4.5. ★★ **(a)** Computing the standard deviation σ_x of N measurements $x_1, \ldots, x_N$ of a single quantity x requires that you compute the sum $\Sigma(x_i - \bar{x})^2$. Prove that this sum can be rewritten as

$$\Sigma[(x_i - \bar{x})^2] = \Sigma[(x_i)^2] - \frac{1}{N}[\Sigma(x_i)]^2. \tag{4.28}$$

This problem is a good exercise in using the Σ notation. Many calculators use the result to compute the standard deviation for the following reason: To use the expression on the left, a calculator must keep track of all the data (which uses a lot of memory) to calculate $\bar{x}$ and then the sum indicated; to use the expression on the right, the machine needs only to keep a running total of $\Sigma(x_i^2)$ and $\Sigma(x_i)$, which uses much less memory. **(b)** Verify the identity (4.28) for the three measurements of Problem 4.1.

4.7. ★★★ Read the first paragraph of Problem 4.6 and then do the following problem: A health physicist is testing a new detector and places it near a weak radioactive sample. In five separate 10-second intervals, the detector counts the following numbers of radioactive emissions:

$$16, 21, 13, 12, 15.$$

(a) Find the mean and standard deviation of these five numbers. **(b)** Compare the standard deviation with its expected value, the square root of the average number. **(c)** Naturally, the two numbers in part (b) do not agree exactly, and we would like to have some way to assess their disagreement. This problem is, in fact, one of error propagation. We have measured the number ν. The expected standard deviation in this number is just $\sqrt{\nu}$, a simple function of ν. Thus, the uncertainty in the standard deviation can be found by error propagation. Show, in this way, that the uncertainty in the SD is 0.5. Do the numbers in part (b) agree within this uncertainty?

4.9. ★ A student measures the period of a pendulum three times and gets the answers 1.6, 1.8, and 1.7, all in seconds. What are the mean and standard deviation? [Use the improved definition (4.9) of the standard deviation.] If the student decides to make a fourth measurement, what is the probability that this new measurement will fall outside the range of 1.6 to 1.8 s? (The numbers here were chosen to "come out right." In Chapter 5, I will explain how to do this kind of problem even when the numbers don't come out right.)

4.13. ★★ **(a)** Calculate the mean and standard deviation for the following 30 measurements of a time t (in seconds):

8.16, 8.14, 8.12, 8.16, 8.18, 8.10, 8.18, 8.18, 8.18, 8.24,
8.16, 8.14, 8.17, 8.18, 8.21, 8.12, 8.12, 8.17, 8.06, 8.10,
8.12, 8.10, 8.14, 8.09, 8.16, 8.16, 8.21, 8.14, 8.16, 8.13.

(You should certainly use the built-in functions on your calculator (or the spreadsheet you created in Problem 4.8 if you did), and you can save some button pushing if you drop all the leading 8s and shift the decimal point two places to the right before doing any calculation.) **(b)** We know that after several measurements, we can expect about 68% of the observed values to be within σ_t of $\bar{t}$ (that is, inside the range $\bar{t} \pm \sigma_t$). For the measurements of part (a), about how many would you expect to lie *outside* the range $\bar{t} \pm \sigma_t$? How many do? **(c)** In Chapter 5, I will show that we can also expect about 95% of the values to be within $2\sigma_t$ of $\bar{t}$ (that is, inside the range $\bar{t} \pm 2\sigma_t$). For the measurements of part (a), about how many would you expect to lie *outside* the range $\bar{t} \pm 2\sigma_t$? How many do?

4.21. ★ Table 4.3 records 10 measurements each of the length l and breadth b of a rectangle. These values were used to calculate the area $A = lb$. If the measurements were made in pairs (one of l and one of b), it would be natural to multiply each pair together to give a value of A—the first l times the first b to give a first value of A, and so on. Calculate the resulting 10 values of A, the mean $\overline{A}$, the SD σ_A, and the SDOM $\sigma_{\overline{A}}$. Compare the answers for $\overline{A}$ and $\sigma_{\overline{A}}$ with the answer (4.18) obtained by calculating the averages $\overline{l}$ and $\overline{b}$ and then taking A to be $\overline{l}\,\overline{b}$, with an uncertainty given by error propagation. (For a large number of measurements, the two methods should agree.)

4.23. ★ A famous example of a systematic error occurred in Millikan's historic measurement of the electron's charge e. He worked on this experiment for several years and had reduced all random errors to a very low level, certainly less than 0.1%. Unfortunately, his answer for e depended on the viscosity of air (denoted η), and the value of η that he used was 0.4% too low. His value for e had the form $e = K\eta^{3/2}$, where K stands for a complicated expression involving several measured parameters but not η. Therefore, the systematic error in η caused a systematic error in e. Given that all other errors (random and systematic) were much less than 0.4%, what was his error in e? (This example is typical of many systematic errors. Until the errors are identified, nothing can be done about them. Once identified, the errors can be eliminated, in this case by using the right value of η.)

4.25. ★★ **(a)** A student measures the speed of sound as $u = f\lambda$, where f is the frequency shown on the dial of an audio oscillator, and λ is the wavelength measured by locating several maxima in a resonant air column. Because there are several measurements of λ, they can be analyzed statistically, and the student concludes that $\lambda = 11.2 \pm 0.5$ cm. Only one measurement has been taken of $f = 3{,}000$ Hz (the setting on the oscillator), and the student has no way to judge its reliability. The instructor says that the oscillator is "certainly 1% reliable"; therefore, the student allows for a 1% systematic error in f (but none in λ). What is the student's answer for u with its uncertainty? Is the possible 1% systematic error from the oscillator's calibration important? **(b)** If the student's measurement had been $\lambda = 11.2 \pm 0.1$ cm and the oscillator calibration had been 3% reliable, what would the answer have been? Is the systematic error important in this case?