ME 221, Fall 2021
University of California, Riverside
Department of Mechanical Engineering

HW 5 (Half Standard HW Pts EC)
Due: 12/1/2021

1. The forward kinematics for the articulate robot shown in Fig. 2 can be described from the following individual transformation matrices:

$$
{}^0T_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^1T_2 = \begin{bmatrix} s\theta_2 & -s\theta_1 & 0 & l_2 c\theta_2 \\ c\theta_2 & c\theta_1 & 0 & l_2 s\theta_2 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^2T_3 = \begin{bmatrix} c\theta_3 & 0 & s\theta_3 & 0 \\ s\theta_3 & 0 & -c\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3T_4 = \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ s\theta_4 & 0 & -c\theta_4 & 0 \\ 0 & -1 & 0 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^4T_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^5T_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Find the 6×6 Jacobian matrix using *Jacobian generating vectors:*

$$
J(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{c}_1(\boldsymbol{q}) & \boldsymbol{c}_2(\boldsymbol{q}) & \cdots & \boldsymbol{c}_6(\boldsymbol{q}) \end{bmatrix}.
$$

2. Consider the geometric Jacobian:

$$
J(\boldsymbol{q}) = \begin{bmatrix} J_D \\ J_R \end{bmatrix} = \begin{bmatrix} -s\theta_1 s\theta_2 d_3 - c\theta_1 l_2 & c\theta_1 c\theta_2 d_3 & c\theta_1 s\theta_2 \\ c\theta_1 s\theta_2 d_3 - s\theta_1 l_2 & s\theta_1 c\theta_2 d_3 & s\theta_1 s\theta_2 \\ 0 & -s_2 d_3 & c\theta_2 \\ 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix}
$$

Find the values of $\boldsymbol{q}$ that result in singularities in the *displacement* Jacobian, J_D.

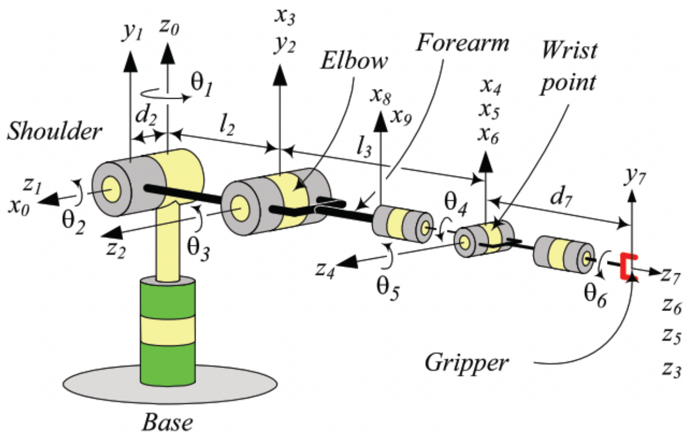

Figure 1: Schematic for Problem 1

3. Suppose you have a dynamic model of a robot in the form:

$$\tilde{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \tilde{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \tilde{q}(\boldsymbol{q}) = \boldsymbol{\tau}$$

   a. Derive an expression for the control variable $\boldsymbol{\tau}$ using the *computed torque controller.*
   b. Draw a block diagram of the controller and plant (robot), with all relevant signals labeled.

4. Show that the robot dynamics can be expressed in terms of the end-effector variables $\boldsymbol{x}$:

$$\Lambda(\boldsymbol{q})\ddot{\boldsymbol{x}} + \Gamma(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{x}} + \eta(\boldsymbol{q}) = \boldsymbol{F}$$

with

$$\Lambda(\boldsymbol{q}) = J^{-T}(\boldsymbol{q})M(\boldsymbol{q})J^{-1}(\boldsymbol{q}),$$
$$\Gamma = J^{-T}(\boldsymbol{q})C(\boldsymbol{q}, \dot{\boldsymbol{q}})J^{-1}(\boldsymbol{q}) - \Lambda(\boldsymbol{q})\dot{J}(\boldsymbol{q})J^{-1}(\boldsymbol{q}),$$
$$\eta(\boldsymbol{q}) = J^{-T}(\boldsymbol{q})g(\boldsymbol{q})$$

and $\boldsymbol{F}$ is the total force (wrench) at the end-effector.

5. Find and plot a polynomial path for a single joint coordinate that satisfies the following conditions:

   a. $q(1) = 0, \quad \dot{q}(1) = 0, \quad q(2) = \pi/2, \quad q(3) = 0 \quad , \dot{q}(3) = 0$
   b. $q(1) = 0, \quad \dot{q}(1) = 0, \quad q(2) = \pi/2, \quad q(2.5) = -\pi/2, \quad q(3) = 0 \quad , \dot{q}(3) = 0$
   c. $q(1) = 0, \quad \dot{q}(1) = 0, \quad q(1.5) = \pi/2, \quad \dot{q}(1.5) = -\pi/16, \quad q(2.5) = -\pi/2, \quad \dot{q}(2.5) = \pi/16, \quad q(3) = 0 \quad , \dot{q}(3) = 0$

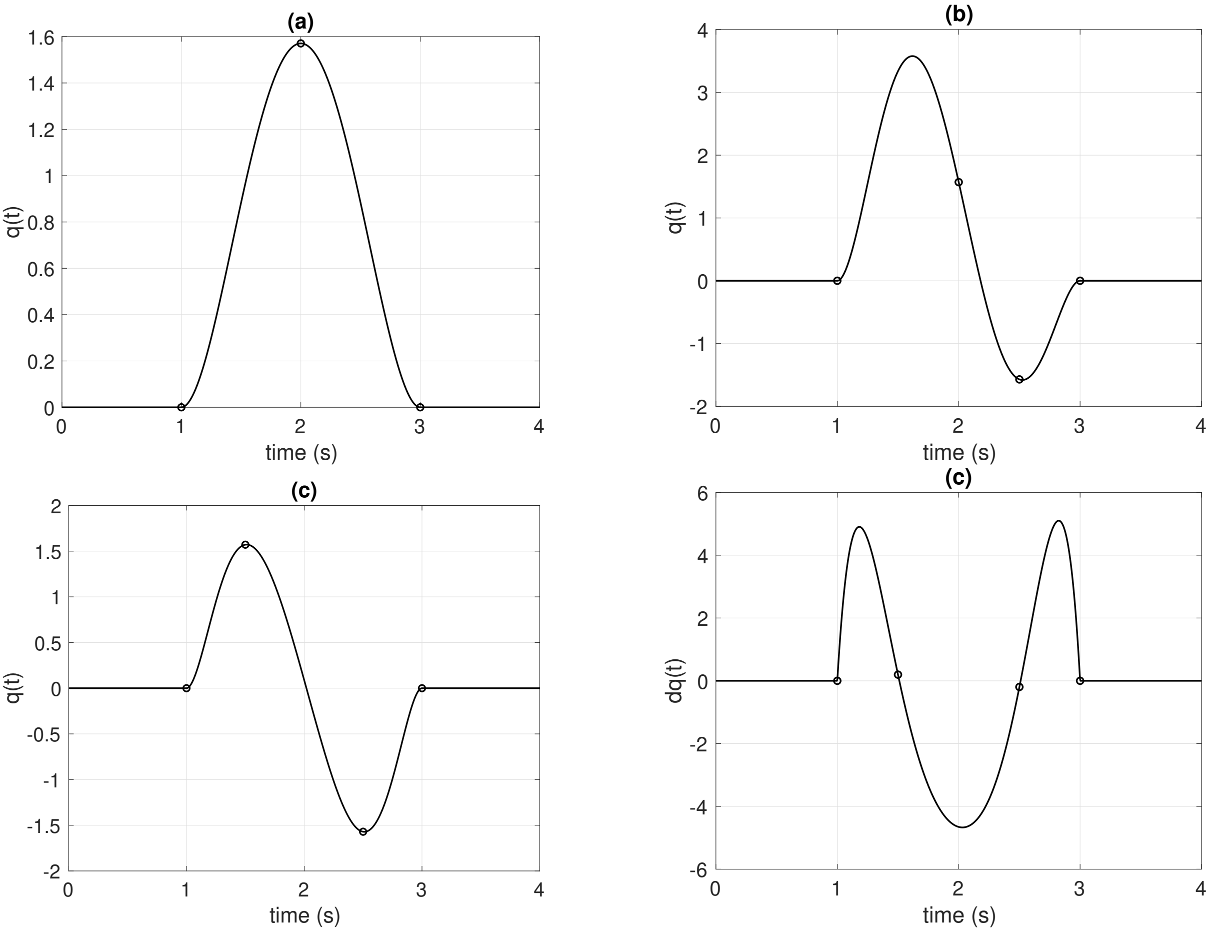

Figure 2: Solution plots for Problem 5