Accuracy of the single-electron pump using an optimized step-like rf drive waveform

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We have performed an optimization of those parameters of a single-electron pump that may influence the accuracy of this device as a standard of dc current. Two types of rf drive were considered: the traditional triangular waveform and a step-like waveform. We have shown that, after optimization, the accuracy of the pump may be improved considerably in a wide range of drive frequencies, especially at low temperatures, using the step-like waveform. For example, the error of a five-junction pump with junction capacitances $C=0.1$ fF at 100 mK and 10 MHz may be as small as 10^{-13}. [S0003-6951(96)00439-1]

The development of controlled transfer of single electrons in nanoscale solid state circuits[1,2] has suggested several applications, notably fundamental standards of dc current. The most important figure of merit of these devices is their accuracy which may be defined as the relative deviation $|(I-nef)/I|$ of the real dc current I from its quantized value nef, where f is the frequency of the rf drive providing for the electron transfer, and n is an integer (usually $n=1$). Our recent theoretical study[3] has suggested that the accuracy may be improved substantially by driving the electrons through the device with step-like waveforms instead of the traditional triangular waveform. The goal of this work is to perform a thorough parameter optimization of the most popular dc current standard, the single-electron pump,[3–7] with the step-like drive for various rf drive frequencies and temperatures, and compare the accuracy with that achievable with the optimized triangular rf waveform. We will consider only the case of zero bias voltage since it has been recently shown[8] that in real applications the bias voltage can be made very small (of the order of 10 μV in that work).

Figure 1 shows the schematics of the M-junction single-electron pump and rf waveforms used in our calculations. The step-like waveform (Fig. 1(b)) differs from the one discussed in Ref. 3 by the number of time steps for each rf drive source (four instead of two). The motivation for the new waveform is as follows. At low temperatures and low frequencies, the main source of error during the operation of the pump is $(M-1)$th order cotunneling[9] in the direction opposite to the desired current (in Fig. 1(a), from right to left instead of from left to right).[3] The cotunneling into the ith island can only occur before the desirable ''classical'' (first-order) tunneling from the $(i-1)$th island takes place, while the ith island is still empty. Cotunneling is substantial only if the energy difference E between the initial and the final states of the electron is not very small (at low temperatures, the Nth order cotunneling rate Γ_{ct} scales as E^{2N-1}). At the first time step, the rf drive signal $Q_i(t)=C_gV_i(t)$ applied to the ith island has amplitude $(-e/2-Q_1)$, where Q_1 is small and positive. Simultaneously, the signal applied to the $(i-1)$th island is $(-e/2+Q_1)$. As a result, the energy difference E (which is the same for cotunneling and for the desired classical transition of a single positive charge to island i) is positive, but small. Thus, both processes are possible, but the rate of cotunneling is very low. By the end of the first step the probability p_c that the classical transition has occurred is close to unity, so that cotunneling during the second step of $Q_i(t)$ is no longer a serious threat.

However, because of the low classical transition rate during the first step, the probability of dynamic error (i.e., the probability $q_c=1-p_c$ that the classical transition has not happened) may not be low enough. To suppress this second source of error, we replace Q_1 by $Q_2>Q_1$ during the second time step. The time duration τ_0 of the second step is an adjustable parameter, which fixes the length of the first time step to $\tau/M-\tau_0$, where $\tau=1/f$ is the length of the whole rf drive period. Finally, at the third and fourth steps of $Q_i(t)$, the signal amplitude is equal to $(-e/2+Q_1)$ and $(-e/2+Q_2)$, respectively, in order to provide similar conditions for tunneling from the ith to the $(i+1)$th island. Note that at every instant the total charge inserted into two neighboring islands is $(-e)$, exactly like in the case of the triangular drive (Fig. 1(c)).[3–7]

In our numerical calculations we have considered a pump containing $M=5$ identical tunnel junctions with capacitance C and tunnel resistance R, with small gate capacitances $C_g \ll C$ and negligible stray capacitances. Since the tunnel junctions are identical, we have assumed that the parameters Q_1 and Q_2 are similar for all islands. In this case, we shall optimize four parameters that may have a strong influence on the device accuracy: the time step length τ_0, the amplitude parameters Q_1 and Q_2, and the tunnel resistance R. The optimal values of those parameters were obtained by a numerical search in the four dimensional parameter space using Powell's method,[10] whereas the device accuracy calculation in each point of the parameter space was performed using the program SENECA described in detail elsewhere.[3,11] Briefly, SENECA was developed for numerical study of the dynamics and statistics of single-electron systems presenting

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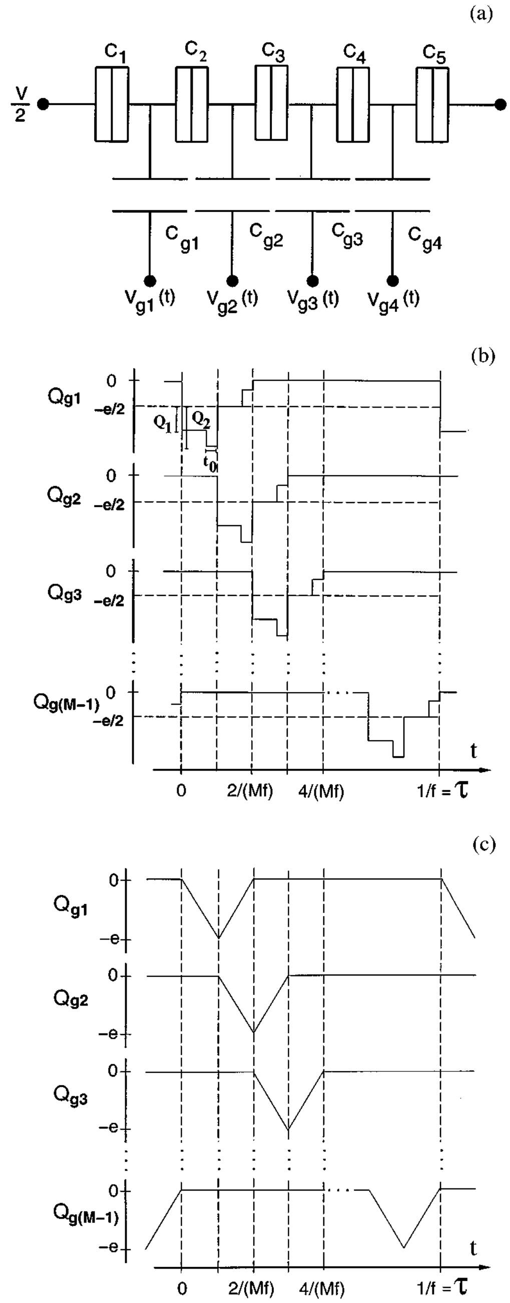

FIG. 1. (a) The five-junction pump and (b),(c) possible waveforms of its rf drive: (b) new step-like signal, (c) usual triangular-shaped signal. For numerical calculations we have used $C_i = C = 0.1$ fF, $C_{gi} = C_g \ll C$, and $V = 0$. Possible offset charges of the islands are assumed to be compensated by dc components of the voltages $V_i(t)$.

arbitrary combinations of small tunnel junctions, capacitances, and voltage sources. The algorithm is based on the numerical solution of a master equation describing the time evolution of the probabilities of the electric charge states of the system, with an iterative account of the most important states. In this way, the method is able to describe very small deviations from the ideal behavior of a system, due to finite speed of applied signals, thermal activation, and cotunneling, while requiring relatively modest computer resources. For example, the optimization of the pump with typical parameters $T = 100$ mK and $f = 10$ MHz takes ~ 2 hours of CPU time on the DEC Alpha 250 4/266 workstation, and uses ~ 15 MB of memory.

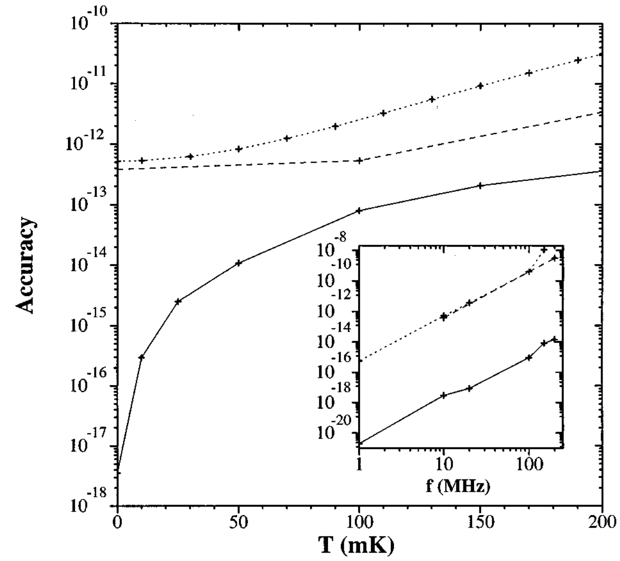

FIG. 2. Accuracy of the five-junction pump driven by the step-like waveform (solid line), triangular waveform with optimized tunnel resistances (dashed line), and triangular waveform with fixed $R = 300$ kΩ (dotted line) as a function of temperature at $f = 10$ MHz. The inset shows the accuracy of the five-junction pump driven by the same waveforms as a function of frequency at $T = 0$.

Figure 2 shows the accuracy of the five-junction pump as a function of temperature at $f = 10$ MHz. The dotted curve corresponds to the case when the triangular waveform is used and R is kept fixed at 300 kΩ. To obtain the data shown by the dashed curve, the resistance was optimized. The results show that the effect of the optimization is negligible at low temperatures, although it may improve performance by an order of magnitude for $T > 100$ mK. The insensitivity of the pump's accuracy to R at low temperatures and low frequencies can be easily understood in terms of the linearity in time of the energy $E(t)$ during a tunneling event, which results from the linearity of the applied signals. In this case, the probability of having a cotunneling event by time t may be estimated as[9]

$$p_{ct}(t) \approx \int_0^t \Gamma_{ct} q_c dt', \Gamma_{ct} \propto \alpha^N (ft)^{2N-1}, \tag{1}$$

where q_c is the probability that classical tunneling has not happened. This probability satisfies the equation[1,2]

$$\frac{dq_c}{dt} \approx -\Gamma_c q_c, \Gamma_c \propto \alpha f t, q_c(0) = 1, \tag{2}$$

where $\alpha = R_K / 4\pi^2 R_t$, $R_K = h/e^2 \approx 25.8$ kΩ is the quantum unit of resistance, and $N = M - 1$ is the lowest possible order of cotunneling in the pump. These equations show that at the end of the time step $(t \sim f^{-1} \gg \Gamma_c^{-1})$

$$p_{ct}(t) \propto \alpha^0 f^{M-2}. \tag{3}$$

This estimate is valid only if cotunneling is the main source of error, which is not the case at very large R when dynamic errors become important, or at high temperatures when errors due to thermal activation are the dominating contribution to the total error.[3] Figure 3 shows the effect of the value of R on the accuracy of the pump driven by the triangular wave-

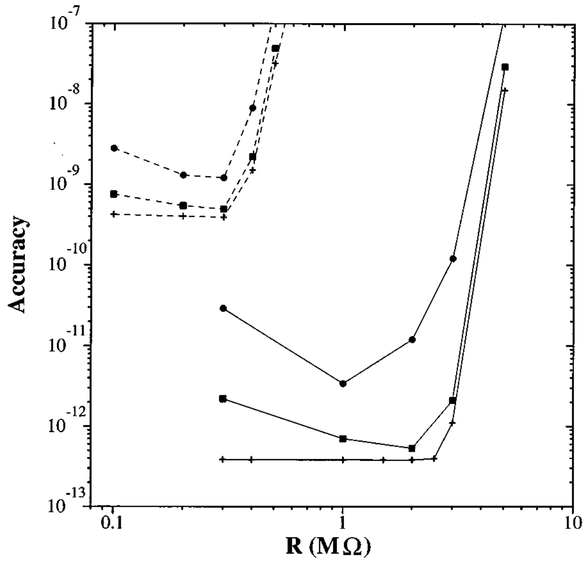

FIG. 3. Accuracy of the five-junction pump driven by the triangular waveform (Fig. 1(c)) as a function of tunnel junction resistance. The two sets of curves correspond to two different rf drive frequencies, 10 MHz (solid lines) and 100 MHz (dashed lines). Temperatures: $T=0$ (crosses); $T=100$ mK (boxes); and $T=200$ mK (circles).

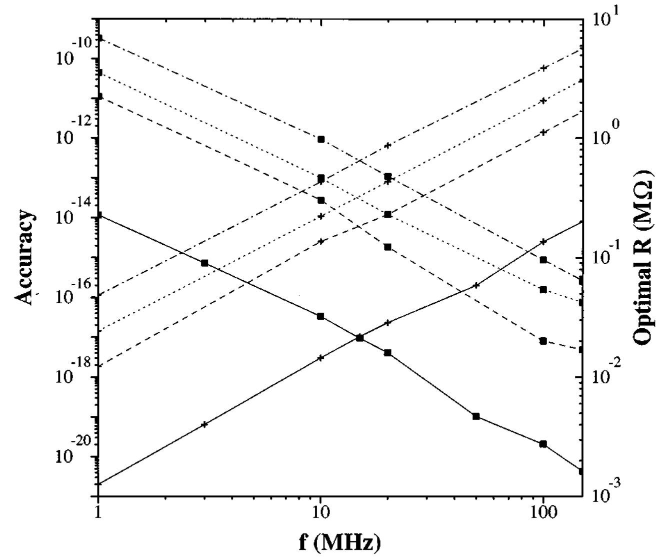

FIG. 4. Accuracy (crosses) and optimal tunnel resistance R (boxes) of the five-junction pump driven by the step-like waveform (Fig. 1(b)) as a function of frequency and temperature. Solid lines: $T=0$; dashed lines: $T=25$ mK; dotted lines: $T=50$ mK; dot-dash lines: $T=100$ mK.

form for two sets of frequencies at three different temperatures. Notice that the accuracy is insensitive to R when $T=0$ and R is not very large, and that the range of acceptable values of R at $T=0$ shifts towards smaller values as the frequency increases (as a consequence of increasing dynamic errors). Thus, if temperature and frequency are relatively high, a proper choice of R may lead to a considerable gain in accuracy even for the triangular waveform of the rf drive.

The solid line in Fig. 2 shows the accuracy of the pump driven by the step-like waveform. The effect of the new waveform is dramatic at very low temperatures (in our example, below ~ 50 mK), where accuracy is $50–10^5$ times better than for the case of the triangular drive. Even at higher T a factor of ~ 10 favors the step-like waveform. In the inset we again compare the accuracy of the pump driven by the triangular waveform (with fixed and optimized R) and the step-like waveform as a function of frequency at $T=0$. The improvement factor of $\sim 10^5$ in accuracy from the triangular to the step-like waveform holds in the entire frequency range. The frequency dependence of the accuracy of the pump driven by the triangular waveform follows the f^3 law (see Eq. 3) very well.

Figure 4 shows the ultimate accuracy of the pump driven by the step-like waveform, as well as the optimal tunnel resistance, as functions of frequency and for several temperatures. For all temperatures both accuracy and optimal R are close to power functions of frequency. We see that this waveform is very efficient in suppressing dynamic and cotunneling errors (although it is relatively sensitive to thermally induced errors). For example, at $T=100$ mK, accuracies of $\simeq 10^{-13}$ and $\simeq 10^{-10}$ may be obtained at $f=10$ MHz and $f=100$ MHz, respectively. Note that some caution is necessary when using our results for very low values of R. The tunneling rates used to calculate the error rates[5] originate from a perturbation theory in α,[9] meaning that our results are strictly correct only at $\alpha \to 0$. We believe there is no large error in our results even for relatively large values of α (e.g., $\alpha \approx 0.06$ for $R=10$ kΩ).

In conclusion, we have performed a full parameter optimization for the operation of a single-electron pump driven by a special step-like waveform and have found that a very substantial improvement in the accuracy of the device (in comparison with the traditional triangular waveform) is quite possible. Even for a simple five-junction pump with $C=0.1$ fF, a relative accuracy better than 10^{-10} (i.e., much better than the current requirements for a useful fundamental standard of dc current[5]) can be reached in a wide frequency range at a feasible temperature of 100 mK. Further improvements are possible using a small dc bias voltage V applied to the pump[3] and more complex rf drive waveforms. However, the effect of a finite bias voltage on the accuracy of pump driven by the proposed step-like waveform relative to the triangular waveform is unknown and will be the subject of a future work.

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