INVERSE KINEMATICS PLANAR EXAMPLE

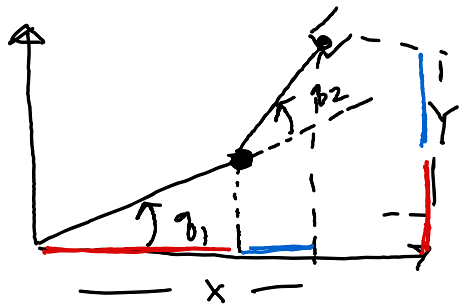
Inverse Kinematics : given $\vec{x}d = [x, y, z, \alpha, \beta, \gamma]$

$$\underbrace{* \text{ find } \vec{q} \rightarrow \vec{x}d \; *}_{\text{goal.}}$$

$${}^{0}\vec{T}_{E}^{d} = \begin{bmatrix} {}^{0}R_{E}^{d} & \begin{matrix} {}^{0}x_e \\ {}^{0}y_e \end{matrix} \\ 0 & \begin{matrix} {}^{0}z_e \\ 1 \end{matrix} \end{bmatrix} \Rightarrow \text{Find } \vec{q} \text{ s.t. } {}^{0}T_{e}(\vec{q}) = \vec{T}d$$

Forward kinematics

Planar Robot



$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ given

$\vec{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ Find

$$x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2) = l_1 c_1 + l_2 c_{12}$$

$$y = l_1 \sin q_1 + l_2 \sin(q_1 + q_2) = l_1 s_1 + l_2 s_{12}$$

$\underline{\cos^2 x + \sin^2 x = 1}$ ← let's take advantage of this

what about $x^2 + y^2 = ?$

$$x = l_1 c_1 + l_2 c_{12}$$
$$y = l_1 s_1 + l_2 s_{12}$$

$$x^2 = l_1^2 c_1^2 + l_2^2 c_{12}^2 + 2 l_1 l_2 c_1 c_{12}$$
$$+ \quad y^2 = l_1^2 s_1^2 + l_2^2 s_{12}^2 + 2 l_1 l_2 s_1 s_{12}$$

$$x^2 + y^2 = l_1^2 (s_1^2 + c_1^2) + l_2^2 (s_{12}^2 + c_{12}^2) + 2 l_1 l_2 (c_1 c_{12} + s_1 s_{12})$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 (c_1 c_{12} + s_1 s_{12})$$

$\underline{c_1 c_{12} + s_1 s_{12}}$ can we simplify?

$$\boxed{\cos(\theta_1 - (\theta_1 + \theta_2))} \rightarrow \cos\theta_2$$

$$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$
$$\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$$

$$c_1 [c_1 c_2 - s_1 s_2] + s_1 [s_1 c_2 + c_1 s_2] = \cos\theta_2$$

$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2$$

$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2$$

$$\boxed{\cos\theta_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}}$$