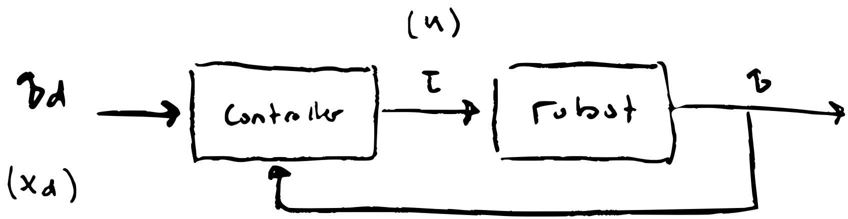
Lecture 15 ME 221 11/17/22

Last time: PID - controller

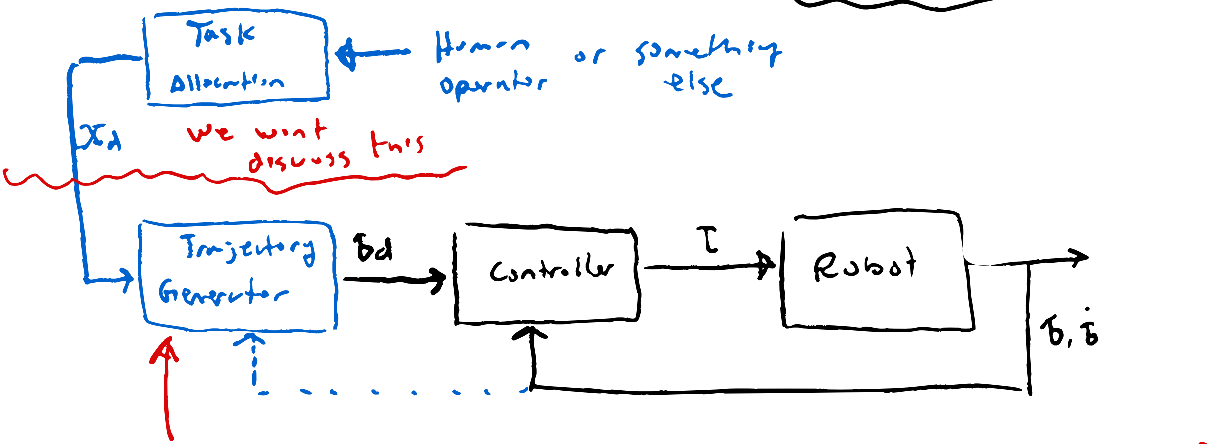


$$\tau = K_p \cdot e + K_I \cdot \int_0^t e(\tau)\,d\tau + K_d \dot{e}$$

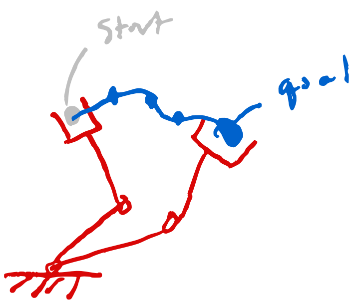
$$e = q_d - q.$$

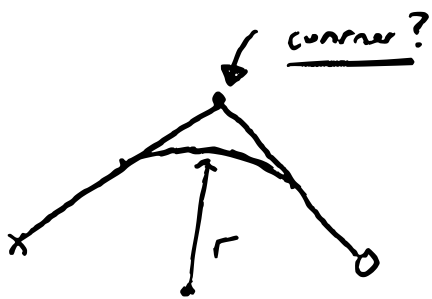
Trajectory Generation

Big Picture



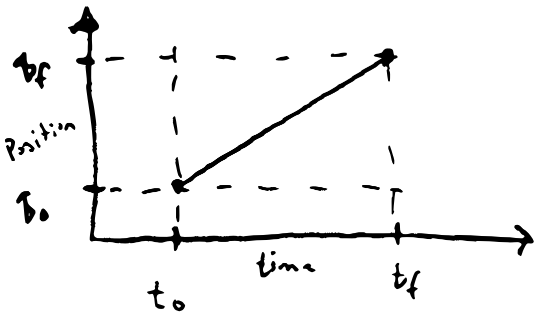
How to plan trajectories, given end-points?





How to design <u>smooth</u> trajectories?

let's look at a single joint:



<u>first order polynomial</u>

* $q(t) = a_0 + a_1 t$

How would we choose a_0 & a_1?

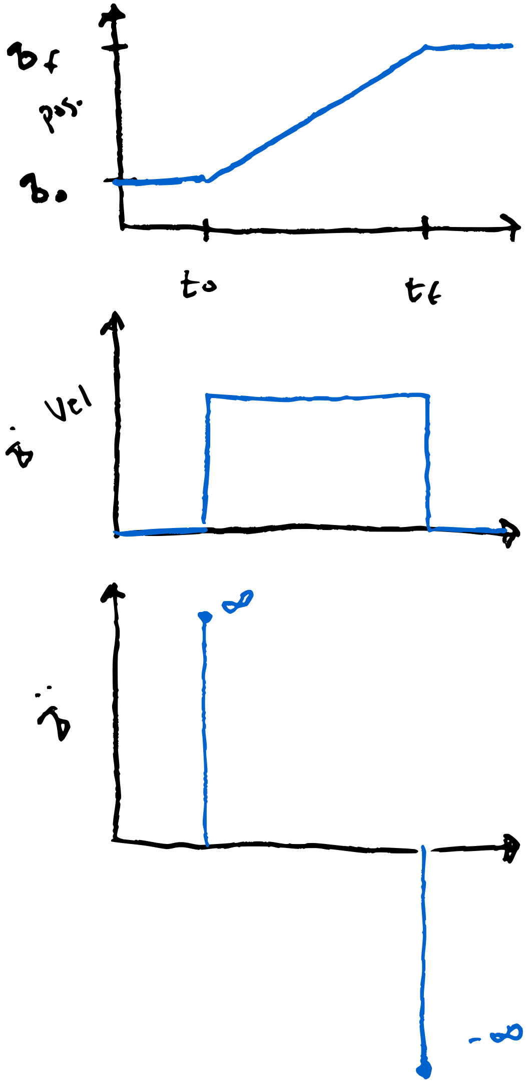
<u>Known</u>

(q_0, t_0)

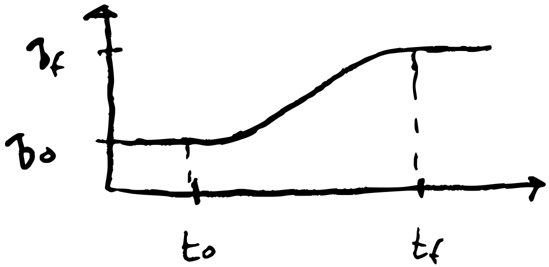
(q_f, t_f)

$$\Rightarrow \underbrace{\begin{bmatrix} q_0 \\ q_f \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix}}_{\text{Known}} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$b = Ax \Rightarrow \boxed{x = A^{-1} b}$$



Moral: first order polynomial $\rightarrow$ infinite acceleration = no good!



cubic polynomial

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

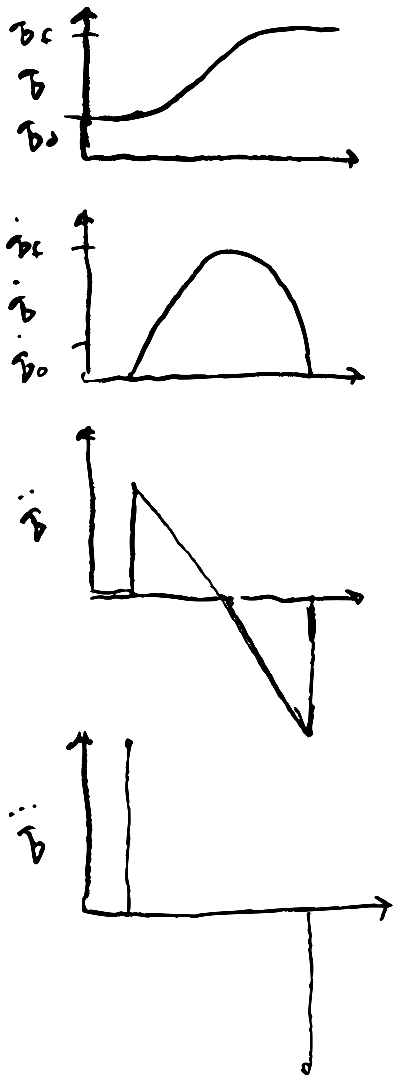
$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Know

$$\left.\begin{array}{l} q_0,\ t_0 \\ q_f,\ t_f \\ \dot{q}_0,\ t_0 \\ \dot{q}_f,\ t_f \end{array}\right\}$$

provide by you!

$$\underbrace{\begin{bmatrix} q_0 \\ \dot{q}_0 \\ q_f \\ \dot{q}_f \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}}_{\text{Known}} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$



jerk has infinites !!

Example: Quintic polynomial

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

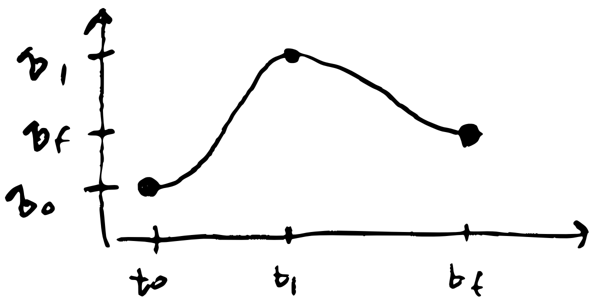
$$\left.\begin{array}{l|l} q(t_0) = q_0 & q(t_f) = q_f \\ \dot{q}(t_0) = \dot{q}_0 & \dot{q}(t_f) = \dot{q}_f \\ \ddot{q}(t_0) = \ddot{q}_0 & \ddot{q}(t_f) = \ddot{q}_f \end{array}\right\}$$ known by you!

$$\begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ q_f \\ \dot{q}_f \\ \ddot{q}_f \end{bmatrix} = A \begin{bmatrix} a_0 \\ \vdots \\ \vdots \\ a_5 \end{bmatrix}$$

Solve for these coefficients

- the number of <u>required conditions</u> determines the degree of the polynomial

- types of conditions
  - positions along the desired trajectory (waypoints or vias)



<u>we want zero velocity at end-points</u>

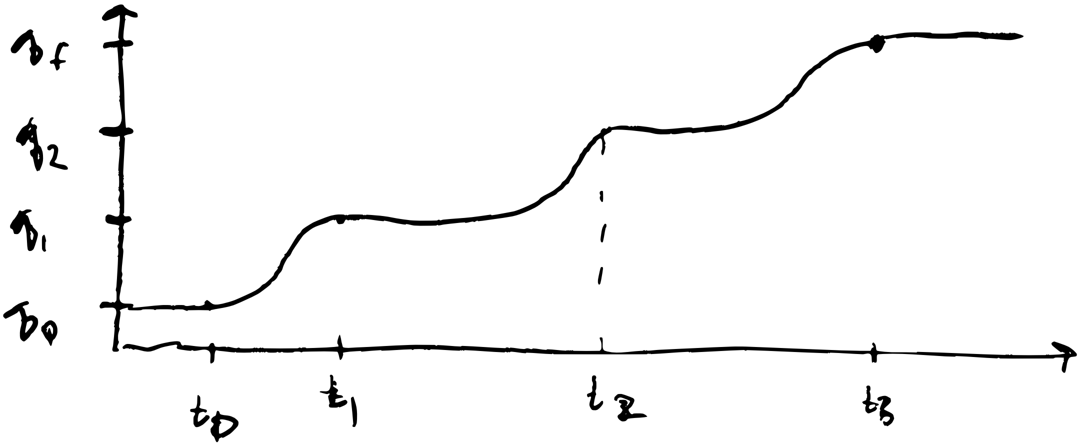
$\theta(t_0) = \theta_0$ $\qquad \dot{\theta}(t_0) = \dot{\theta}_0$

$\theta(t_1) = \theta_1$ $\qquad \dot{\theta}(t_f) = \dot{\theta}_f$

$\theta(t_f) = \theta_f$

5 constraints $\longrightarrow$ 4^{th} polynomial.

Example: double polynomial.



$$q(t_0) = q_0$$
$$\dot{q}(t_0) = 0$$
$$q(t_1) = q_1$$
$$\dot{q}(t_1) = \dot{q}_1$$

3-order poly.

$$q(t_1) = q_1$$
$$\dot{q}(t_1) = \dot{q}_1$$
$$q(t_2) = q$$
$$\dot{q}(t_2) = 0$$

3-order

same thing

Time Scaling in Task space

- separate $\underbrace{x, y, z}_{d}$, R

$$X(s) = X_0 + s \cdot (X_f - X_0) \qquad s \in [0, 1]$$

let s be a polynomial *
so we can use the same
tools!

* $s = a_0 + a_1 t + a_2 t^2 \ldots$

$$\dot{X}(s) = \frac{dx}{ds} \dot{s}$$

$$\ddot{X}(s) = \frac{dx}{ds} \ddot{s} + \frac{d^2 x}{ds^2} \dot{s}^2$$

$$R(s) = R_0 \underbrace{e^{\log(R_0^T R_f)s}}_{\Delta R}$$

time scale the orientation.