6.1. ★ An enthusiastic student makes 50 measurements of the heat Q released in a certain reaction. Her average and standard deviation are

$$\overline{Q} = 4.8 \quad \text{and} \quad \sigma_Q = 0.4,$$

both in kilocalories. **(a)** Assuming her measurements are governed by the normal distribution, find the probability that any one measurement would differ from $\overline{Q}$ by 0.8 kcal or more. How many of her 50 measurements should she expect to differ from $\overline{Q}$ by 0.8 kcal or more? If one of her measurements is 4.0 kcal and she decides to use Chauvenet's criterion, would she reject this measurement? **(b)** Would she reject a measurement of 6.0 kcal?

6.3. ★ A student makes 14 measurements of the period of a damped oscillator and obtains these results (in tenths of a second):

$$7,\ 3,\ 9,\ 3,\ 6,\ 9,\ 8,\ 7,\ 8,\ 12,\ 5,\ 9,\ 9,\ 3.$$

Believing that the result 12 is suspiciously high, she decides to apply Chauvenet's criterion. How many results should she expect to find as far from the mean as 12 is? Should she reject the suspect result?

6.5. ★★ In the course of a couple of hours, a nuclear engineer makes 12 measurements of the strength of a long-lived radioactive source with the following results, in millicuries:

$$12, 34, 22, 14, 22, 17, 24, 22, 18, 14, 18, 12.$$

(Because the source has a long life, its activity should not change appreciably during the time all the measurements are made.)
(a) What are his mean and standard deviation? (Use the built-in functions on your calculator.) **(b)** According to Chauvenet's criterion, would he be justified in rejecting the value 34 as a mistake? Explain your reasoning clearly. **(c)** If he does reject this measurement, what does he get for his new mean and standard deviation? (Read the hint to Problem 6.4.)

7.1. ★ Find the best estimate and its uncertainty based on the following four measurements of a certain voltage:

$$1.4 \pm 0.5, \quad 1.2 \pm 0.2, \quad 1.0 \pm 0.25, \quad 1.3 \pm 0.2.$$

7.3. ★ **(a)** Two measurements of the speed of sound u give the answers 334 ± 1 and 336 ± 2 (both in m/s). Would you consider them consistent? If so, calculate the best estimate for u and its uncertainty. **(b)** Repeat part (a) for the results 334 ± 1 and 336 ± 5. Is the second measurement worth including in this case?

7.5. ★★ Two students measure a resistance by different methods. Each makes 10 measurements and computes the mean and its standard deviation, and their final results are as follows:

$$\text{Student A:} \quad R = 72 \pm 8 \text{ ohms}$$

$$\text{Student B:} \quad R = 78 \pm 5 \text{ ohms.}$$

(a) Including both measurements, what are the best estimate of R and its uncertainty? **(b)** Approximately how many measurements (using his same technique) would student A need to make to give his result the same weight as B's? (Remember that each student's final uncertainty is the SDOM, which is equal to the SD/$\sqrt{N}$.)

8.1. ★ Use the method of least squares to find the line $y = A + Bx$ that best fits the three points (1, 6), (3, 5), and (5, 1). Using squared paper, plot the three points and your line. Your calculator probably has a built-in function to calculate A and B; if you don't know how to use it, take a moment to learn and then check your own answers to this problem.

8.3. ★ The best estimates for the constants A and B are determined by Equations (8.8) and (8.9). The solutions to these equations were given in Equations (8.10) through (8.12). Verify that these are indeed the solutions of (8.8) and (8.9).

8.5. ★★ Line Through the Origin. Suppose two variables x and y are known to satisfy a relation $y = Bx$. That is, $y \propto x$, and a graph of y vs x is a line *through the origin.* (For example, Ohm's law, $V = RI$, tells us that a graph of voltage V vs current I should be a straight line through the origin.) Suppose further that you have N measurements (x_i, y_i) and that the uncertainties in x are negligible and those in y are all equal. Using arguments similar to those of Section 8.2, prove that the best estimate for B is

$$B = \frac{\sum xy}{\sum x^2}.$$

8.7. ★★ To find the spring constant of a spring, a student loads it with various masses M and measures the corresponding lengths l. Her results are shown in Table 8.4. Because the force $mg = k(l - l_o)$, where l_o is the unstretched length of the spring, these data should fit a straight line, $l = l_o + (g/k)m$. Make a least-squares fit to this line for the given data, and find the best estimates for the unstretched length l_o and the spring constant k. Do the calculations yourself, and then check your answers using the built-in functions on your calculator.

Table 8.4. Length versus load for a spring; for Problem 8.7.

"x": Load m (grams)	200	300	400	500	600	700	800	900
"y": Length l (cm)	5.1	5.5	5.9	6.8	7.4	7.5	8.6	9.4

8.9. ★★ Weighted Least Squares. Suppose we measure N pairs of values (x_i, y_i) of two variables x and y that are supposed to satisfy a linear relation $y = A + Bx$. Suppose the x_i have negligible uncertainty and the y_i have *different* uncertainties σ_i. (That is, y_1 has uncertainty σ_1, while y_2 has uncertainty σ_2, and so on.) As in Chapter 7, we can define the *weight* of the ith measurement as $w_i = 1/\sigma_i^2$. Review the derivation of the least-squares fit in Section 8.2 and generalize it to cover this situation, where the measurements of the y_i have different weights. Show that the best estimates of A and B are

$$A = \frac{\Sigma wx^2 \, \Sigma wy - \Sigma wx \, \Sigma wxy}{\Delta} \tag{8.37}$$

and

$$B = \frac{\Sigma w \Sigma wxy - \Sigma wx \, \Sigma wy}{\Delta} \tag{8.38}$$

where

$$\Delta = \Sigma w \, \Sigma wx^2 - (\Sigma wx)^2. \tag{8.39}$$

Obviously, this method of *weighted least squares* can be applied only when the uncertainties σ_i (or at least their relative sizes) are known.

8.11. ★★ **(a)** If you have access to a spreadsheet program such as Lotus 123 or Excel, create a spreadsheet that will calculate the coefficients A and B for the least-squares fit for up to 10 points $(x_1, y_1), \ldots$. Use the layout of Table 8.1. **(b)** Test your spreadsheet with the data of Problems 8.1 and 8.7.

8.15. ★★ Kundt's tube is a device for measuring the wavelength λ of sound. The experimenter sets up a standing wave inside a glass tube in which he or she has sprinkled a light powder. The vibration of the air causes the powder to move and eventually to collect in small piles at the displacement nodes of the standing wave, as shown in Figure 8.5. Because the distance between the nodes is λ/2, this lets the

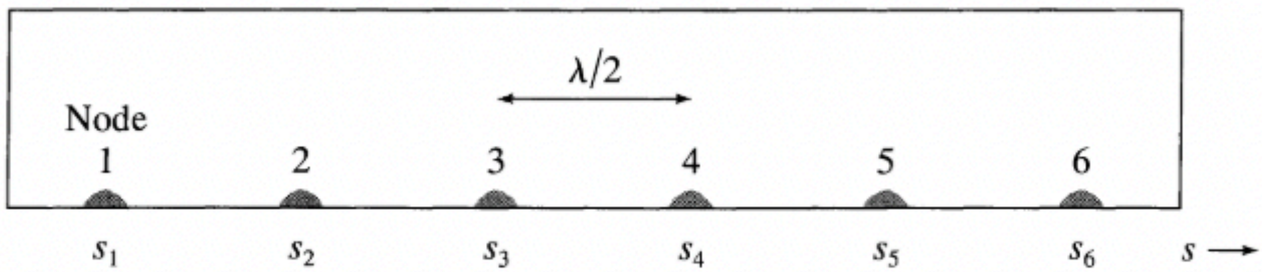

Figure 8.5. Kundt's tube, with small piles of powder at the nodes of a standing wave; for Problem 8.15.

experimenter find λ. A student finds six nodes (numbered $n = 1, \ldots, 6$) as shown in Table 8.7. Because the nodes should be equal distances λ/2 apart, their positions should satisfy $s_n = A + Bn$, where $B = \lambda/2$. Use the method of least squares to fit the data to this line and find the wavelength and its uncertainty.

Table 8.7. Positions of nodes in Kundt's tube; for Problem 8.15.

"x": Node number n	1	2	3	4	5	6
"y": Position s_n (cm)	5.0	14.4	23.1	32.3	41.0	50.4

8.21. ★★ Consider the problem of fitting a set of measurements (x_i, y_i), with $i = 1, \ldots, N$, to the polynomial $y = A + Bx + Cx^2$. Use the principle of maximum likelihood to show that the best estimates for A, B, and C based on the data are given by Equations (8.27). Follow the arguments outlined between Equations (8.24) and (8.27).

8.25. ★★ The rate at which a sample of radioactive material emits radiation decreases exponentially as the material is depleted. To study this exponential decay, a student places a counter near a sample and records the number of decays in 15 seconds. He repeats this five times at 10-minute intervals and obtains the results shown in Table 8.11. (Notice that, because it takes nearly 10 minutes to prepare the equipment, his first measurement is made at $t = 10$ min.)

Table 8.11. Number $\nu(t)$ of emissions in a 15-second interval versus total time elapsed t (in minutes); for Problem 8.25.

"x": Elapsed time t	10	20	30	40	50
"y": Number $\nu(t)$	409	304	260	192	170

If the sample does decay exponentially, the number $\nu(t)$ should satisfy

$$\nu(t) = \nu_o e^{-t/\tau}, \tag{8.42}$$

where τ is the (unknown) mean life of the material in question and ν_o is another unknown constant. To find the mean life τ, the student takes the natural log of Equation (8.42) to give the linear relation

$$z = \ln(\nu) = \ln(\nu_o) - t/\tau \tag{8.43}$$

and makes a least-squares fit to this line. What is his answer for the mean life τ? How many decays would he have counted in 15 seconds at $t = 0$?