3.1. ★ To measure the activity of a radioactive sample, two students count the alpha particles it emits. Student A watches for 3 minutes and counts 28 particles; Student B watches for 30 minutes and counts 310 particles. **(a)** What should Student A report for the average number emitted in 3 minutes, with his uncertainty? **(b)** What should Student B report for the average number emitted in 30 minutes, with her uncertainty? **(c)** What are the fractional uncertainties in the two measurements? Comment.

3.3. ★ Most of the ideas of error analysis have important applications in many different fields. This applicability is especially true for the square-root rule (3.2) for counting experiments, as the following example illustrates. The normal average incidence of a certain kind of cancer has been established as 2 cases per 10,000 people per year. The suspicion has been aired that a certain town (population 20,000) suffers a high incidence of this cancer because of a nearby chemical dump. To test this claim, a reporter investigates the town's records for the past 4 years and finds 20 cases of the cancer. He calculates that the expected number is 16 (check this) and concludes that the observed rate is 25% more than expected. Is he justified in claiming that this result proves that the town has a higher than normal rate for this cancer?

3.11. ★ With a good stopwatch and some practice, you can measure times ranging from approximately 1 second up to many minutes with an uncertainty of 0.1 second or so. Suppose that we wish to find the period τ of a pendulum with $\tau \approx 0.5$ s. If we time 1 oscillation, we have an uncertainty of approximately 20%; but by timing several oscillations together, we can do much better, as the following questions illustrate:

(a) If we measure the total time for 5 oscillations and get 2.4 ± 0.1 s, what is our final answer for τ, with its absolute and percent uncertainties? [Remember the rule (3.9).]

(b) What if we measure 20 oscillations and get 9.4 ± 0.1 s?

(c) Could the uncertainty in τ be improved indefinitely by timing more oscillations?

3.13. ★ If I have measured the radius of a sphere as $r = 2.0 \pm 0.1$ m, what should I report for the sphere's volume?

3.21. ★ **(a)** To find the velocity of a cart on a horizontal air track, a student measures the distance d it travels and the time taken t as

$$d = 5.10 \pm 0.01 \text{ m} \qquad \text{and} \qquad t = 6.02 \pm 0.02 \text{ s}.$$

What is his result for $v = d/t$, with its uncertainty? **(b)** If he measures the cart's mass as $m = 0.711 \pm 0.002$ kg, what would be his answer for the momentum $p = mv = md/t$? (Assume all errors are random and independent.)

3.29. ★ **(a)** An experiment to measure Planck's constant h gives it in the form $h = K\lambda^{1/3}$ where K is a constant known exactly and λ is the measured wavelength emitted by a hydrogen lamp. If a student has measured λ with a fractional uncertainty she estimates as 0.3%, what will be the fractional uncertainty in her answer for h? Comment. **(b)** If the student's best estimate for h is 6.644×10^{-34} J·s, is her result in satisfactory agreement with the accepted value of 6.626×10^{-34} J·s?

3.39. ★★ **(a)** The glider on a horizontal air track is attached to a spring that causes it to oscillate back and forth. The total energy of the system is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, where m is the glider's mass, v is its velocity, k is the spring's force constant, and x is the extension of the spring from equilibrium. A student makes the following measurements:

$$m = 0.230 \pm 0.001 \text{ kg}, \qquad v = 0.89 \pm 0.01 \text{ m/s},$$
$$k = 1.03 \pm 0.01 \text{ N/m}, \qquad x = 0.551 \pm 0.005 \text{ m}.$$

What is her answer for the total energy E? **(b)** She next measures the position x_{max} of the glider at the extreme end of its oscillation, where $v = 0$, as

$$x_{max} = 0.698 \pm 0.002 \text{ m}.$$

What is her value for the energy at the end point? **(c)** Are her results consistent with conservation of energy, which requires that these two energies should be the same?

3.45. ★ **(a)** For the function $q(x, y) = xy$, write the partial derivatives $\partial q/\partial x$ and $\partial q/\partial y$. Suppose we measure x and y with uncertainties δx and δy and then calculate $q(x, y)$. Using the general rules (3.47) and (3.48), write the uncertainty δq both for the case when δx and δy are independent and random, and for the case when they are not. Divide through by $|q| = |xy|$, and show that you recover the simple rules (3.18) and (3.19) for the fractional uncertainty in a product. **(b)** Repeat part (a) for the function $q(x, y) = x^n y^m$, where n and m are known fixed numbers. **(c)** What do Equations (3.47) and (3.48) become when $q(x)$ depends on only one variable?

3.49. ★★★ If an object is placed at a distance p from a lens and an image is formed at a distance q from the lens, the lens's focal length can be found as

$$f = \frac{pq}{p + q}. \tag{3.56}$$

[This equation follows from the "lens equation," $1/f = (1/p) + (1/q)$.] **(a)** Use the general rule (3.47) to derive a formula for the uncertainty δf in terms of p, q, and their uncertainties. **(b)** Starting from (3.56) directly, you cannot find δf in steps because p and q both appear in numerator and denominator. Show, however, that f can be rewritten as

$$f = \frac{1}{(1/p) + (1/q)}.$$

Starting from this form, you *can* evaluate δf in steps. Do so, and verify that you get the same answer as in part (a).