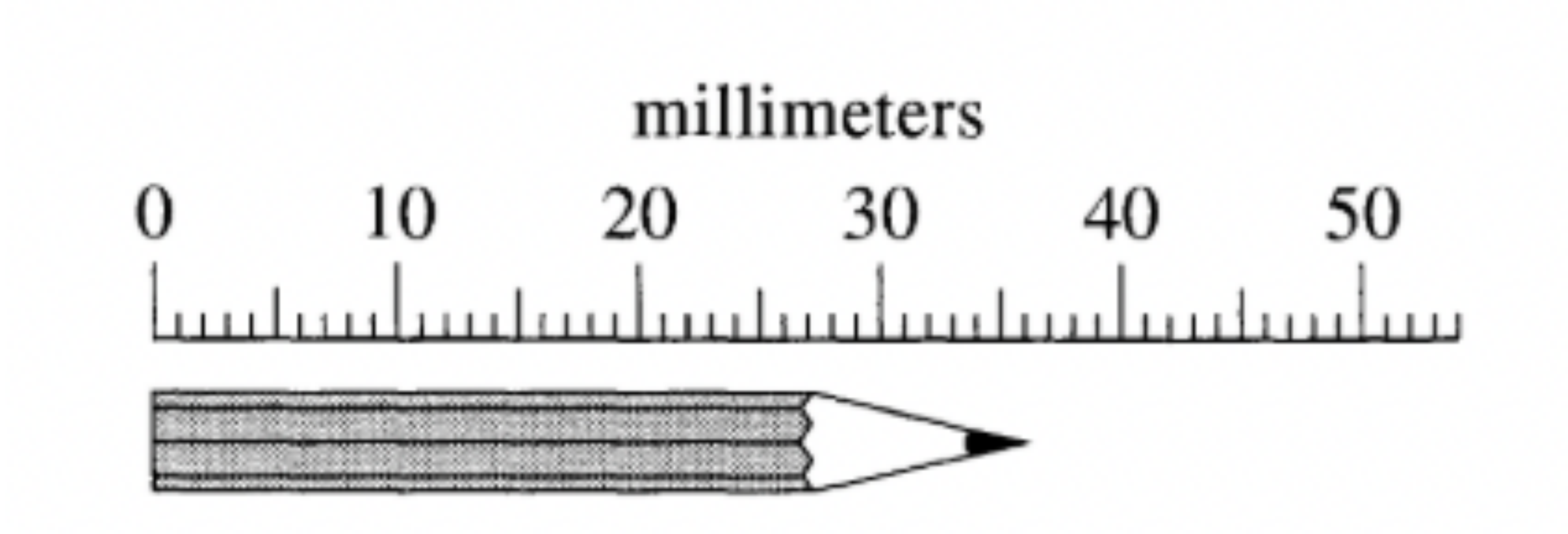
ME170b Lecture 2 1/19/24

Experimental Techniques

Last time:
> Hypotheses
> Lab report
> Uncertainties

Today:
> How to Report and Use Uncertainties (CH2)
> Propagation of Uncertainties (CH3)

How should uncertainties in an experiment be reported?



standard form

measured value $x = x_{best} \pm \delta x$

experimenters best estimate

uncertainty:

highest & lowest probable value.

("95% confidence interval")

Quick Check[1] 2.1. **(a)** A student measures the length of a simple pendulum and reports his best estimate as 110 mm and the range in which the length probably lies as 108 to 112 mm. Rewrite this result in the standard form (2.3). **(b)** If another student reports her measurement of a current as $I = 3.05 \pm 0.03$ amps, what is the range within which I probably lies?

---

a) 110 ± 2 mm

b.) 3.02 – 3.08 Amps

How many significant figures for uncertainty?

what's wrong with this estimate:

$$(\text{measured } g) = 9.82 \pm 0.02385 \text{ m/s}^2.$$

δx is an estimate - should have too much precision.

General Rule: δx should 1 significant figure

$$\boxed{9.82 \pm 0.02 \text{ m/s}^2}$$

How many significant figures for uncertainty?

what's wrong with this estimate:

$$(\text{measured } g) = 9.82 \pm 0.02385 \text{ m/s}^2.$$

dx is an <u>estimate</u> — should not be stated with too much precision

Rule for Stating Uncertainties

Experimental uncertainties should almost always be rounded to one significant figure.

rewrite the gravity estimate:

These are ‘rules of thumb’ — should use best judgment

If our estimate is: $\delta x = 0.14.$

rounding to one sigfig: $\delta x = 0.1$

That’s nearly 50% difference!

If leading digit is 1 (or 2) it’s okay to use two significant figures

Significant figures of the estimate is determined <u>after</u> uncertainty

what's wrong with this estimate:

$$\text{measured speed} = 6051.78 \pm 30 \text{ m/s}$$

$6050. \pm 30$ m/s

Significant figures of the estimate is determined after uncertainty

what's wrong with this estimate:

$$\text{measured speed} = 6051.78 \pm 30 \text{ m/s}$$

$$\text{measured speed} = 6050 \pm 30 \text{ m/s}.$$

Rule for Stating Answers

The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.

One caveat

An important qualification to rules (2.5) and (2.9) is as follows: To reduce inaccuracies caused by rounding, *any numbers to be used in subsequent calculations should normally retain at least one significant figure more than is finally justified.*

Scientific Notations

$$\text{measured charge} = 1.61 \times 10^{-19} \pm 5 \times 10^{-21} \text{ coulombs.}$$

$$= (1.61 \pm 0.05) \times 10^{-19} \text{ coulombs}$$

Simpler & clearer to put best estimate & uncertainty in same form.

Quick Check 2.2. Rewrite each of the following measurements in its most appropriate form:

(a) $v = 8.123456 \pm 0.0312$ m/s
(b) $x = 3.1234 \times 10^4 \pm 2$ m
(c) $m = 5.6789 \times 10^{-7} \pm 3 \times 10^{-9}$ kg.

a. 8.12 ± 0.03 m/s

b. 31234 ± 2 m or $3.1234 \pm 0.0002 \times 10^4$ m

c. 5.6789 ± 0.003 kg →

$$\boxed{5.679 \pm 0.003 \text{ kg}}$$

Discrepancy

The difference between two measured values of the same quantity

Student A: 15 ± 1 ohms
Student B: 25 ± 2 ohms,

What is the discrepancy?

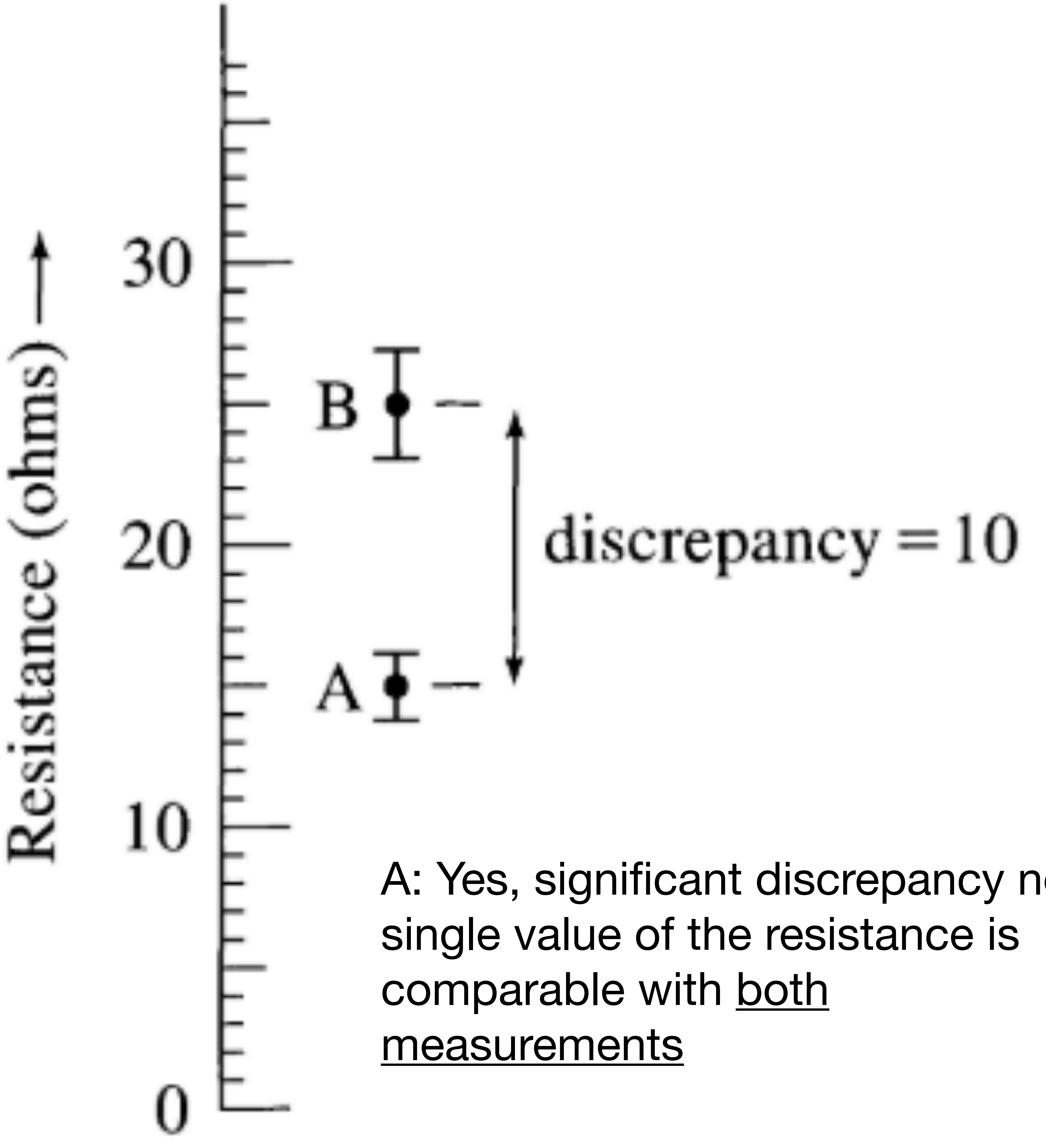
Are the discrepancy *significant*?

$d_{AB} = 25 - 15\,\Omega = 10\,\Omega$

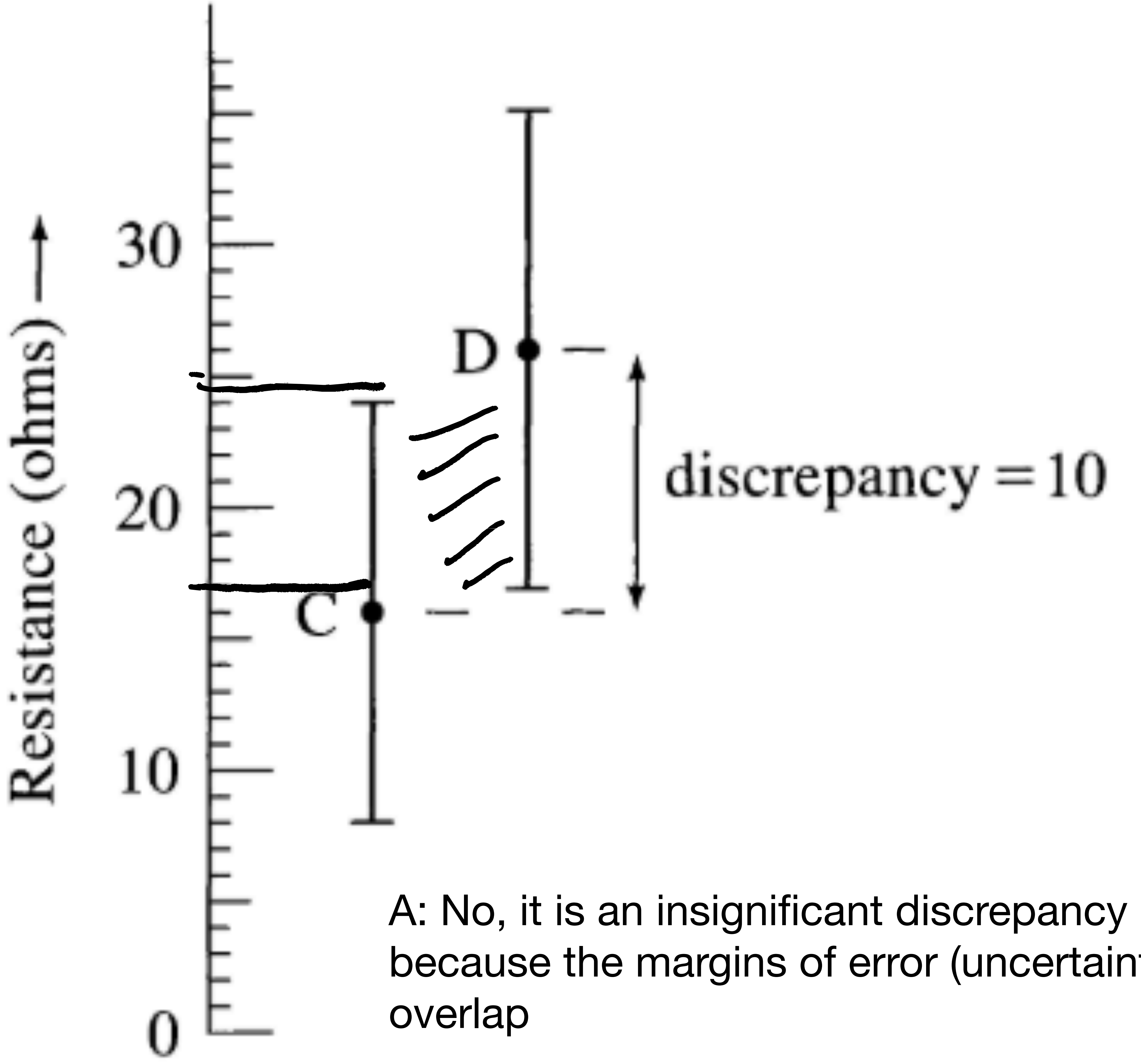
Student C: 16 ± 8 ohms
Student D: 26 ± 9 ohms.

$d_{CD} = 26 - 16 = 10$

Are the discrepancy significant? Why or why not?



A: Yes, significant discrepancy no single value of the resistance is comparable with <u>both measurements</u>



A: No, it is an insignificant discrepancy because the margins of error (uncertainty) overlap

True Error

the difference between a measured value and the true value

the reality is that true value can almost never be known!

example: currently the accepted value of universal gas constant:

$8.31451 \pm 0.00007 \; J/(mol \cdot K)$

Can you think of something in which the true value is known?

- amount of people in a room

$\pi = \frac{C}{D}$

True Error

the difference between a measured value and the true value

the reality is that true value can <u>almost never</u> be known

example: currently <u>accepted value</u> of the universal gas constant

$$(\text{accepted } R) = 8.31451 \pm 0.00007 \text{ J/(mol} \cdot \text{K)}.$$

Can you think of something in which the true value is known?

Experiments without drawing a conclusion (testing a hypothesis) has little merit.

A <u>single measurement</u> for an experiment, by itself is also uninteresting

Experiments should <u>compare two or more numbers</u>

Two major types of experiments:

1. comparing a measured value to the accepted value (H: The measured value will equal the accepted value)

2. several measurements and you want to relate the measurements to physical law (H: The measurement will follow some theoretical law)

Example: Experiment for measuring a quantity with a <u>known value</u>

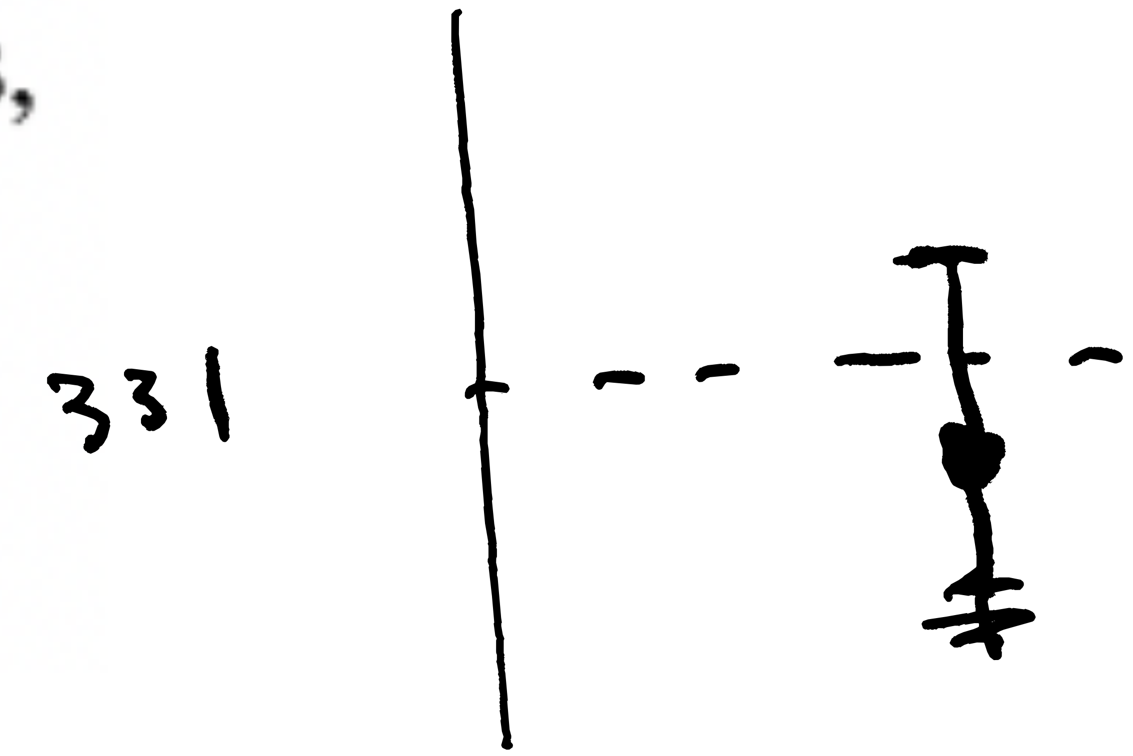
H: the measurement with be equal to the known value

Procedure:

1. measure quantity
2. estimate the experimental uncertainty
3. compare with the accepted value (test our hypothesis)

$$\text{A's measured speed} = 329 \pm 5 \text{ m/s},$$

$$\text{accepted speed} = 331 \text{ m/s}.$$



How to present the result?

Example: Experiment for measuring a quantity with a known value

H: the measurement with be equal to the known value

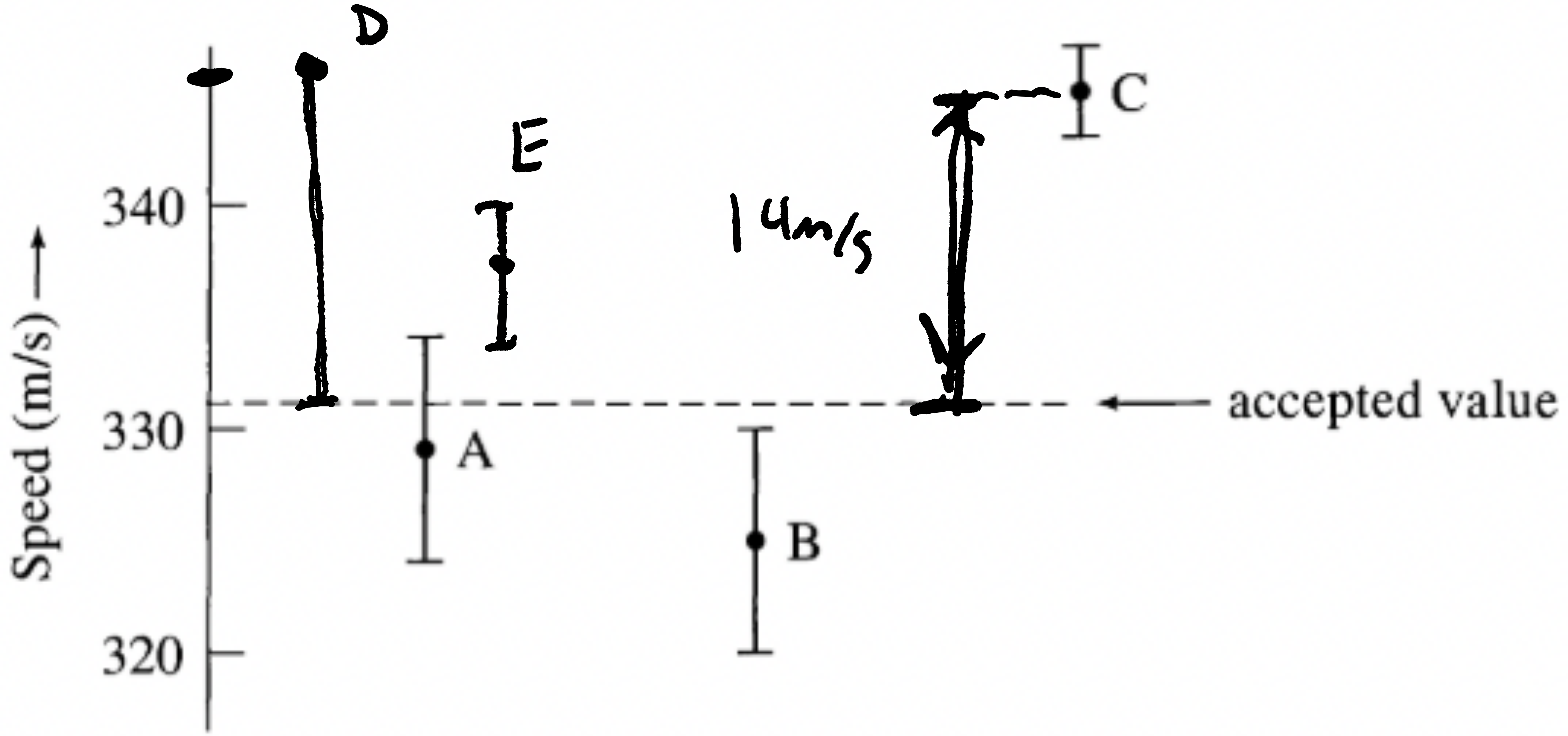
D = 345 ± 14

A's measured speed = 329 ± 5 m/s,

B's measured speed = 325 ± 5 m/s,

C's measured speed = 345 ± 2 m/s

E = 334



A ~~& B~~: "The excepted value lies within our margin of error, therefore the data supports our hypothesis"

C: "The discrepancy is 14 m/s which is approximately 7 times the uncertainty of our measurement. Our hypothesis is reject, however, it's possible our measurement is wrong."

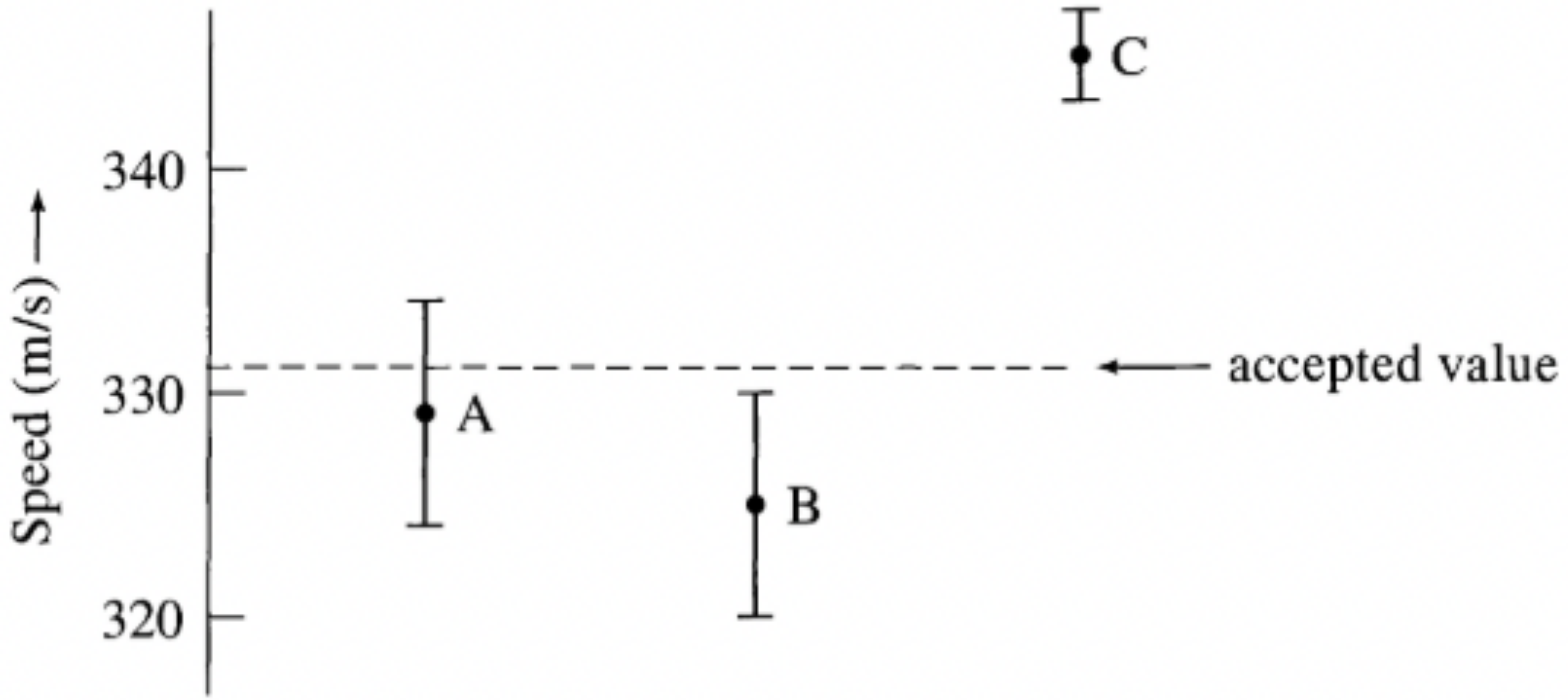
Example: Experiment for measuring a quantity with a known value

H: the measurement with be equal to the known value

A's measured speed $= 329 \pm 5$ m/s,

B's measured speed $= 325 \pm 5$ m/s,

C's measured speed $= 345 \pm 2$ m/s



Example: An experiment comparing two measurements

H: the measurements with be equal

Conservation of momentum states total momentum of isolated system is constant

We can use an experiment with two measurements to confirm theory. How?

$$p = m \cdot v$$

Example: An experiment comparing two measurements

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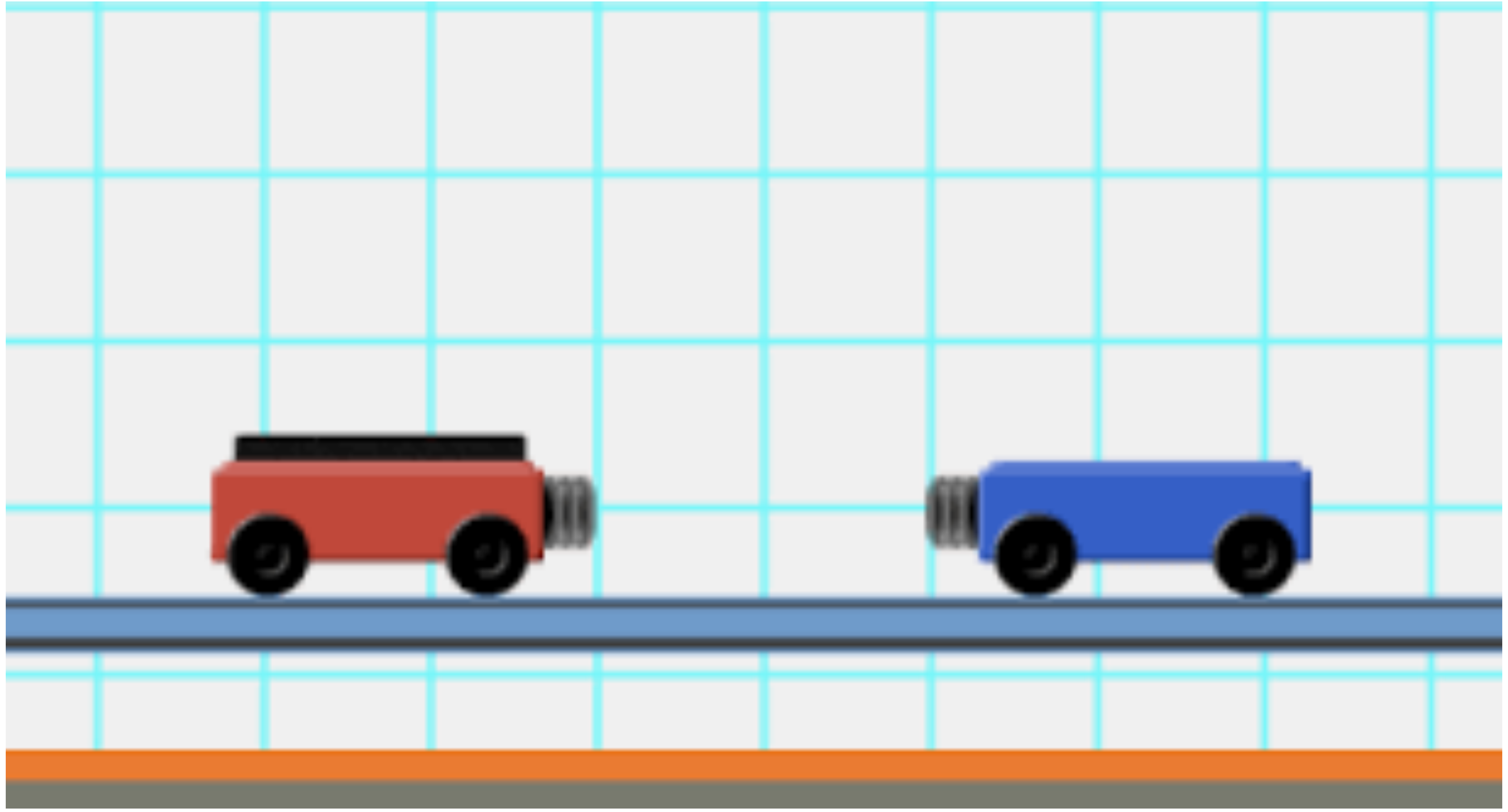
Conservation of momentum states total momentum of isolated system is constant

We can use an experiment with two measurements to confirm theory.



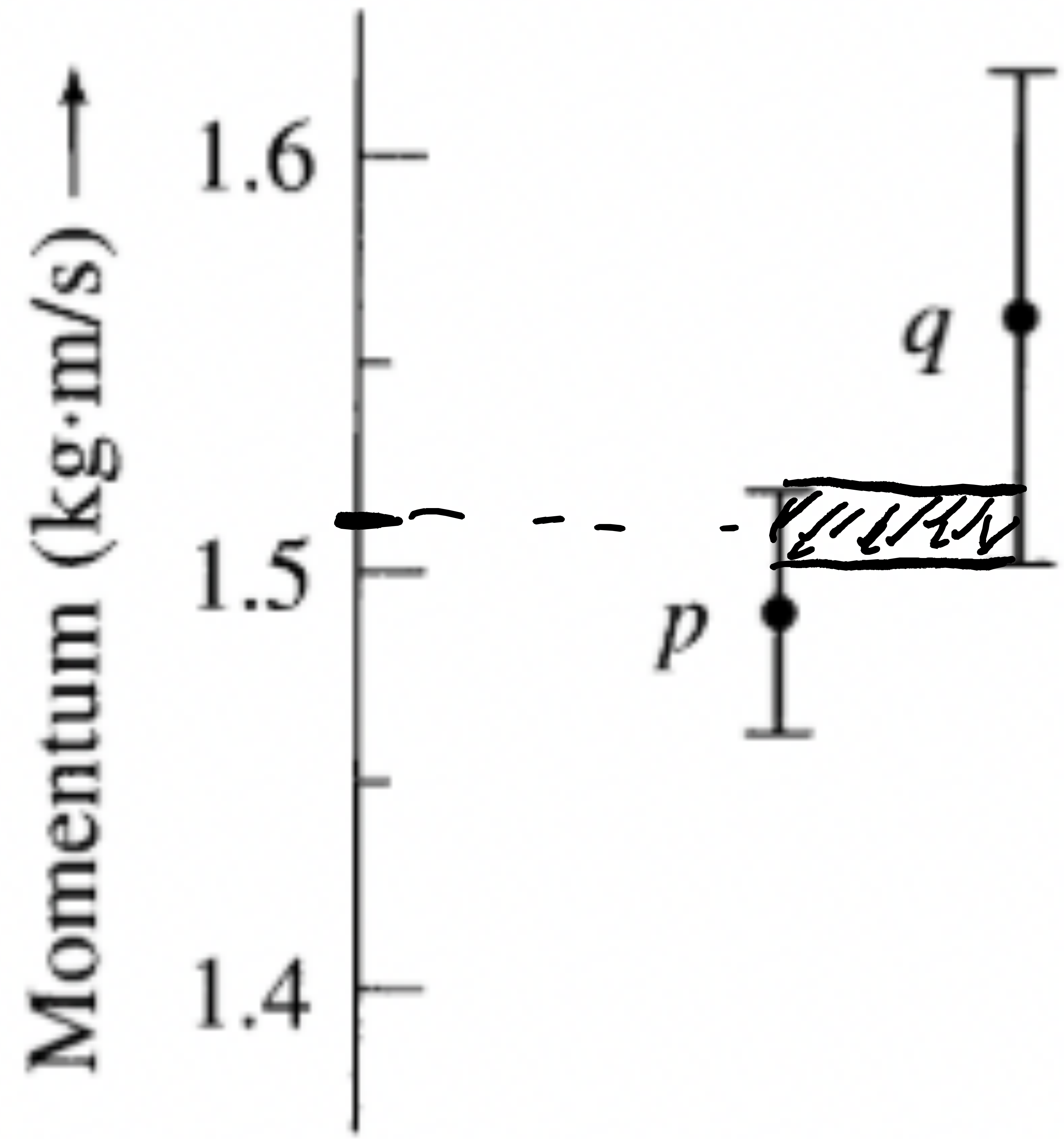
measure rho before and after collision — theory says they will be the same!

Example: An experiment comparing two measurements

H: the measurements with be equal

initial momentum $p = 1.49 \pm 0.03$ kg·m/s

final momentum $q = 1.56 \pm 0.06$ kg·m/s.



Does the data support our hypothesis? Why/why not?

The measurements overlap! – supports our Hypothesis

↳ the measurement are consistent w/ conservation of momentum.

Repeated measures

Table 2.1. Measured momenta (kg·m/s).

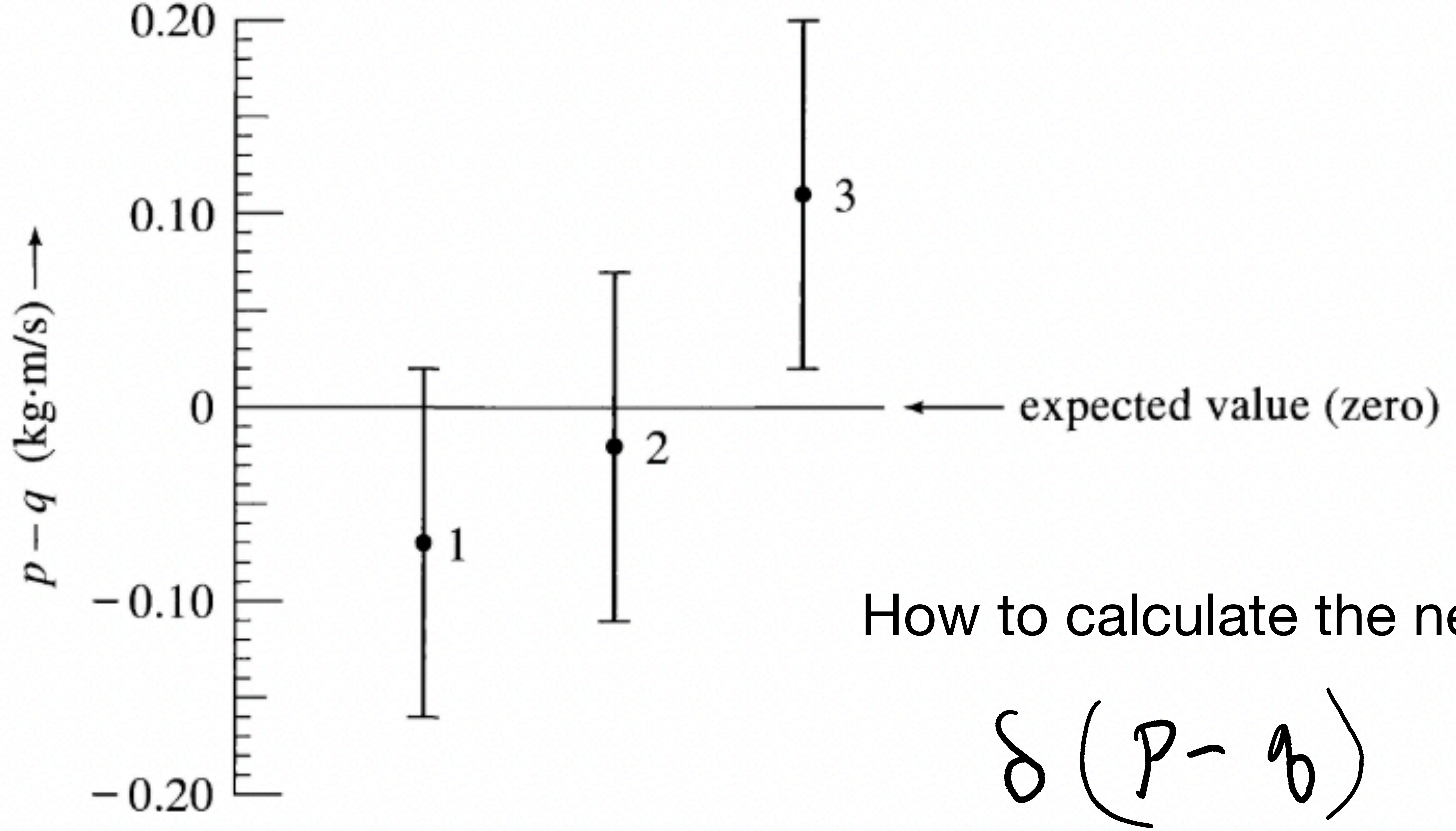
Trial number	Initial momentum p (all ± 0.03)	Final momentum q (all ± 0.06)
1	1.49	1.56
2	3.10	3.12
3	2.16	2.05
etc.		

differen

another way to think about it: p - q -> 0

Example: An experiment comparing two measurements

Another way is to subtract p - q — the result should be zero!



How to calculate the new uncertainty?

$\delta(p-q)$

How to find uncertain of $p - q$

$p = p_{best} \pm \delta p$

$q = q_{best} \pm \delta q$

Highest value : $p = p_{best} + \delta p$

$q = q_{best} - \delta q$

Highest : $(p_{best} - q_{best}) + (\delta p + \delta q)$

lowest value : $p = p_{best} - \delta p$

$q = q_{best} + \delta q$

lowest value : $(p_{best} - q_{best}) - (\delta p + \delta q)$

How to calculate the new uncertainty when subtracting?

Check the extreme values!

highest probable value:

p = p_best + dp (largest probable value)

q = q_best - dq (smallest probable value)

hight estimate = (p_best - q_best) + (dp + dq)

lowest probable value:

p = p_best - dp (smallest probable value)

q = q_best + dq (largest probable value)

low estimate = (p_best - q_best) - (dp + dq)

Uncertainty in Difference (provisional rule)

Uncertainty in a Difference
(Provisional Rule)

If two quantities x and y are measured with uncertainties δx and δy, and if the measured values x and y are used to calculate the difference $q = x - y$, the *uncertainty in q* is the *sum of the uncertainties in x and y*:

$$\delta q \approx \delta x + \delta y.$$

Provisional because we will update with better uncertainty estimate

Quick Check 2.3. In an experiment to measure the latent heat of ice, a student adds a chunk of ice to water in a styrofoam cup and observes the change in temperature as the ice melts. To determine the mass of ice added, she weighs the cup of water before and after she adds the ice and then takes the difference. If her two measurements were

$$(\text{mass of cup \& water}) = m_1 = 203 \pm 2 \text{ grams}$$

and

$$(\text{mass of cup, water, \& ice}) = m_2 = 246 \pm 3 \text{ grams},$$

find her answer for the mass of ice, $m_2 - m_1$, with its uncertainty, as given by the provisional rule (2.18).

$$\boxed{43 \pm 5 \text{ g.}}$$

Relate to measurements through a physical law: Graphical Methods

Physical Laws typically imply a relationship between quantities

Example: Hookes Law
> let’s design an experiment to confirm Hookes Law
> What is the hypothesis?

$$\cancel{F = kx, \quad x = F/k}$$

$$F = Kx$$

Graphical Methods

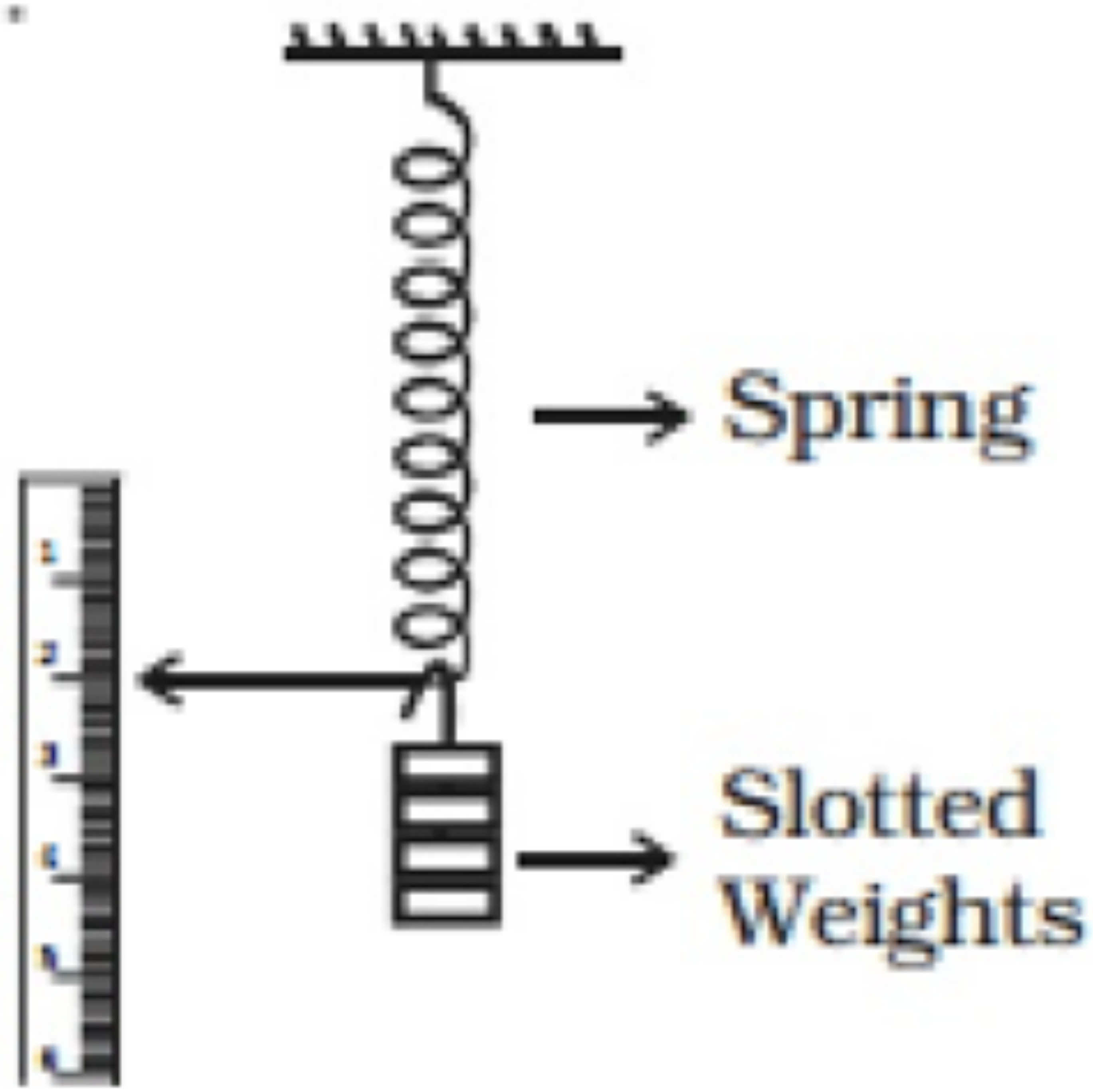


Fig. Experimental setup to verify Hooke's law

$$x = \frac{mg}{k} = \left(\frac{g}{k}\right)m.$$

displacement

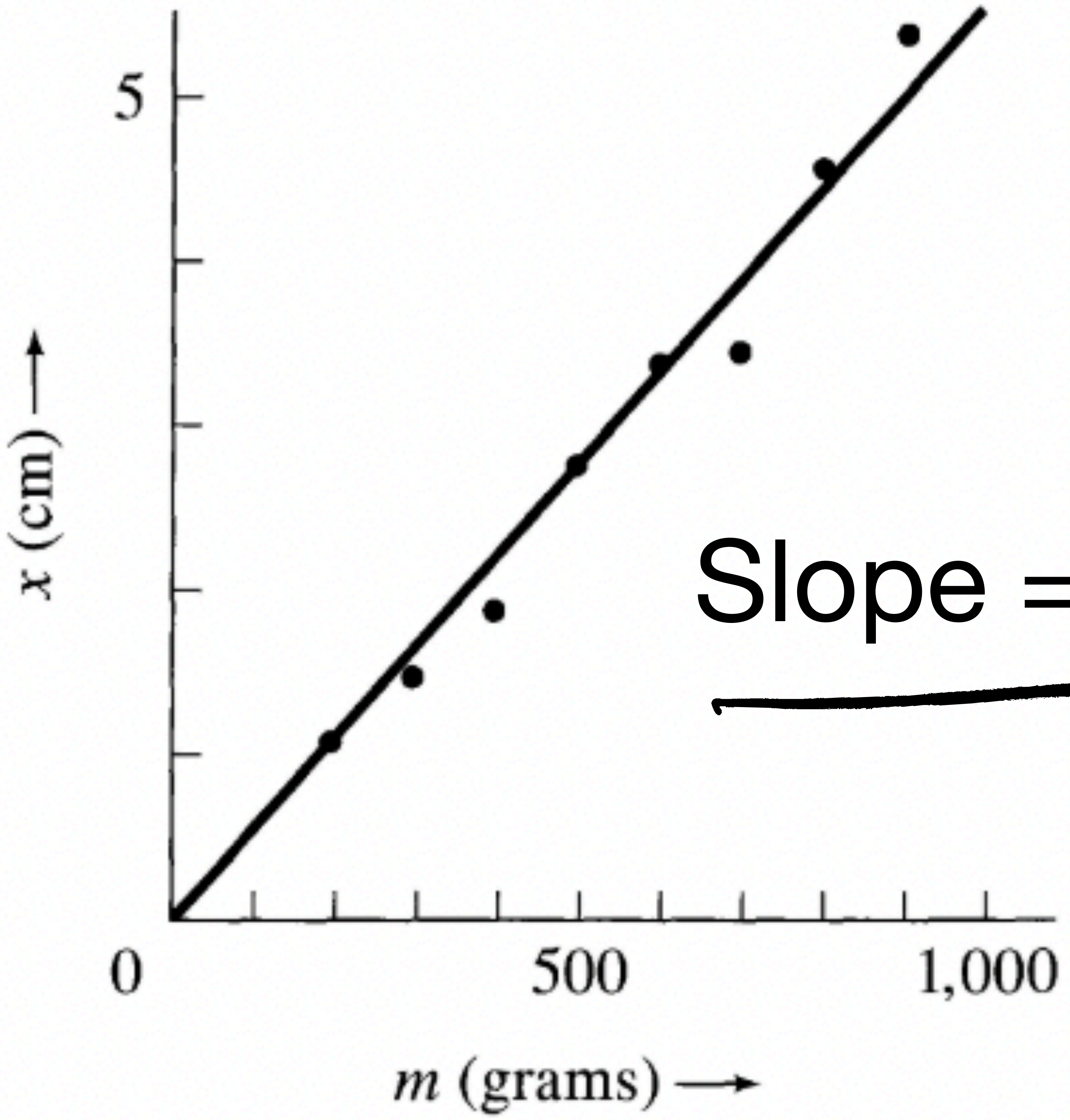
mass

slope

Graphical Methods

Table 2.3. Load and extension.

Load m (grams) (δm negligible)	200	300	400	500	600	700	800	900
Extension x (cm) (all ± 0.3)	1.1	1.5	1.9	2.8	3.4	3.5	4.6	5.4

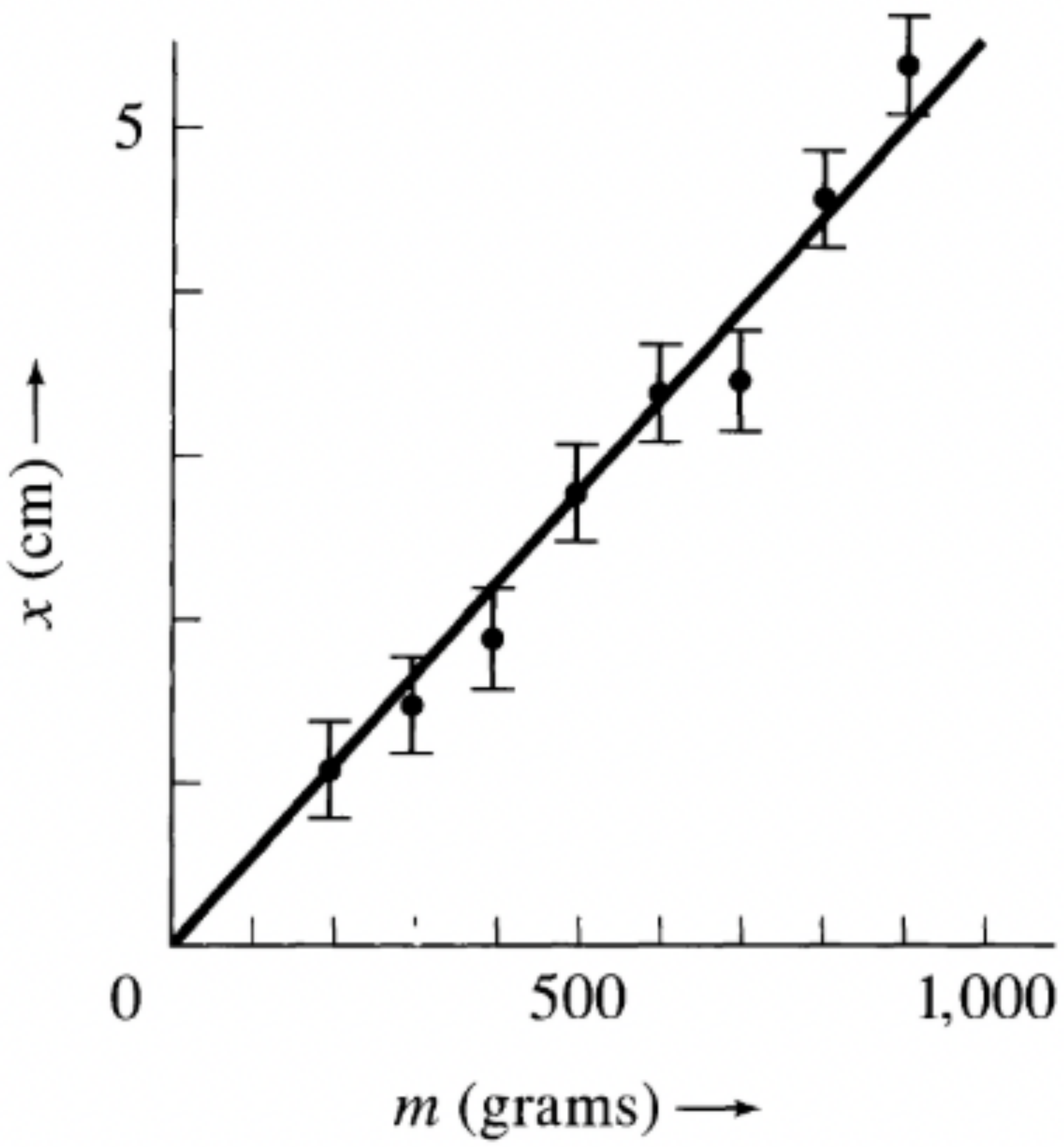


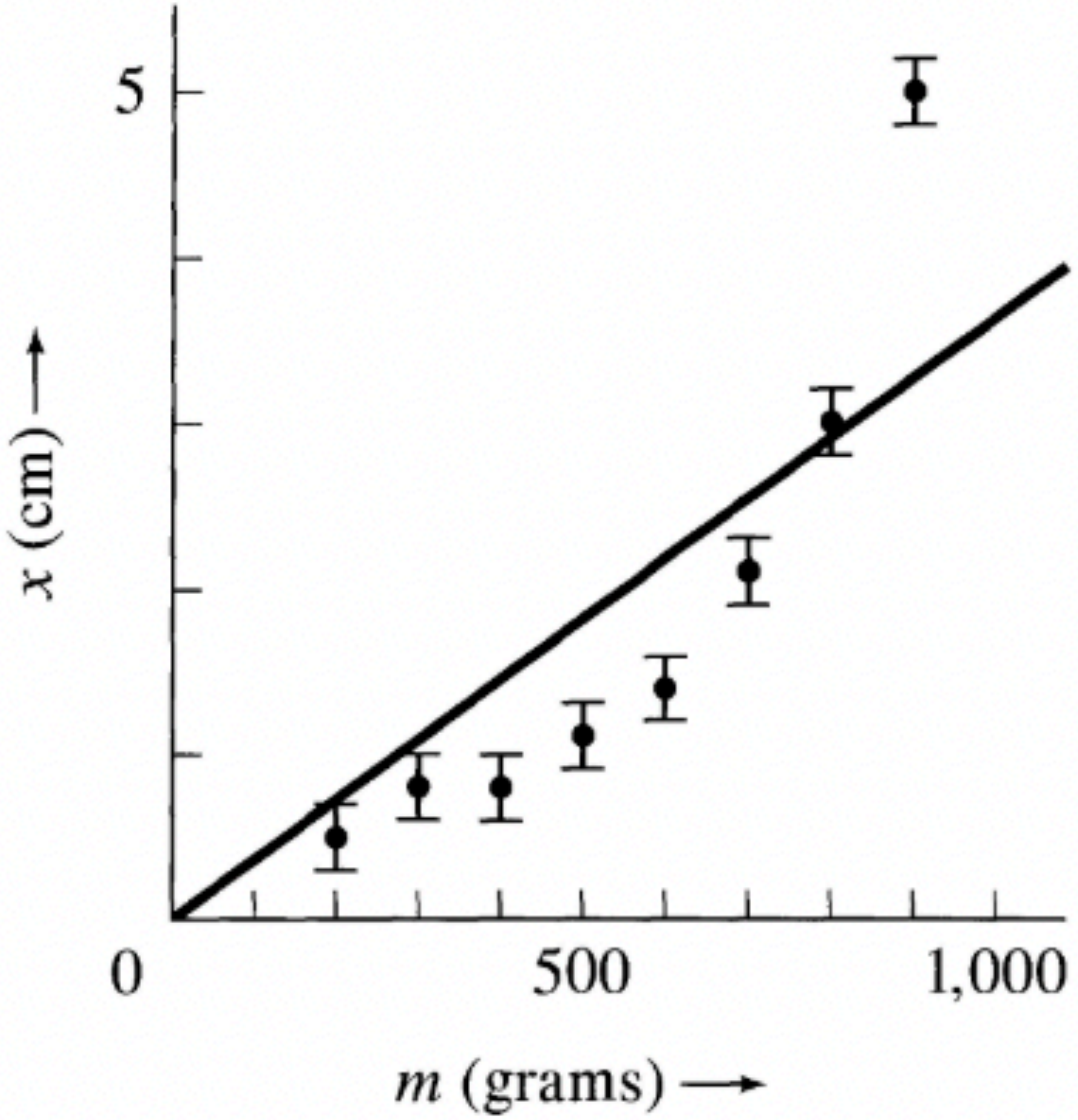
Slope = g/k

Easy to visually draw 'best fit' line
which will confirm the linear relationship

Graphical Methods

Better to include error bars

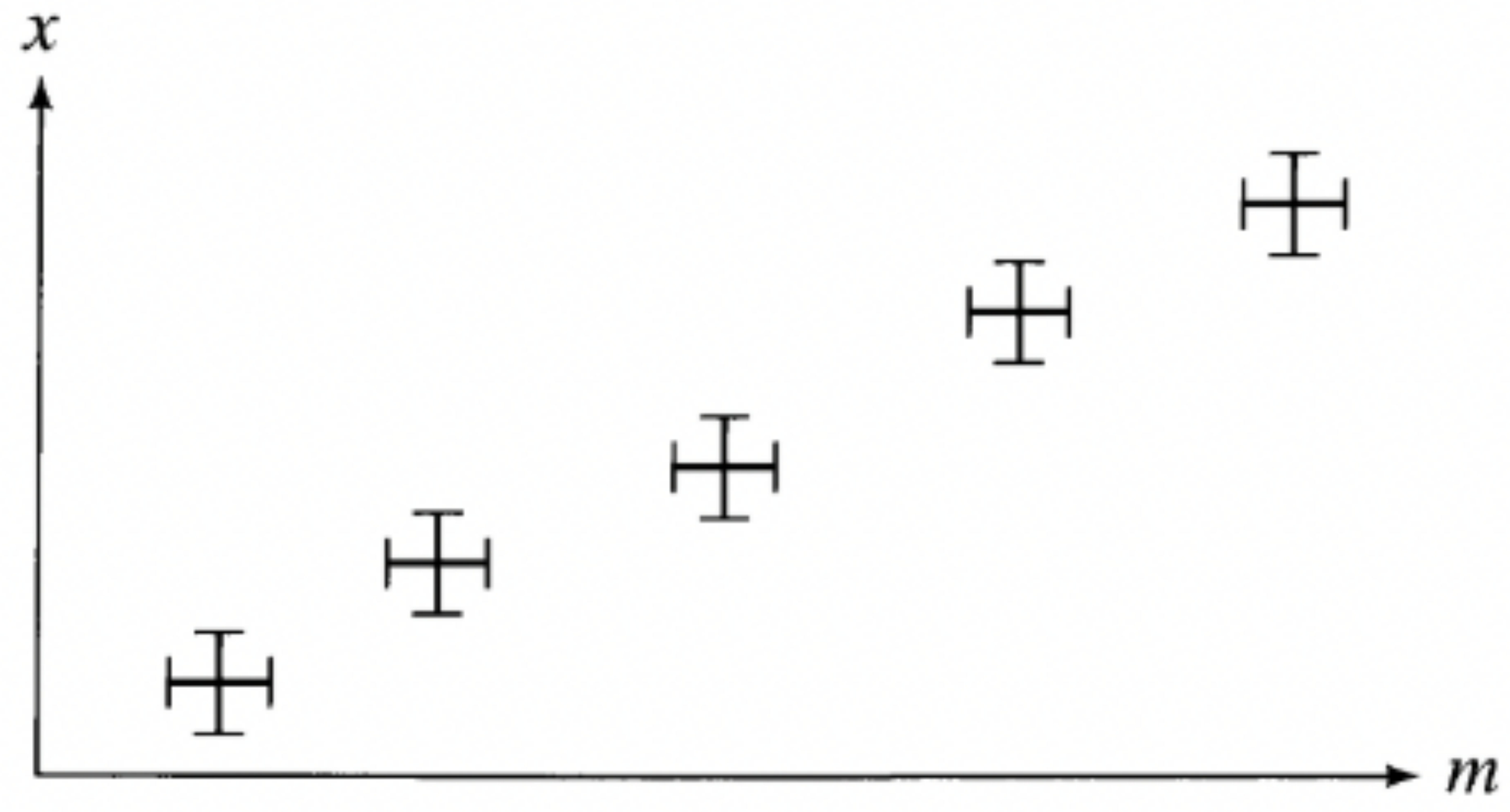




Right plot does not support hypothesis (k = constant)

Graphical Methods

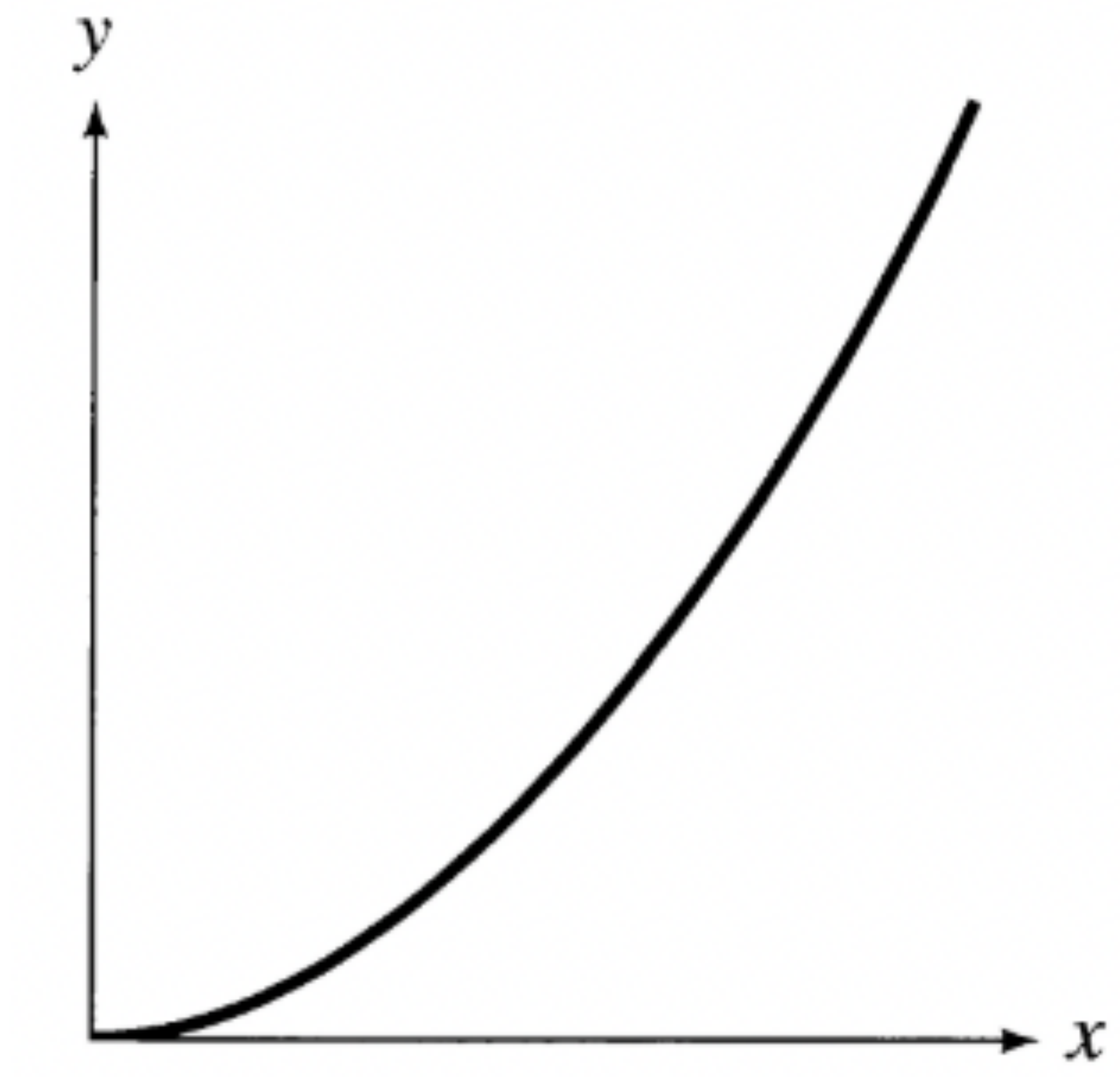
What if mass also has uncertainty?



Graphical Methods

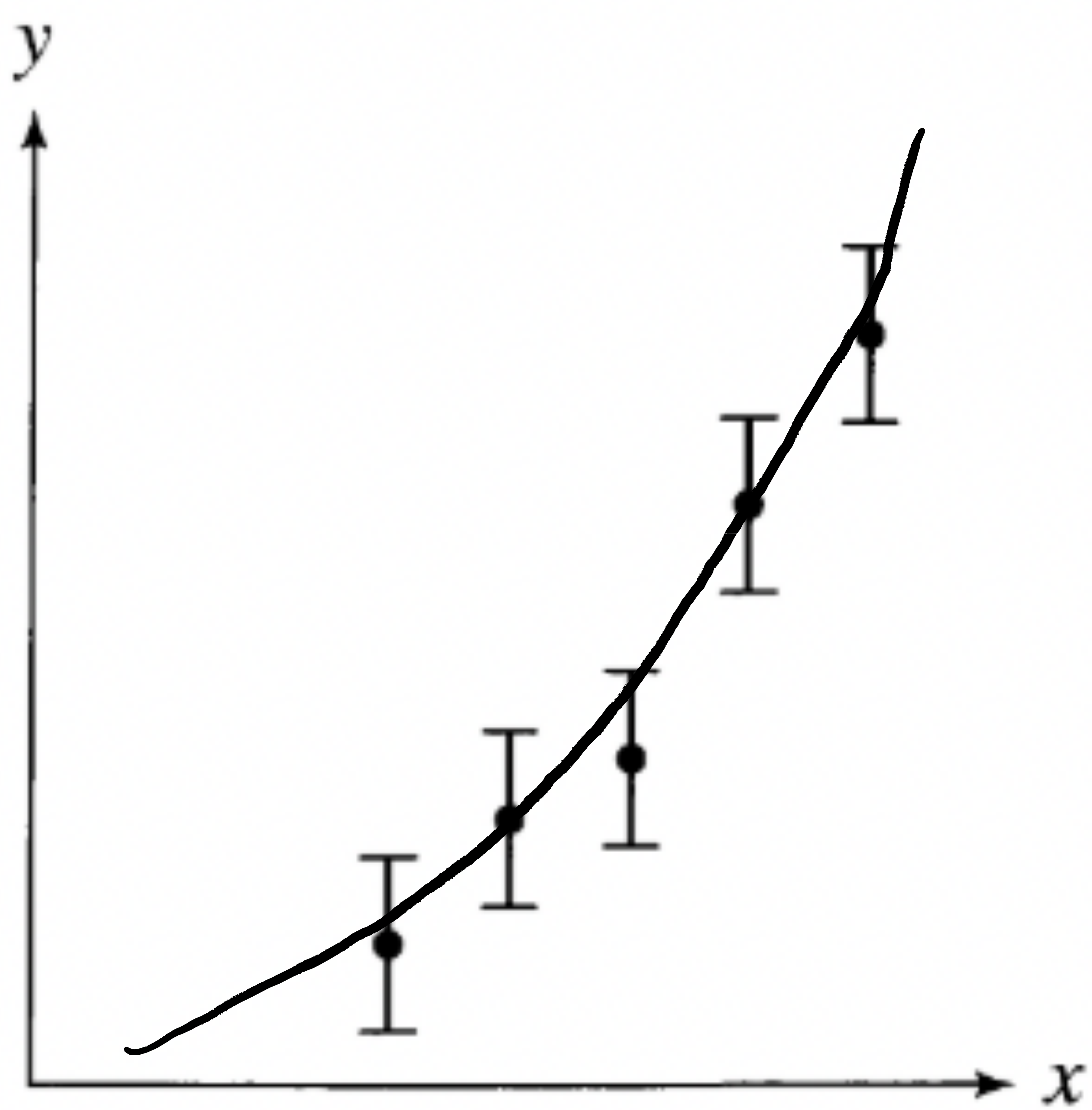
Different ways to show data for clarity

$$y = Ax^2,$$



Graphical Methods

Different ways to show data for clarity



If we plot the data directly, is hard to determine visually if the data follows the theoretical (y = Ax^2)

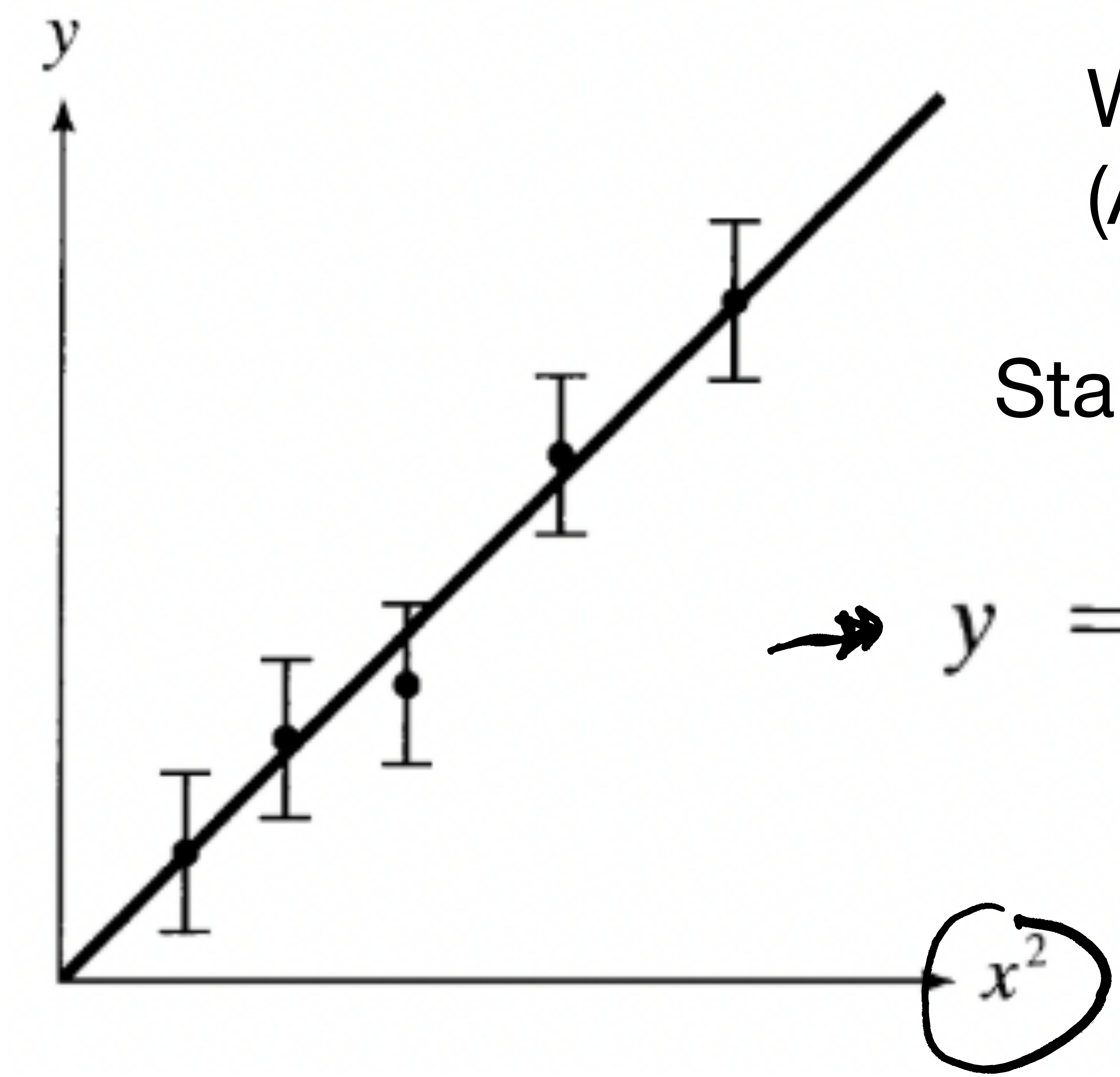
Is there something that can be done?

$$\boxed{y = A z}$$

x^2

Graphical Methods

Different ways to show data for clarity



We can plot x^2 vs y which has linear (A) relationship

Standard approach, used all the time

$$y = Ae^{Bx}.$$

ln(y) = ln(A) + Bx

but what about uncertainty?

Fractional uncertainty

same units as "best"

$$(\text{measured } x) = x_{\text{best}} \pm \delta x,$$

indicates the reliability or precision of the measurement.

indicates the reliability or precision of the measurement. The uncertainty δx by itself does not tell the whole story, however. An uncertainty of one inch in a distance of one mile would indicate an unusually precise measurement, whereas an uncertainty of one inch in a distance of three inches would indicate a rather crude estimate.

$$\boxed{\text{fractional uncertainty} = \frac{\delta x}{|x_{\text{best}}|}.}$$

also called the *relative uncertainty* or the *precision*.

Fractional uncertainty allows a more intuitive sense of uncertainty

$$\text{length } l = 50 \pm 1 \text{ cm}$$

$$\frac{\delta l}{|l_{\text{best}}|} = \frac{1 \text{ cm}}{50 \text{ cm}} = 0.02$$

$$\text{length } l = \underline{50 \text{ cm} \pm 2\%}.$$

The fractional uncertainty is an approximate indication of the quality of a measurement, whatever the size of the quantity measured. Fractional uncertainties of 10% or so are usually characteristic of fairly rough measurements. (A rough mea-

Quick Check 2.4. Convert the errors in the following measurements of the velocities of two carts on a track into fractional errors and percent errors: **(a)** $v = 55 \pm 2$ cm/s; **(b)** $u = -20 \pm 2$ cm/s. **(c)** A cart's kinetic energy is measured as $K = 4.58$ J $\pm$ 2%; rewrite this finding in terms of its absolute uncertainty. (Because the uncertainties should be given to one significant figure, you ought to be able to do the calculations in your head.)

[redacted].58 +- 0.09 J

$v = 55 \pm \frac{2}{|55|}\,\%$ cm/s

$\boxed{55 \pm 4\%\ \text{cm/s}}$

$u = -20 \pm 10\%$

$c = 4.58 \pm 0.09$ J

Fractional Uncertainty and Significant Figures

The concept of fractional uncertainty is closely related to the familiar notion of significant figures. In fact, the number of significant figures in a quantity is an approximate indicator of the fractional uncertainty in that quantity. To clarify this

$x = 21$ to two significant figures means

unambiguously that x is closer to 21 than to either 20 or 22; thus, the number 21,

two significant figures, means 21 ± 0.5.

Fractional Uncertainty and Significant Figures

Table 2.4. Approximate correspondence between significant figures and fractional uncertainties.

Number of significant figures	Corresponding fractional uncertainty is	
	between	or roughly
1	10% and 100%	50%
2	1% and 10%	5%
3	0.1% and 1%	0.5%

Multiplying measurements

$$(\text{measured value of } x) = x_{\text{best}} \pm \delta x$$

in terms of the fractional uncertainty, as

$$(\text{measured value of } x) = x_{\text{best}}\left(1 \pm \frac{\delta x}{|x_{\text{best}}|}\right).$$

Let us now return to our problem of calculating $p = mv$, when m and v have been measured, a

$$(\text{measured } m) = m_{\text{best}}\left(1 \pm \frac{\delta m}{|m_{\text{best}}|}\right)$$

$$(\text{measured } v) = v_{\text{best}}\left(1 \pm \frac{\delta v}{|v_{\text{best}}|}\right)$$

Multiplying measurements

$$\text{(best estimate for } p) = p_{\text{best}} = m_{\text{best}} v_{\text{best}}.$$

$$\text{(largest value for } p) = m_{\text{best}} v_{\text{best}} \left(1 + \frac{\delta m}{|m_{\text{best}}|}\right)\left(1 + \frac{\delta v}{|v_{\text{best}}|}\right).$$

$<<1$ $<<1$

$$\left(1 + \frac{\delta m}{|m_{\text{best}}|}\right)\left(1 + \frac{\delta v}{|v_{\text{best}}|}\right) = 1 + \frac{\delta m}{|m_{\text{best}}|} + \frac{\delta v}{|v_{\text{best}}|} + \frac{\delta m}{|m_{\text{best}}|}\frac{\delta v}{|v_{\text{best}}|}.$$

neglect

$$\text{(largest value of } p) = m_{\text{best}} v_{\text{best}} \left(1 + \frac{\delta m}{|m_{\text{best}}|} + \frac{\delta v}{|v_{\text{best}}|}\right).$$

Multiplying measurements

$$(\text{value of } p) = m_{\text{best}} v_{\text{best}} \left(1 \pm \left[\frac{\delta m}{|m_{\text{best}}|} + \frac{\delta v}{|v_{\text{best}}|} \right] \right).$$

Uncertainty in a Product
(Provisional Rule)

If two quantities x and y have been measured with small fractional uncertainties $\delta x/|x_{\text{best}}|$ and $\delta y/|y_{\text{best}}|$, and if the measured values of x and y are used to calculate the product $q = xy$, then the *fractional uncertainty in q is the sum of the fractional uncertainties in x and y,*

$$\frac{\delta q}{|q_{\text{best}}|} \approx \frac{\delta x}{|x_{\text{best}}|} + \frac{\delta y}{|y_{\text{best}}|}.$$

Multiplying measurements

Quick Check 2.5. To find the area of a rectangular plate, a student measures its sides as $l = 9.1 \pm 0.1$ cm and $b = 3.3 \pm 0.1$ cm. Express these uncertainties as percent uncertainties and then find the student's answer for the area $A = lb$ with its uncertainty. (Find the latter as a percent uncertainty first and then convert to an absolute uncertainty. Do all error calculations in your head.)

1% and 3%; area = 30 cm^2 +- 4% or 30 +- 1cm^2

$l = 9.1 \pm \frac{0.1}{9.1}$

$= 9.1 \pm 1\%$

$b = 3.3 \pm \frac{0.1}{3.3}$

$= 3.3 \pm 3\%$

$= 30.03 \pm 4\%$

$= 30 \pm 4\%$