ME 221 Lecture 17 11/29/22

Topics on Final

1. Jacobian

$$x = f(q)$$ forward kinematics

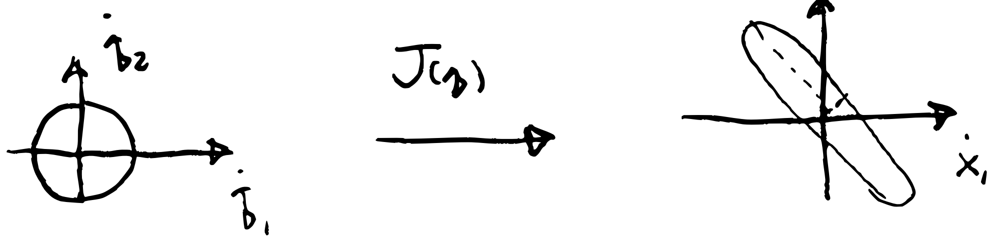
$$\downarrow$$

$$\dot{x} = J(q)\,\dot{q}$$ Jacobian relates joint space velocity to end-effector velocity.

$$J(q) = \left[\frac{\partial f}{\partial q_1} \cdots \frac{\partial f}{\partial q_n} \right]$$

$$J(q) = \dot{T}\,T^{-1}$$

2. Manipulability



- How to find the manipulability ellipsoid given a pose q ?
- How to interpret ?

3. Singularities

$\rightarrow$ occure when Jacobian looses rank.

- How to find ? $\det(J(q)) = 0 \iff \text{rank} < n$ @ a singularity !!!

4. Statics

$$\tau = \underbrace{J(q)^T}_{\substack{\text{maps end-effector force to} \\ \text{joint torques.}}} F_{ext}$$

- why?

5. Dynamics

- basics $\quad \dot{x} = f(x)$

- Lagrangian Mechanics

$$\mathcal{L} = \underbrace{T}_{\text{Kinetic Energy}} - \underbrace{V}_{\text{Potential energy}} \implies \boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = \overbrace{F}^{\text{* all external forces!}}}$$

Equations of motion

* non-conservative forces

For multibody

$$\mathcal{L} = \sum_{i}^{n} T_i - \sum_{i}^{n} V_i$$

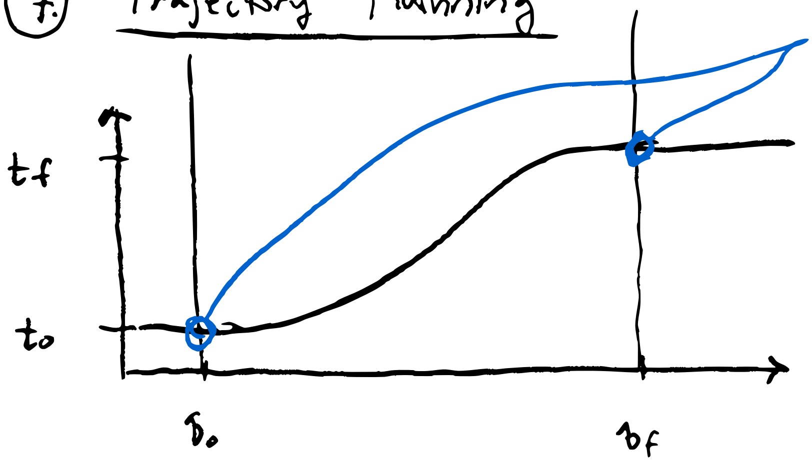
n: links

$$\frac{d}{dt}\begin{bmatrix} \frac{\partial \mathcal{L}}{\partial \dot{q}_1} \\ \vdots \\ \frac{\partial \mathcal{L}}{\partial \dot{q}_n} \end{bmatrix} - \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial q_1} \\ \vdots \\ \frac{\partial \mathcal{L}}{\partial q_n} \end{bmatrix} = \begin{bmatrix} F_1 \\ \vdots \\ F_2 \end{bmatrix}$$

6. PID - Controllers

$$\tau = K_p(q_d - q) + K_d(\dot{q}_d - \dot{q}) + K_I \int_0^t q_d - q \, dt$$

7. Trajectory Planning



constraints

$$q(t) = a_0 + a_1 t + a_2 t^2 \cdots$$

$$\underbrace{\begin{bmatrix} q_0 \\ q_f \\ \vdots \\ \vdots \end{bmatrix}}_{\text{constraints}} = \begin{bmatrix} 1 & t & t^2 \cdots \\ & \ddots & \\ & & \ddots \end{bmatrix} \underbrace{\begin{bmatrix} a_0 \\ \vdots \\ \vdots \\ a_n \end{bmatrix}}_{\text{Parameters!}}$$

8. Robot Dynamics

$$\underbrace{M(q)}_{\text{mass matrix}}\ddot{q} + \underbrace{C(q,\dot{q})}_{\text{Coriolis Matrix}}\dot{q} + \underbrace{g(q)}_{\text{gravity vector}} = \overbrace{\tau}^{\text{Joint torques}} - \underbrace{J^T(q)}_{\text{Jacobian}}\overbrace{F_{ext}}^{\text{external forces.}}$$

9. Control Architectures

- feedforward (Inverse) control
- Computed torque controller (feedback linearization)
- Force Control
- Task space controller $\longrightarrow$ Impedance Controller
- Task space dynamics

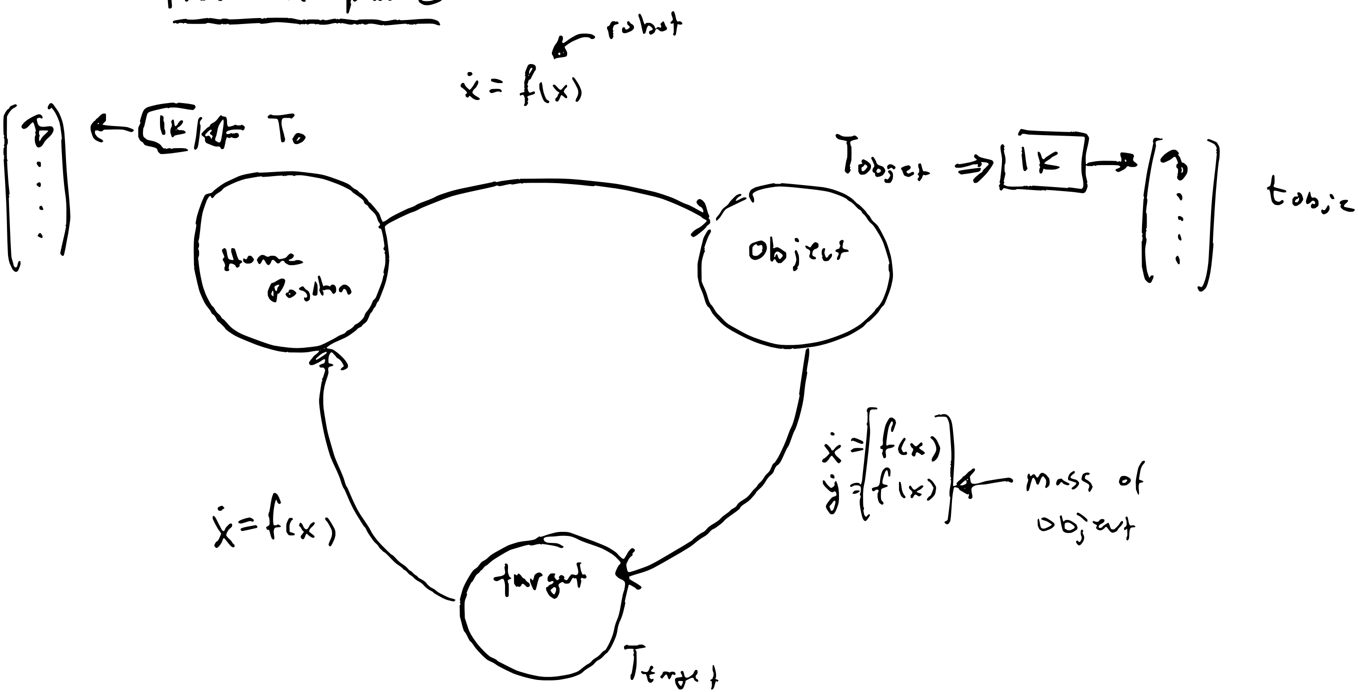
$$\Lambda(q)\ddot{x} + \Gamma(q,\dot{q})\dot{x} + \eta(q) = F$$

- How to derive?

Simulation Advice

$t_{Home \to object}$ | $\theta_{Home \to object} = f(t)$

Pick-n-place



End-effector position

$$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

R

lets do for one dimension:

$x_0, x_f \Rightarrow$ known or user specified

$$x(s) = x_0 + s \cdot (x_f - x_0)$$

$$\underline{s = a_0 + a_1 t + a_2 t^2 \cdots} \qquad s \in [0, 1]$$

$$\dot{x} = \frac{dx}{ds} \dot{s}$$

$\vdots$

$$R(s) = R_0 e^{\log(R_0^T R_f) \cdot s}$$