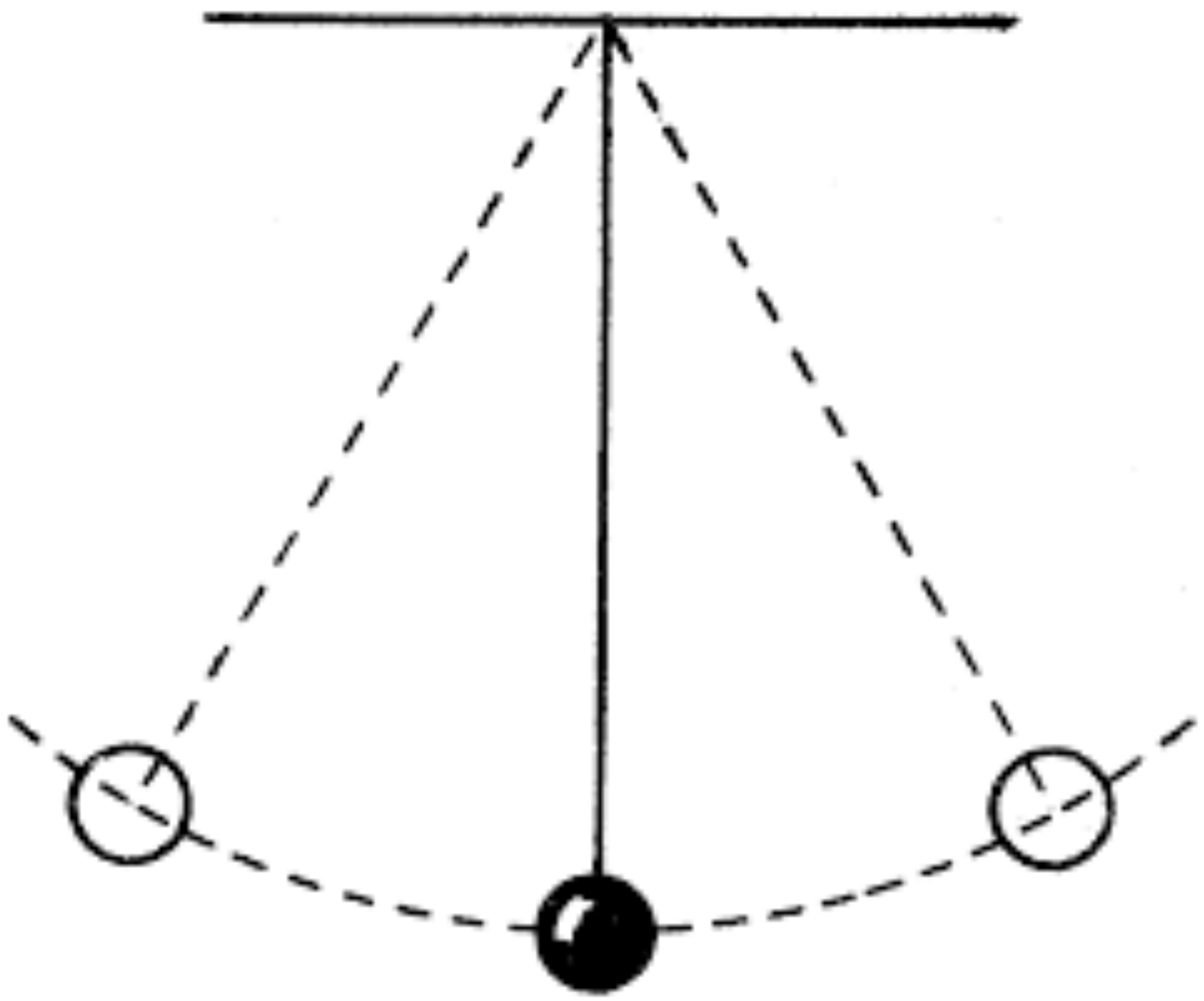
$$g = 4\pi^2 l / T^2$$

Provisional method:

- factional uncertainties add under multiplication

a little better. $\longrightarrow$ Quardrature method: $\sqrt{(\ )^2 + (\ )^2}$

Checking Understanding: Propagation of Uncertainties



How can we track error propagation?

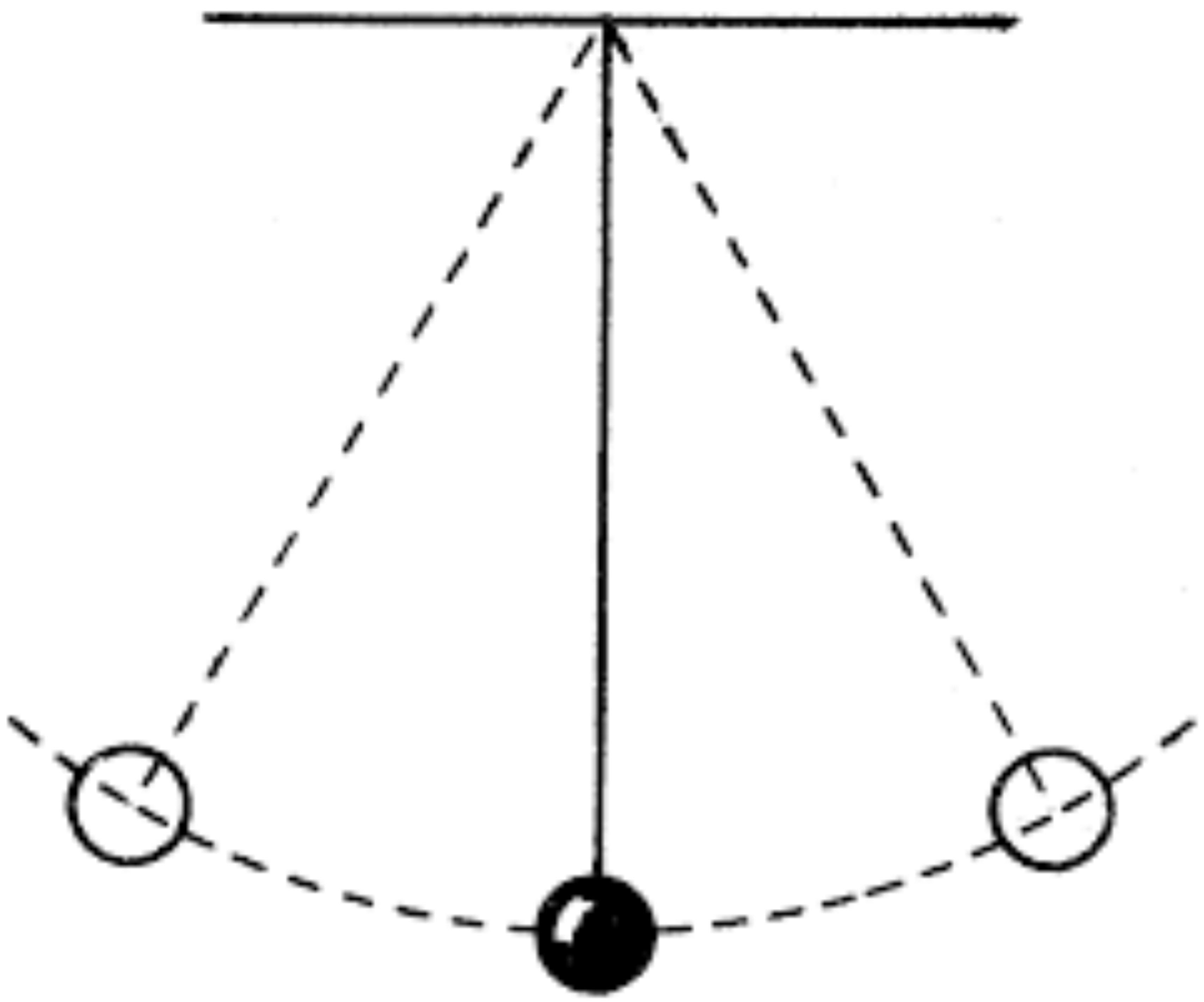
$$g = 4\pi^2 l/T^2.$$

l (cm) all ± 0.1	T (sec) all ± 0.001	g (cm/s^2)	$\delta l/l$ (%)	$\delta T/T$ (%)	$\delta g/g$ (%)	answer $g \pm \delta g$
93.8	1.944	980	0.1	0.05	0.14	980 ± 1.4

propagate

We can apply general formula ~~#~~ best method thus far.

Checking Understanding: Propagation of Uncertainties



How can we track error propagation?

$$g = 4\pi^2 l/T^2.$$

General Formula for Error Propagation: If $q = q(x, \ldots, z)$ is any function of $x, \ldots, z$, then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\,\delta x\right)^2 + \cdots + \left(\frac{\partial q}{\partial z}\,\delta z\right)^2}$$

(provided all errors are independent and random)

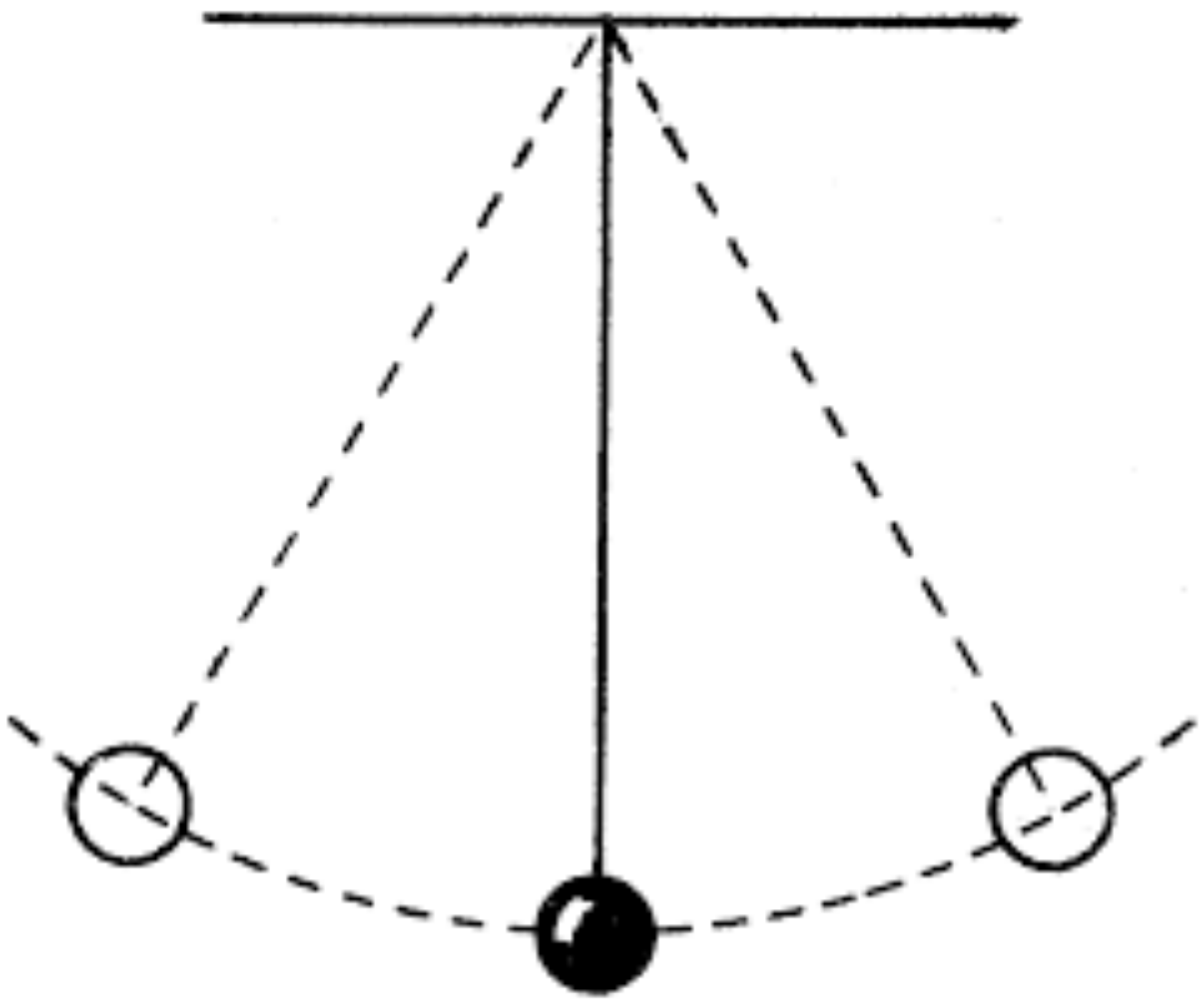
and

$$\delta q \leqslant \left|\frac{\partial q}{\partial x}\right| \delta x + \cdots + \left|\frac{\partial q}{\partial z}\right| \delta z$$

(always). [See (3.47) & (3.48)]

We can apply the general formula

Checking Understanding: Propagation of Uncertainties



How can we track error propagation?

$$g = 4\pi^2 l/T^2.$$

l (cm) all ± 0.1	T (sec) all ± 0.001	g (cm/s^2)
93.8	1.944	980
70.3	1.681	
45.7	1.358	
21.2	0.922	

- How can we leverage repeated measures to estimate uncertainty directly from data? A: Statistics!

First, which types are errors can be estimated statistically?

Random Errors: uncertainties that can be revealed by <u>repeating the measurements</u>

Systematic Errors: errors that are not random are systematic

Consider timing the revolutions of a turntable using a stop watch

maybe we want to test whether a vintage record playing is at the correct RPM

Where do we expect errors to come from?

- reaction time
  → random error
- RPM could vary.
  → random or systematic
- accuracy of stop watch
  → systematic

we can take of through "sampling"

Conceptual example: ‘experiment’ is series of ‘shots’ at target
accurate measurement == center of target. qualify systematic and random error for each (small or large)

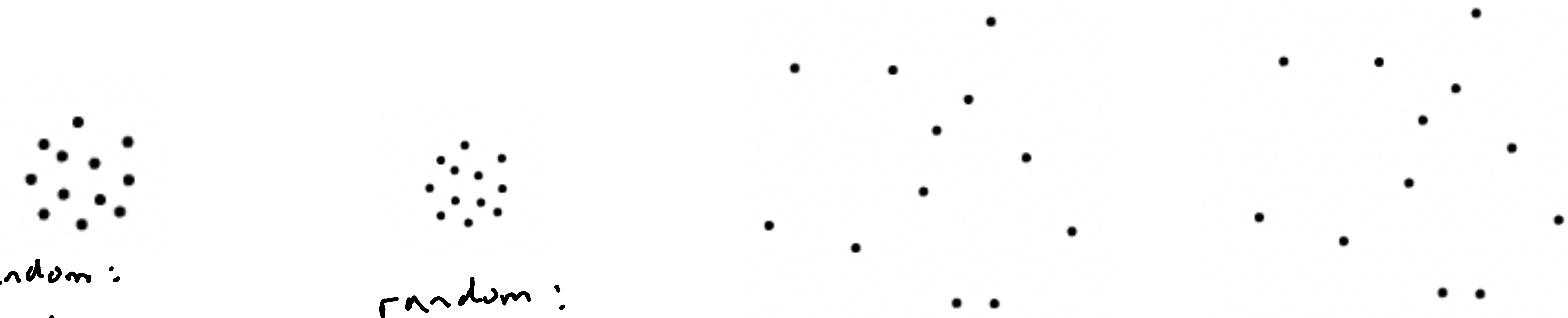
random: small
systematic: small

random: small
systematic: large

random: large
systematic: small

large!

In reality we don't know the target!



random: small

random: small

random: large

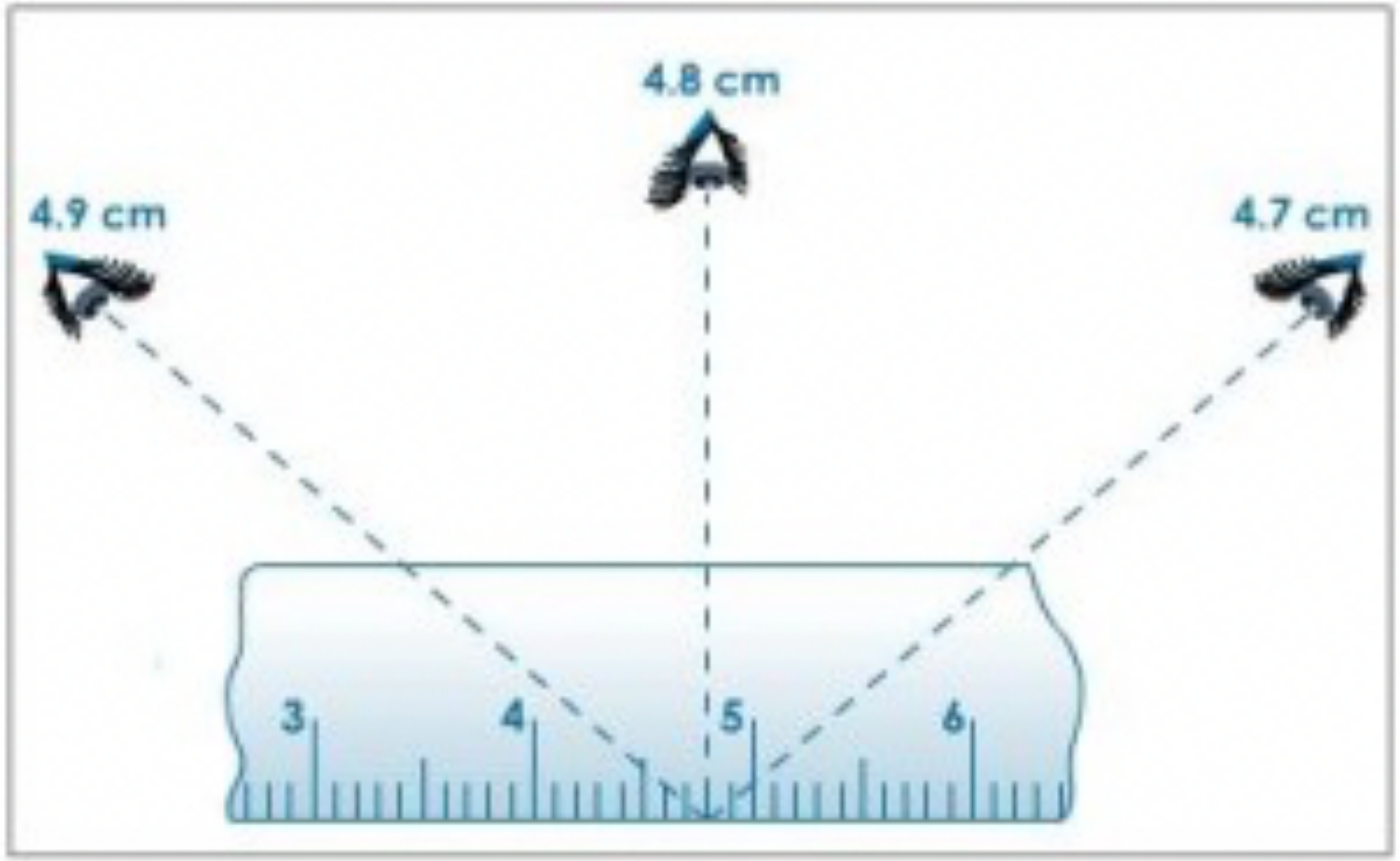
random: large

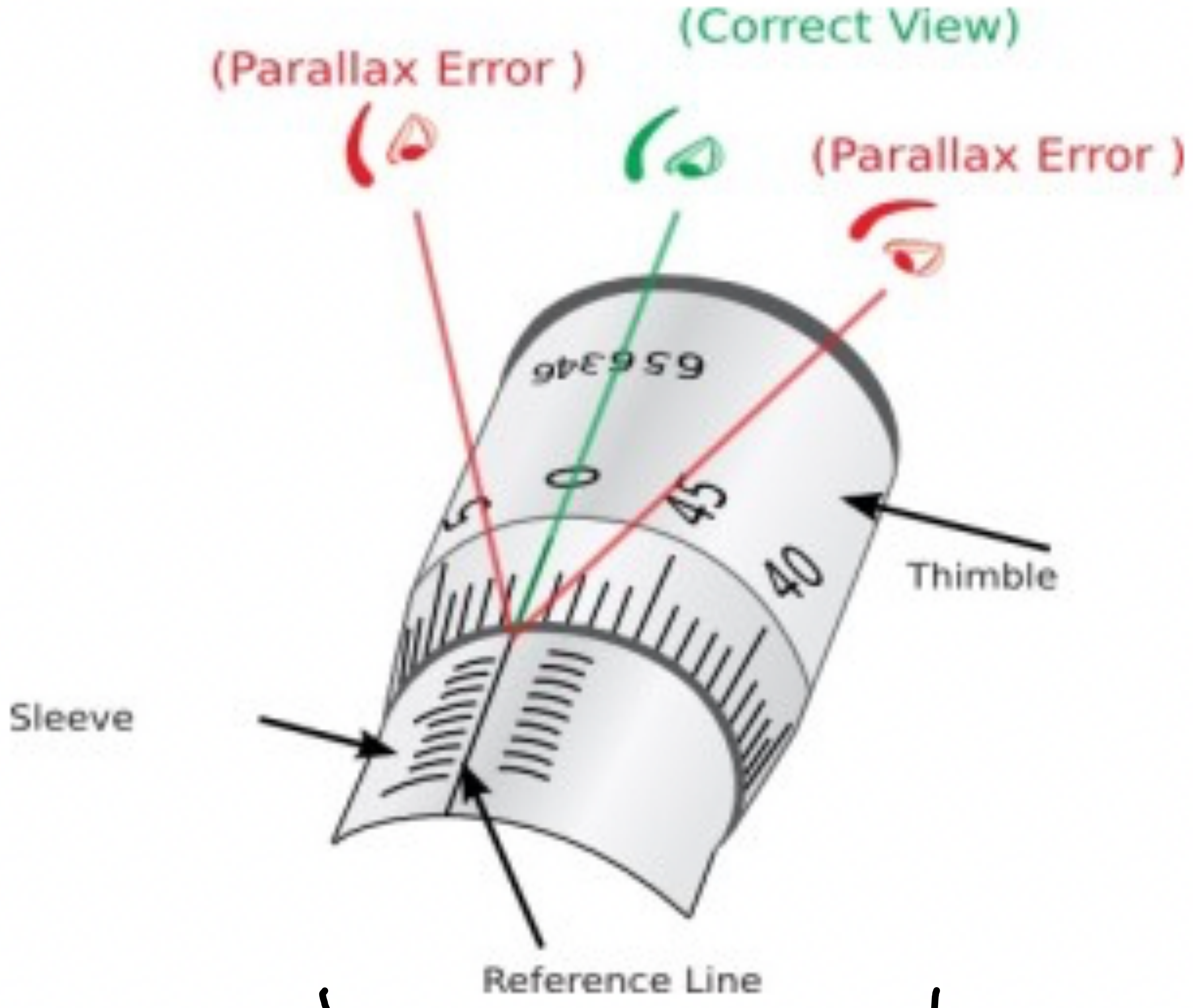
We can't see systematic error!

Key idea: we <u>can</u> identify random error even if we don't know the target.

Errors are not always clear cut

random errors in one experiment may produce systematic errors in another





this is usually random — but can be systematic in certain situations : fixed dial above experimenter.

Dealing with errors

(1) Random errors are easier to deal with.
→ we have statistical tools to
reliable estimate them.

(2) Systematic errors are hard to evaluate
& to detect. Experimenter should carefully
anticipate & eliminate sources of systematic
error! (calibrate device, control the environment)

Statistics the very basics

> context: we have minimized systematic error and therefore all uncertainty are <u>random</u>.

→ repeated measures!

71, 72, 72, 73, 71

what should we use as x_best = ?

$$x_{best} = \overline{x} = \frac{71 + 72 + 72 + 73 + 71}{5}$$

$$= 71.8$$

Statistics the very basics

71, 72, 72, 73, 71

How can we estimate uncertainty from our five measurements?

~~Standard deviation can help us!~~

What is a standard deviation?

71, 72, 72, 73, 71

x_1 x_2 x_3 x_4 x_5

$d_i = x_i - \bar{x}$ deviation of the i^{th} measurement

(residual)

can we just average the deviations?

Trial number i	Measured value x_i	Deviation $d_i = x_i - \bar{x}$
1	71	-0.8
2	72	0.2
3	72	0.2
4	73	1.2
5	71	-0.8
	$\Sigma x_i = 359$	$\Sigma d_i = 0$

$$\text{mean, } \bar{x} = \Sigma x_i / N = 359/5 = \underline{71.8}$$

Definition of standard deviation

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(d_i)^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$\sigma_x^2 = \frac{1}{N}\sum d_i^2$$

<u>variance</u>

Bessels correction

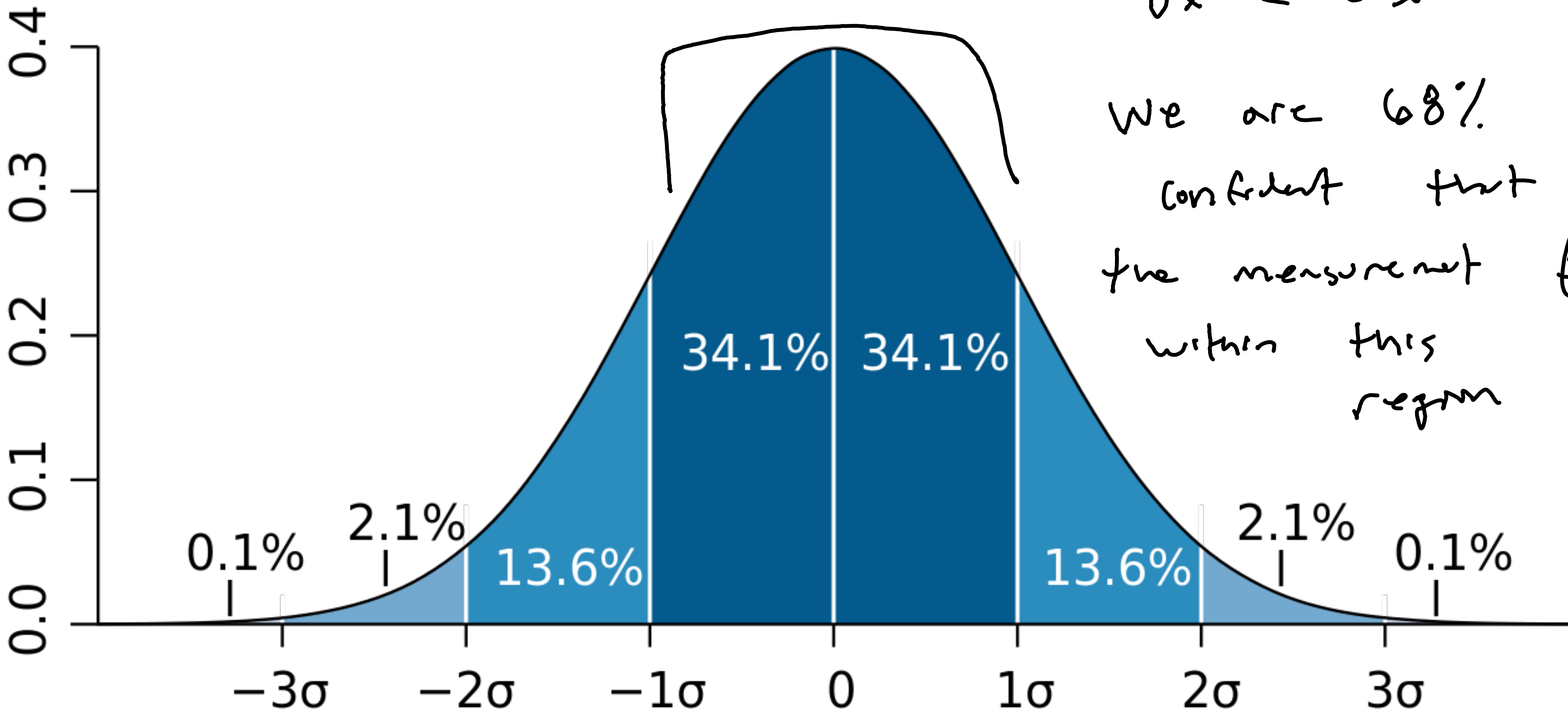
$$\sigma_x = \sqrt{\frac{1}{N-1} \sum (x_i - \bar{x})^2}$$

Sample deviation – most appropriate for our purposes.

Standard deviation as uncertainty of a single measurement

assumption — errors are random normally distributed

$\delta x = \sigma x$



We are 68% confident that the measurement falls within this region

Still big.

Standard deviation of the mean (standard error)

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

this is good estimate for the uncertainty using all measurements

$$\boxed{x = \bar{x} \pm \sigma_{\bar{x}}}$$

* we can prove (ch.5)

* making more measurements doesn't really change SD.

* SE, on the other hand, will decrease with increasing measurements

Histograms and Distributions

By now, it should be clear: serious uncertainty analysis, via statistics, requires experiments that make many measurements.

→ We need tools to display these measurements

26, 24, 26, 28, 23, 24, 25, 24, 26, 25.

Written out, the data conveys very little info.

Histograms and Distributions

23, 24, 24, 24, 25, 25, 26, 26, 26, 28. we can order them.

Table 5.1. Measured lengths x and their numbers of occurrences.

Different values, x_k	23	24	25	26	27	28
Number of times found, n_k	1	3	2	3	0	1

Instead of saying "$x = 24$ was obtained three times",

"$x = 24$ occurred $\underline{3/10}$ fraction."

$$F_k = \frac{n_k}{N}$$ ← number x_k appears

Histograms and Distributions

Table 5.1. Measured lengths x and their numbers of occurrences.

Different values, x_k	23	24	25	26	27	28
Number of times found, n_k	1	3	2	3	0	1

$$\bar{x} = \sum_k x_k F_k$$

: weighted sum of all different values x_k where each x_k is weighted by the fraction of times it occured.

$$\sum F_k = 1$$

Now we are ready to display data in a Histogram.

Histograms and Distributions

Table 5.1. Measured lengths x and their numbers of occurrences.

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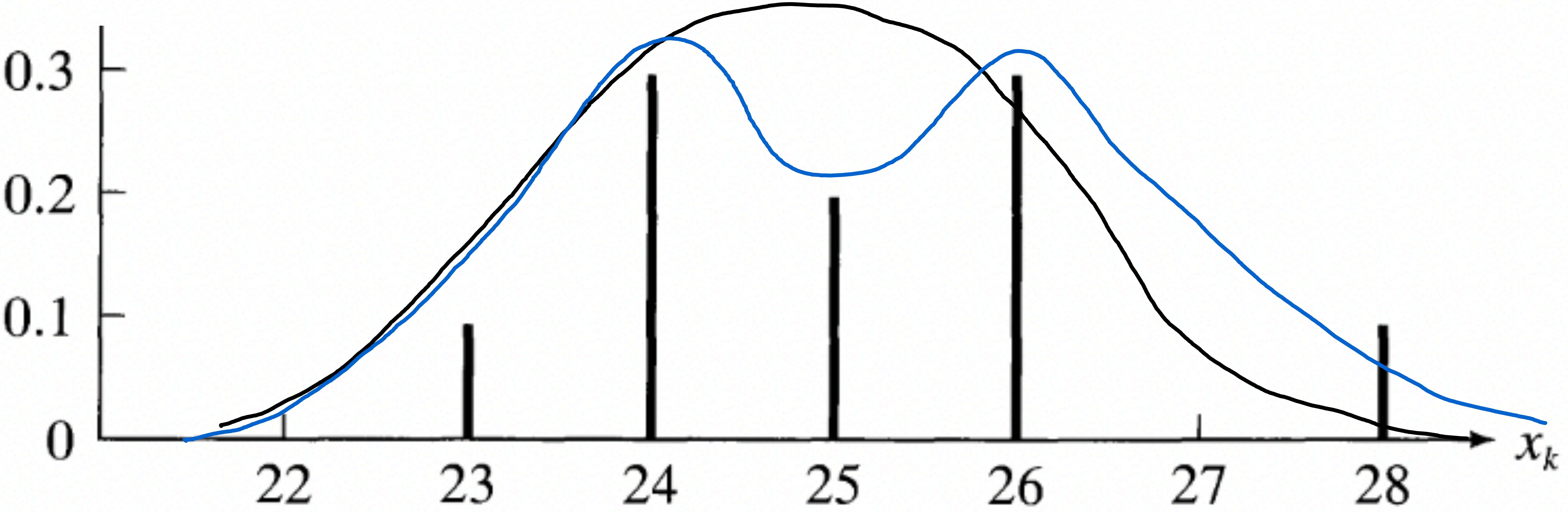


Histograms and Distributions

What about for ‘untidy data’

26.4, 23.9, 25.1, 24.6, 22.7, 23.8, 25.1, 23.9, 25.3, 25.4.

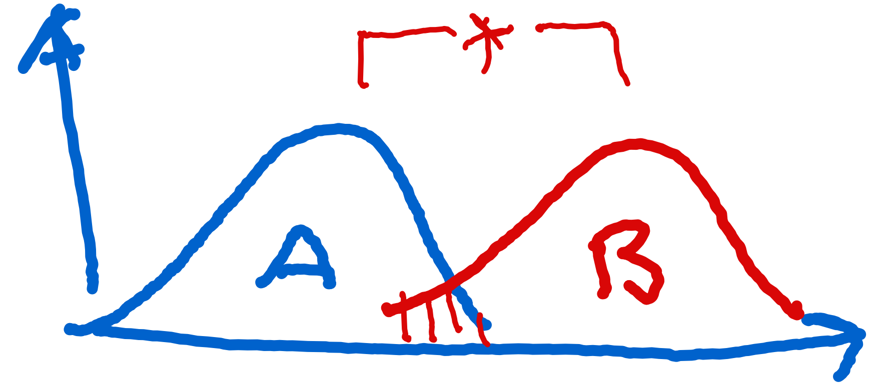
Table 5.2. The 10 measurements (5.9) grouped in bins.

Bin	22 to 23	23 to 24	24 to 25	25 to 26	26 to 27	27 to 28
Observations in bin	1	3	1	4	1	0

Histograms and Distributions

What about for ‘untidy data’

26.4, 23.9, 25.1, 24.6, 22.7, 23.8, 25.1, 23.9, 25.3, 25.4.

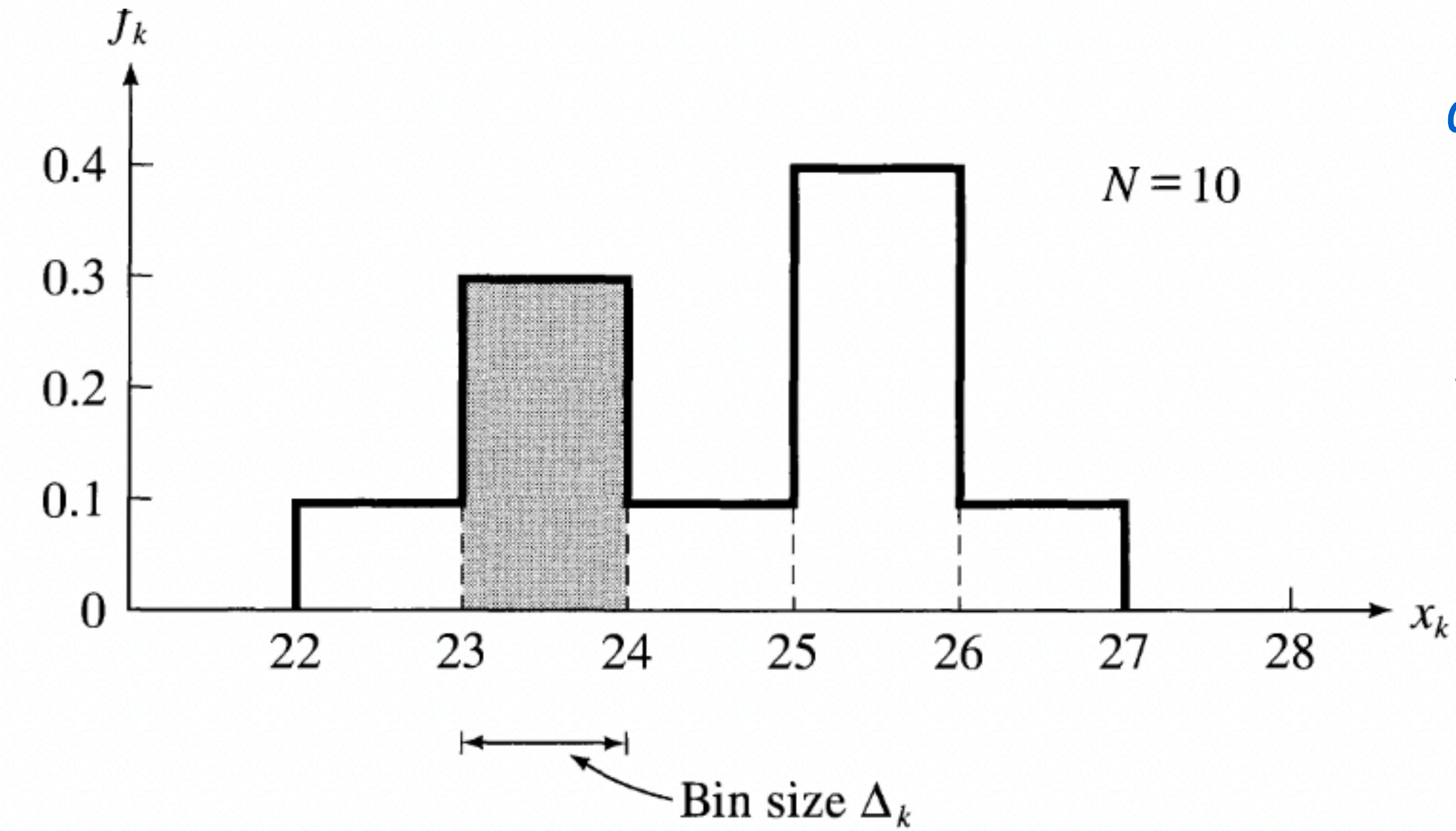


Histograms and Distributions

$f_k \Delta_k$ = fraction of measurements in the kth bin.

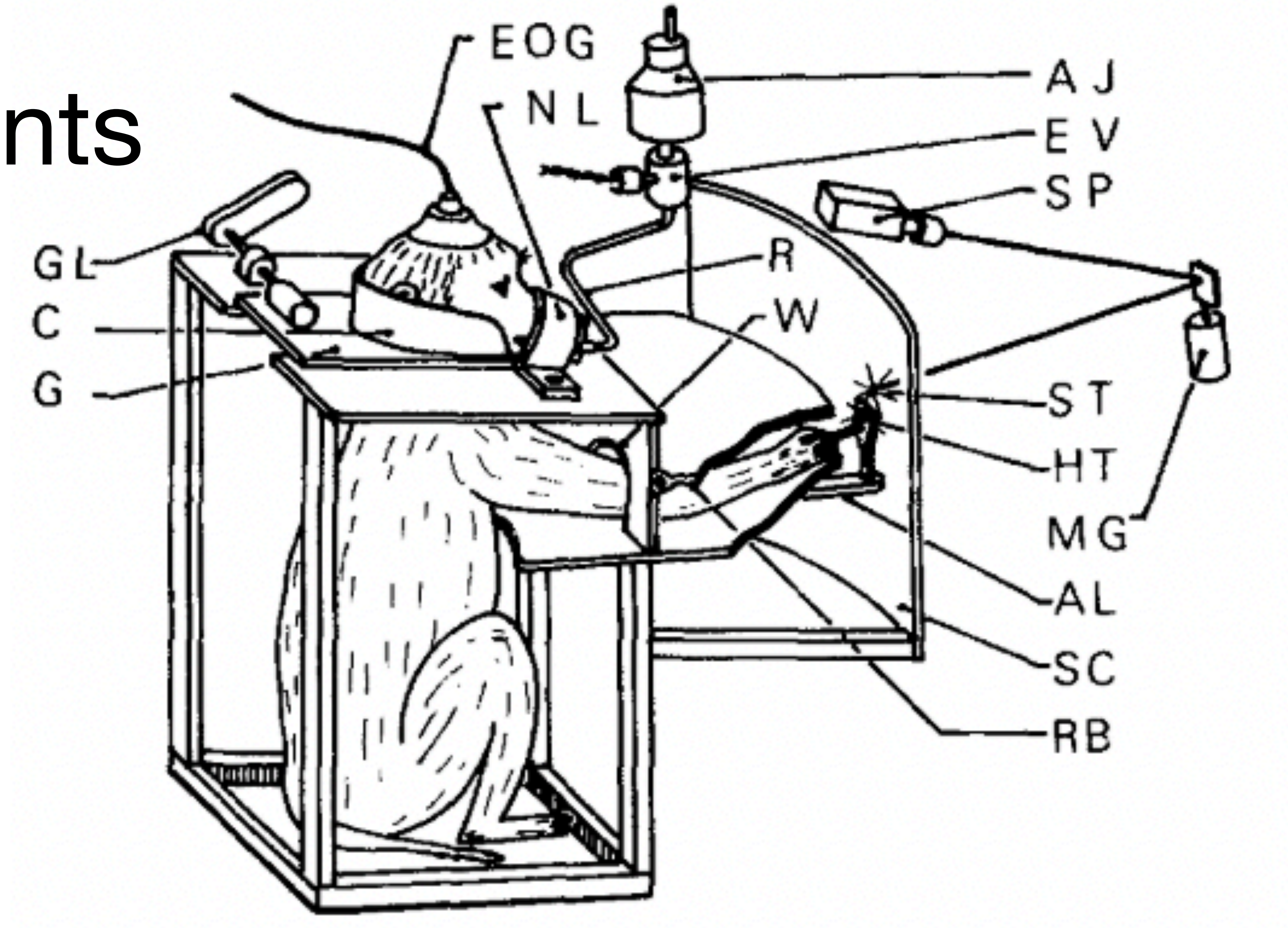
Table 5.2. The 10 measurements (5.9) grouped in bins.

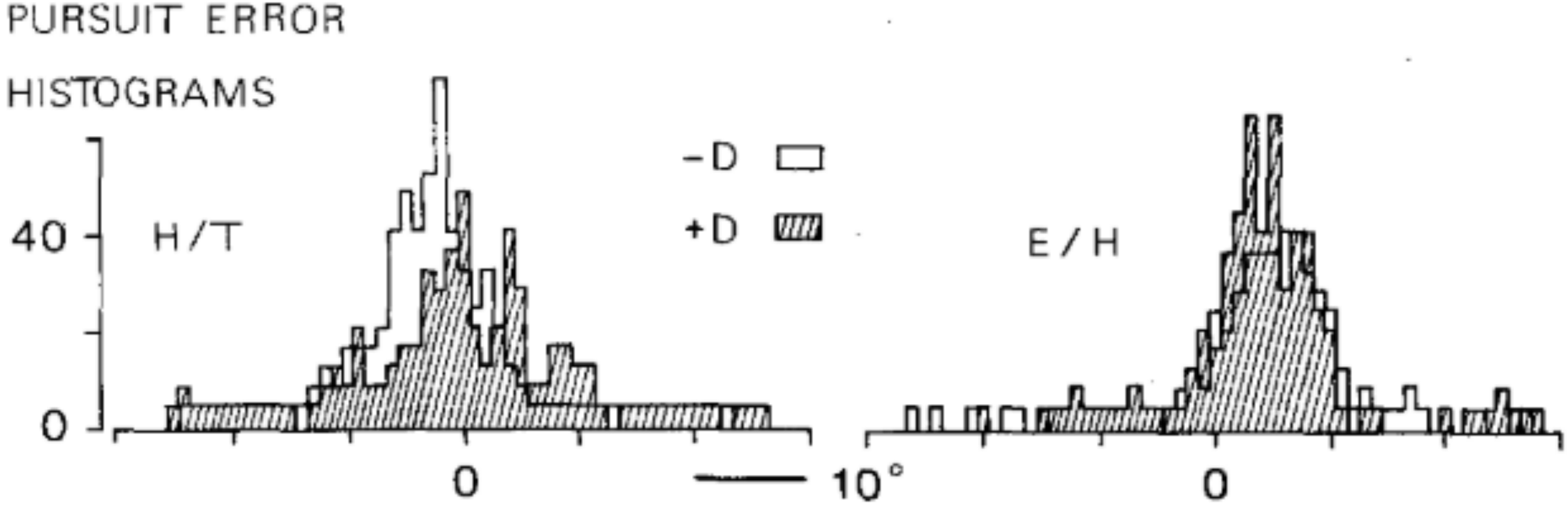
Bin	22 to 23	23 to 24	24 to 25	25 to 26	26 to 27	27 to 28
Observations in bin	1	3	1	4	1	0



area $f_k \Delta_k$ corresponds to F_k in the bar histogram

Example of histograms from real experiments





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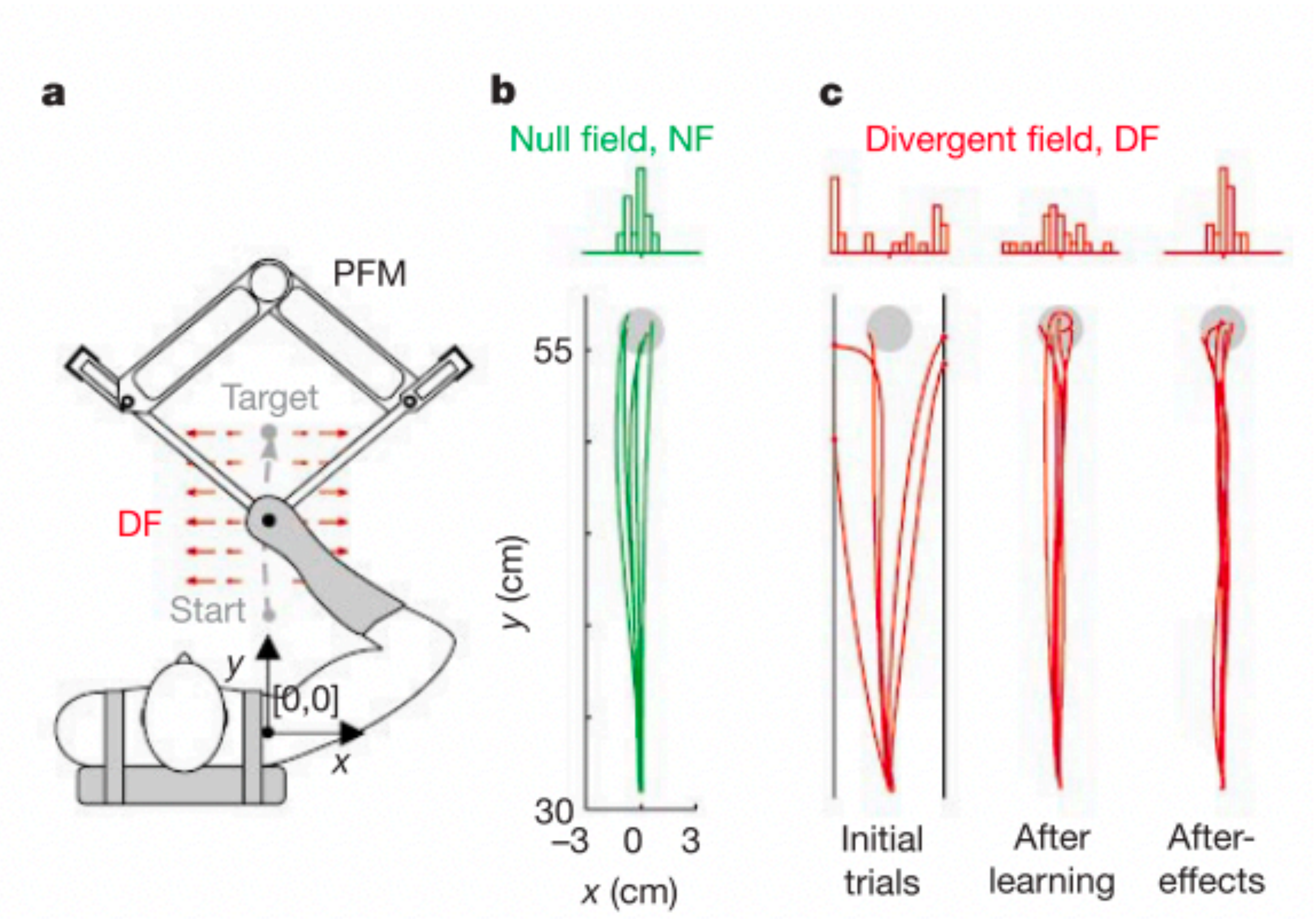
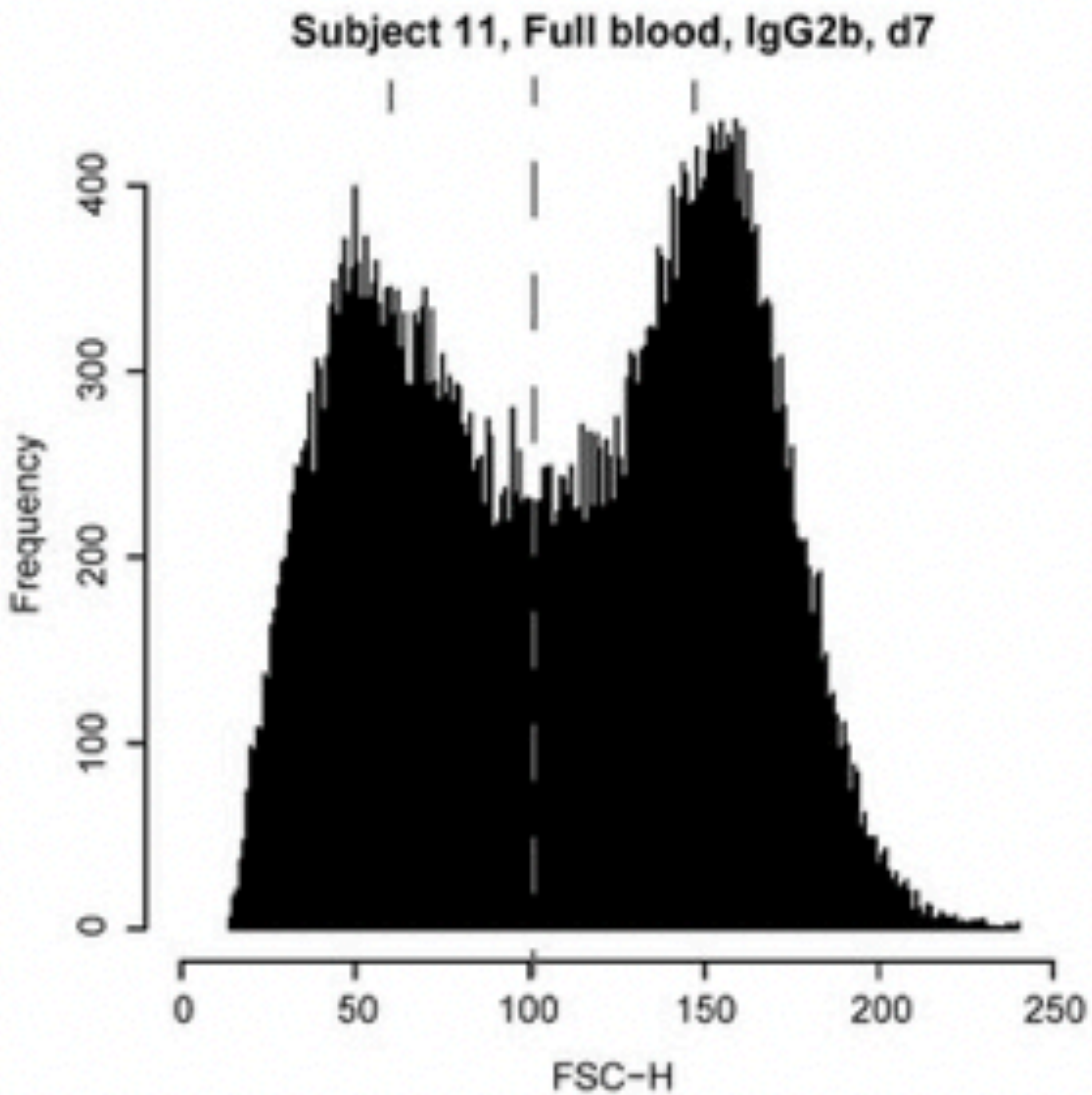
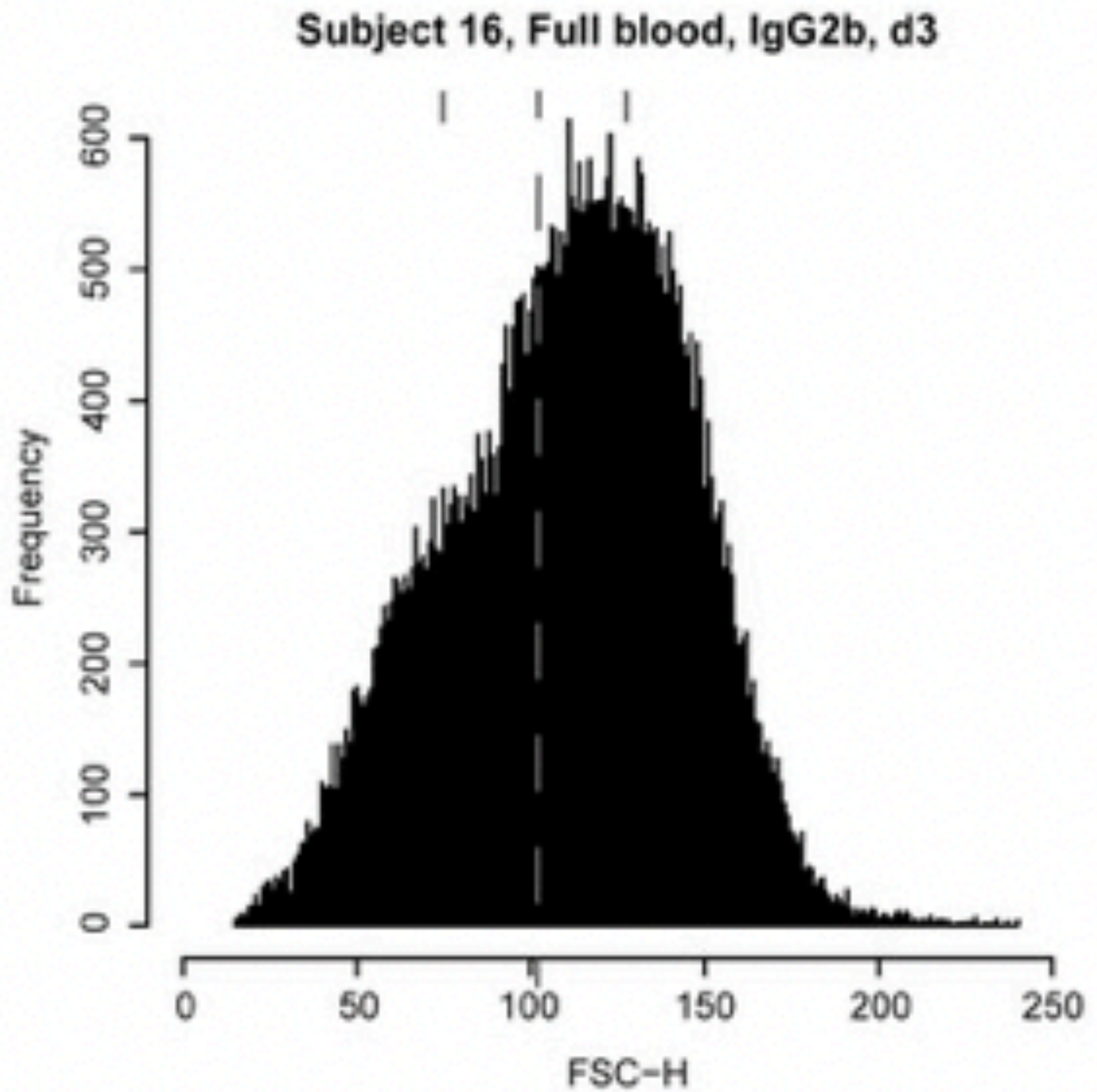
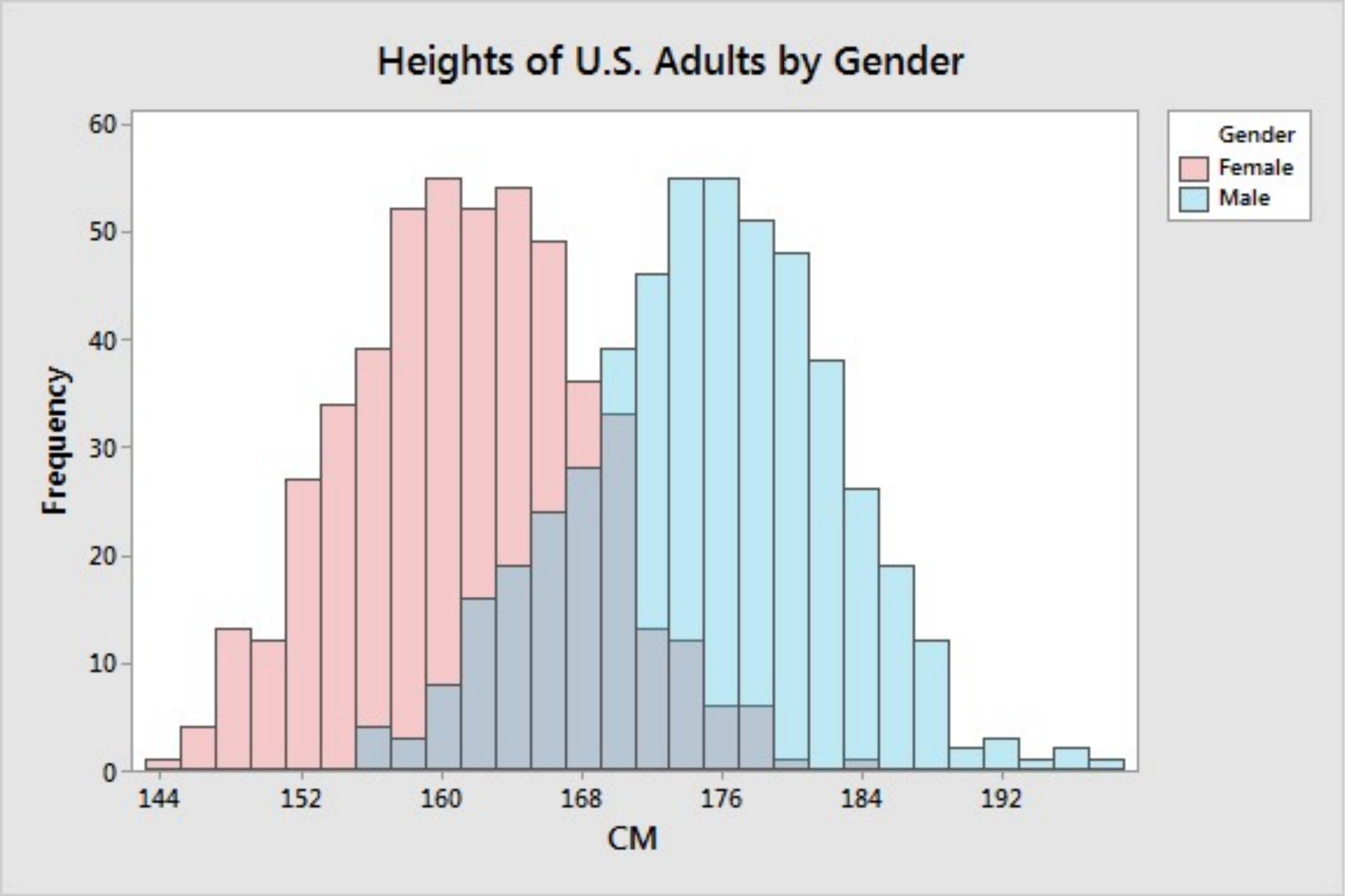


Example of histograms from real experiments





It’s really easy to visually test hypothesis with histograms



Histograms with matlab

histogram

Histogram plot

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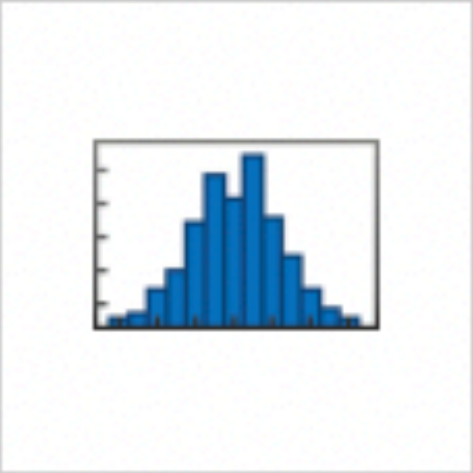
expand all in page



Description

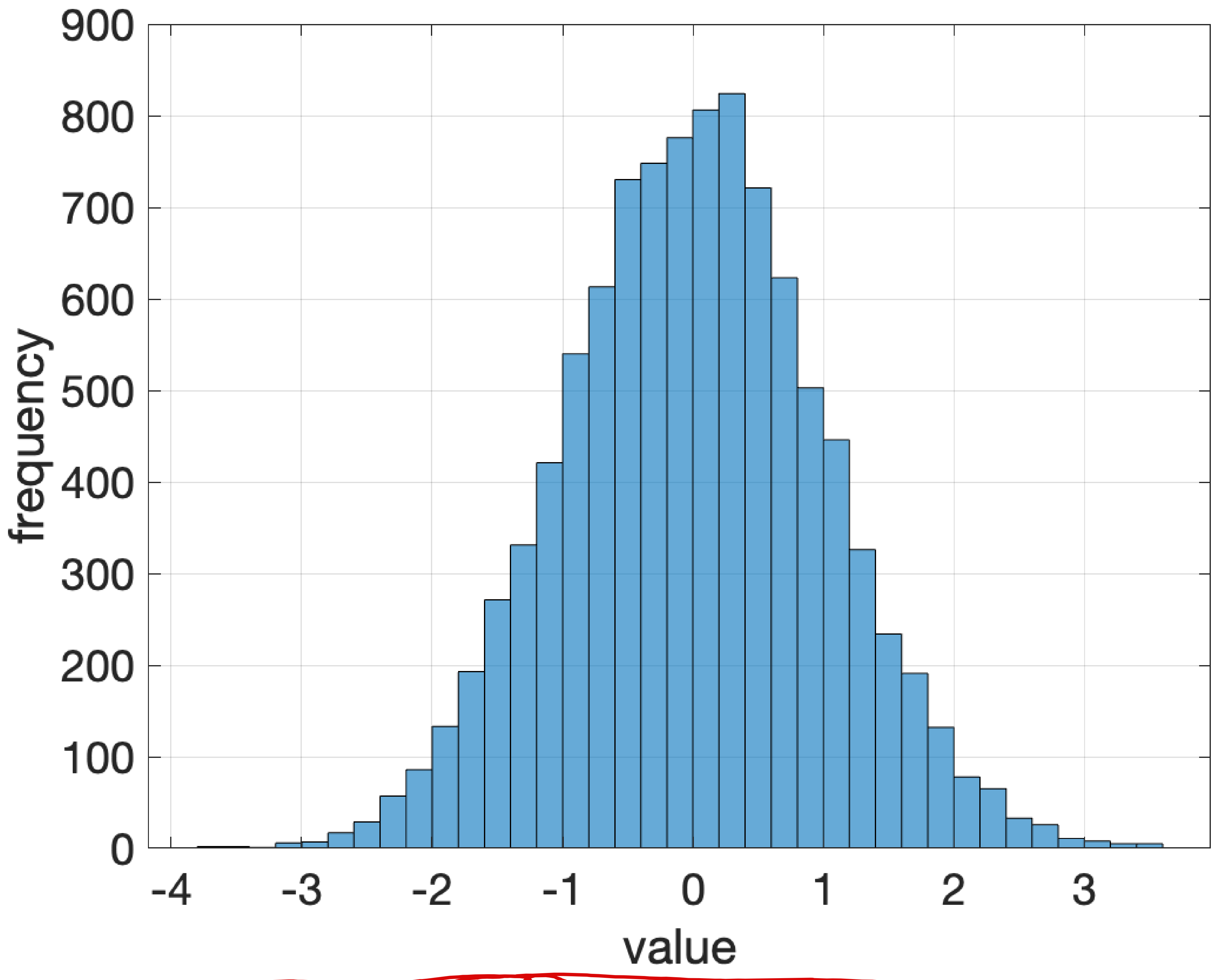
Histograms are a type of bar plot for numeric data that group the data into bins. After you create a `Histogram` object, you can modify aspects of the histogram by changing its property values. This is particularly useful for quickly modifying the properties of the bins or changing the display.

Creation

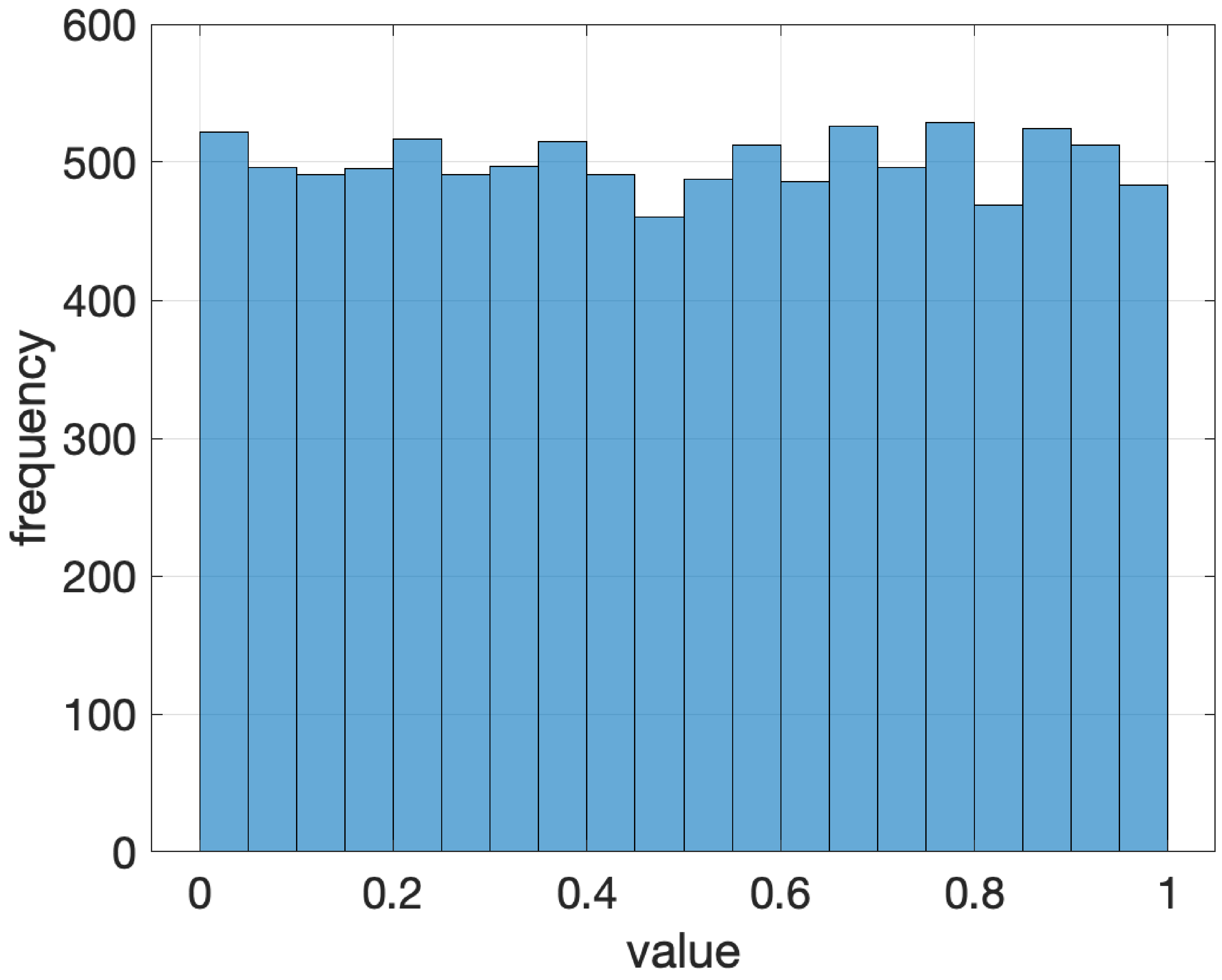
Syntax

```
histogram(X)
histogram(X,nbins)
histogram(X,edges)
histogram('BinEdges',edges,'BinCounts',counts)
```

Histograms with matlab




```
data = randn(10000,1);
figure; histogram(data);
        xlabel('value');
        ylabel('frequency');
```

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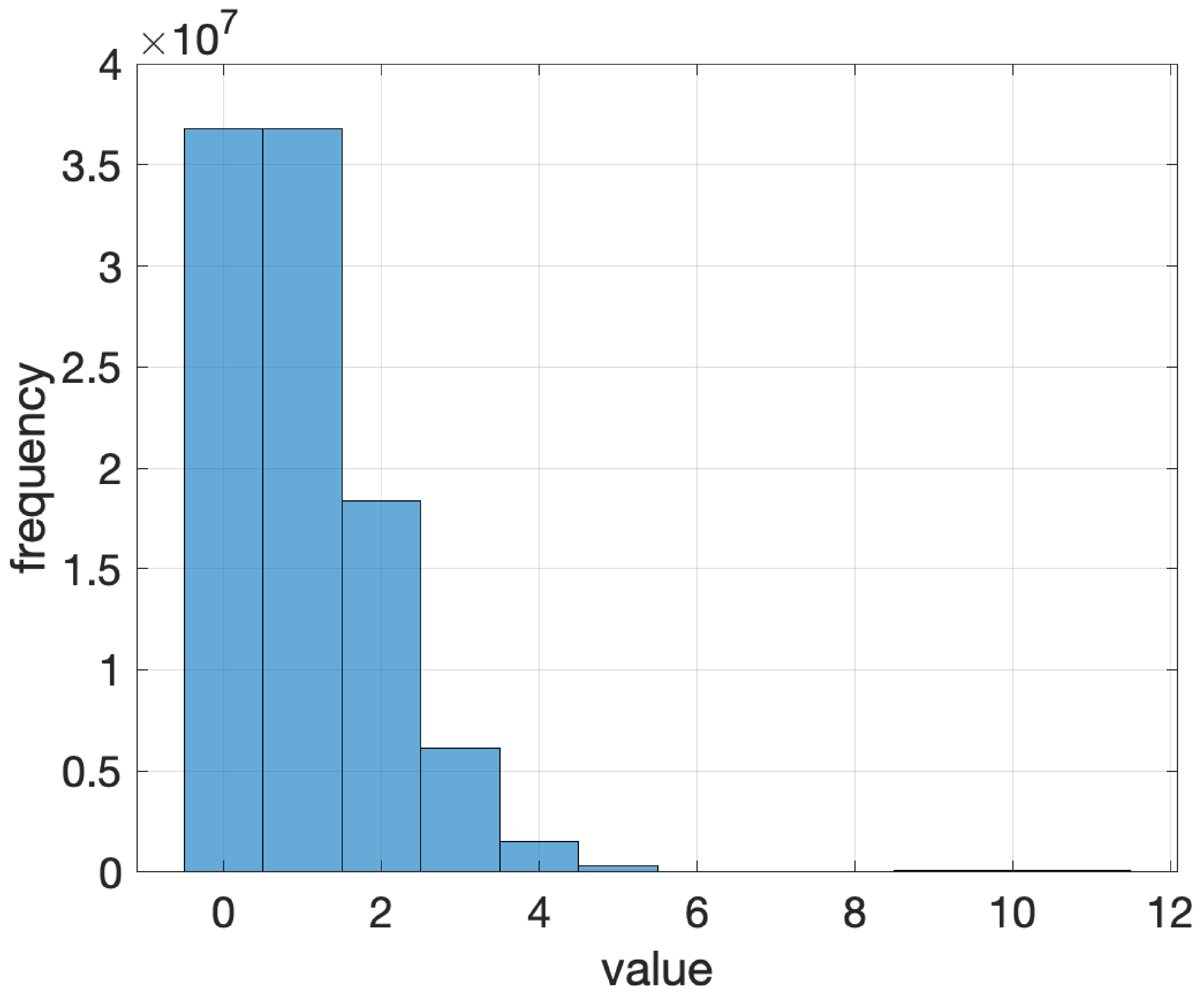
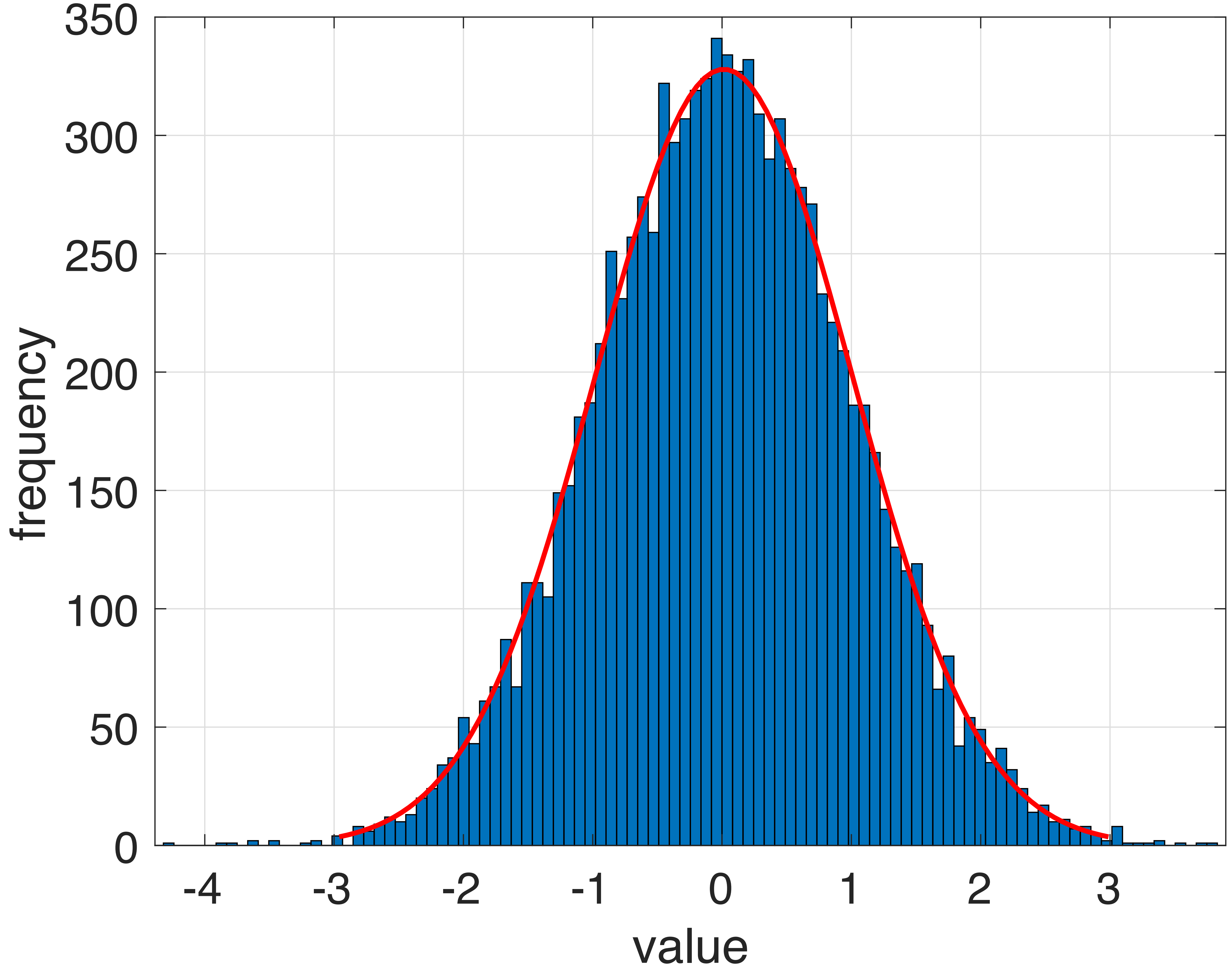
Histograms with matlab



data = poissrnd(1,10000)

```
figure; histogram(data);
        xlabel('value');
        ylabel('frequency');
```

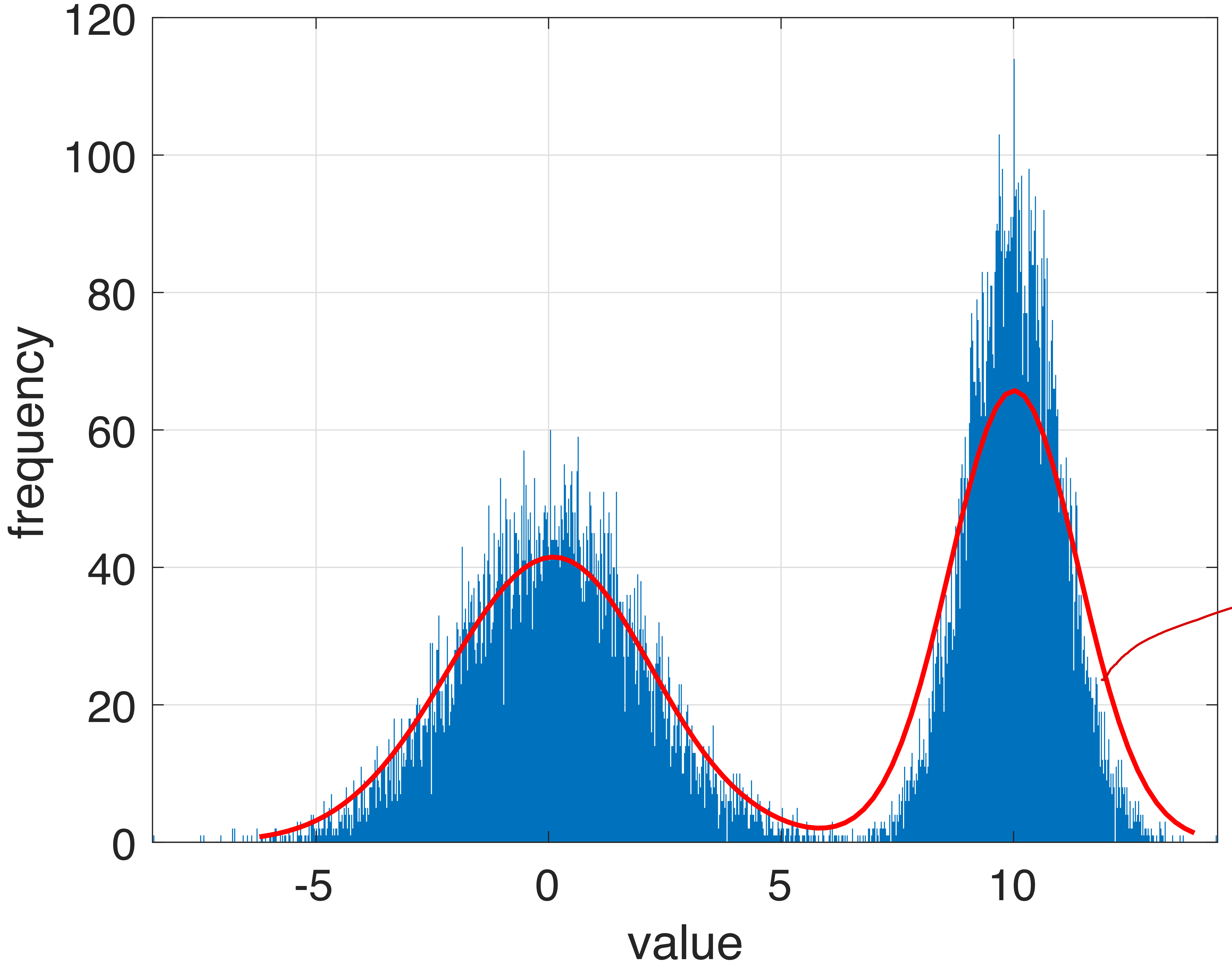
Histograms with matlab




```
data = randn(10000,1);
figure; histfit(data);
                xlabel(‘value');
                ylabel('frequency');
```

Histograms with matlab

data = [(randn(10000,1) + 10); 2*randn(10000,1)]



figure; histfit(data,1000,’kernel’);
xlabel(‘value');
ylabel('frequency');

function