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# Non-ideal quantum feedback of a solid-state qubit

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**Abstract.** We have analyzed the operation of the Bayesian quantum feedback of a solid-state qubit, designed to maintain perfect coherent oscillations in the qubit for arbitrarily long time. In particular, we have studied the feedback efficiency in presence of dephasing environment and detector nonideality. Also, we have investigated the effect of qubit parameter deviation.

## 1. Introduction

With recent experimental demonstration [1] of quantum feedback in optics, the issue of quantum feedback of solid-state systems becomes especially interesting. The use of quantum feedback to maintain coherent (Rabi) oscillations in a qubit for arbitrarily long time has been proposed and analyzed in Refs. [2] and [3]. The basic idea is to monitor the qubit state via the output of a weakly coupled detector (using Bayesian formalism [4] to translate noisy detector output into the qubit evolution), then compare the qubit oscillation phase with the desired value, and then slightly change the qubit barrier height in order to reduce the phase difference. As has been shown in Ref. [2], the fidelity of such feedback loop can be close to 100% in the ideal case, while the fidelity decreases because of detector nonideality and/or significant interaction with environment, as well as in the case of insufficient bandwidth of the line carrying the signal from detector. In the present paper we study in more detail the operation of the quantum feedback loop in presence of extra dephasing due to environment and non-ideal detector, and also analyze the effect of qubit parameter deviation on the feedback loop performance.

## 2. Model

We consider the quantum feedback loop shown in Fig. 1, which controls the qubit characterized by the Hamiltonian $\mathcal{H}_{qb} = (\varepsilon/2)(c_2^\dagger c_2 - c_1^\dagger c_1) + H_{fb}(c_1^\dagger c_2 + c_2^\dagger c_1)$, where $c_{1,2}^\dagger$ and $c_{1,2}$ are creation and annihilation operators in the measurement basis, $\varepsilon$ is the qubit energy asymmetry, and tunneling amplitude $H_{fb}$ can be controlled by the feedback loop: $H_{fb} = H + \Delta H_{fb}$. We consider a "charge" qubit continuously measured by QPC or SET, so that the measurement setup is similar to what has been studied theoretically, e.g. in Refs. [4, 5, 6, 7, 8]. The evolution of the qubit density matrix $\rho$ is described by the quantum Bayesian equations [4]

$$\dot\rho_{11} = -(2H_{fb}/\hbar)\,\mathrm{Im}\rho_{12} + (2\Delta I/S_I)\,\rho_{11}\rho_{22}\,[I(t)-I_0], \tag{1}$$

$$\dot\rho_{12} = i\,(\varepsilon/\hbar)\,\rho_{12} + i\,(H_{fb}/\hbar)\,(\rho_{11}-\rho_{22}) - (\Delta I/S_I)\,(\rho_{11}-\rho_{22})\,[I(t)-I_0]\,\rho_{12} - \gamma\rho_{12}\,, \tag{2}$$

where $I(t) = I_0 + (\Delta I/2)(\rho_{11}-\rho_{22}) + \xi(t)$ is the noisy detector current , $S_I$ is the current spectral density, $\Delta I = I_1 - I_2$, $I_0 = (I_1+I_2)/2$, and $I_{1,2}$ are two average detector currents corresponding to two states of the qubit. The dephasing rate $\gamma = \gamma_d + \gamma_{env}$ has the contribution $\gamma_d$ due to detector nonideality, $\gamma_d = (\eta^{-1} - 1)(\Delta I)^2/4S_I$ (here $\eta \leq 1$ is the detector quantum efficiency [4, 7, 8]) and contribution $\gamma_{env}$ due to interaction with extra environment. We focus on the case of weak coupling between qubit and detector: $\mathcal{C} \lesssim 1$ where $\mathcal{C} = \hbar(\Delta I)^2/S_I H$.

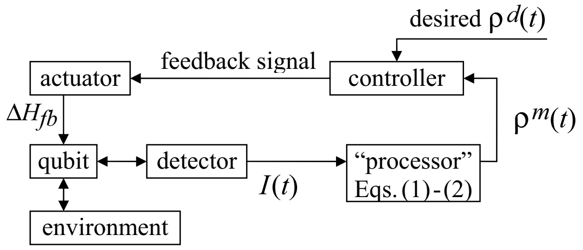


**Figure 1.** Schematic of the quantum feedback loop. By comparing the monitored qubit state $\rho^m$ with the desired state $\rho^d$, a certain algorithm (controller) produces the feedback signal which changes the qubit tunneling amplitude $H_{fb}$ in order to reduce the difference between $\rho^m$ and $\rho^d$.

We study the feedback loop (Fig. 1), which goal is to maintain perfect coherent oscillations in the qubit, so the desired evolution is

$$\rho^d_{11}(t) = (1 + \cos \Omega_0 t)/2, \qquad \rho^d_{12}(t) = (i \sin \Omega_0 t)/2, \tag{3}$$

with frequency $\Omega_0 = 2H/\hbar$ equal to Rabi frequency $\Omega = \sqrt{4H^2 + \varepsilon^2}/\hbar$ in the case $\varepsilon = 0$. We assume simple linear feedback control:

$$\Delta H_{fb} = -FH\Delta\phi_m, \tag{4}$$

where $\Delta\phi_m = \phi_m(t) - \Omega_0 t \pmod{2\pi}$, $\phi_m(t) = \arctan[2\,\mathrm{Im}\rho^m_{12}/(\rho^m_{11} - \rho^m_{22})]$, and $F$ is feedback strength. The monitored qubit state $\rho^m$ may differ from the actual state $\rho$ because of imperfections. The controller (4) is supposed to decrease the phase difference: if the monitored phase $\phi_m(t)$ is ahead of the desired value, then negative $\Delta H_{fb}$ slows down qubit oscillations; if $\phi_m(t)$ is behind the desired value, the oscillation frequency increases to catch up. We characterize the feedback efficiency (fidelity) $D$ by the average scalar product of two Bloch vectors corresponding to the desired and actual states; an equivalent definition is $D = 2\langle \mathrm{Tr}\rho\rho^d \rangle - 1$.

## 3. Ideal case

The starting point is the case of ideal detector ($\eta = 1$, e.g. QPC), no extra environment ($\gamma_{env} = 0$), and symmetric qubit ($\varepsilon = 0$). We also assume infinite bandwidth of the line between detector and processor (then $\rho^m = \rho$) and no time delay in the feedback loop. As shown in Ref. [2], in this case the feedback fidelity $D$ can be made arbitrarily close to 1. Approximate analytical formula can be derived for weak coupling and sufficiently efficient feedback ($\mathcal{C} \lesssim 1$, $D \gtrsim 1/2$), then $D \approx \exp(-\mathcal{C}/32F)$. This formula has been confirmed by numerical calculations using Monte Carlo method [4]. Notice that $|\Delta H_{fb}|/H < \pi F$, and $F$ scales with coupling $\mathcal{C}$. Therefore, in the experimentally realistic case $\mathcal{C} \ll 1$ a typical amount of the parameter change due to feedback is small, $|\Delta H_{fb}| \ll H$.

## 4. Effect of imperfect detector and extra dephasing

Various nonidealities reduce the feedback fidelity, preventing $D$ from approaching 100%. In this Section we consider the effects of imperfect quantum efficiency of the detector ($\eta < 1$) and extra qubit dephasing $\gamma_{env}$ due to coupling to environment. Both effects contribute to the total qubit dephasing rate $\gamma = \gamma_{env} + (\eta^{-1} - 1)(\Delta I)^2/4S_I$ in Eq. (2) and can be characterized by effective quantum efficiency of the qubit detection $\eta_e = [1 + 4\gamma S_I/(\Delta I)^2]^{-1} = [\eta^{-1} + 4\gamma_{env} S_I/(\Delta I)^2]^{-1}$.

The dots in Fig. 2 show the Monte Carlo result for the feedback efficiency $D_{max}(\eta_e)$ maximized over the feedback factor $F$ in the case of weak coupling $\mathcal{C}$ (there is practically no dependence on $\mathcal{C}$ if $\mathcal{C}/\eta_e \leq 1$). The maximum is still reached at large $F$, similar to the ideal case.

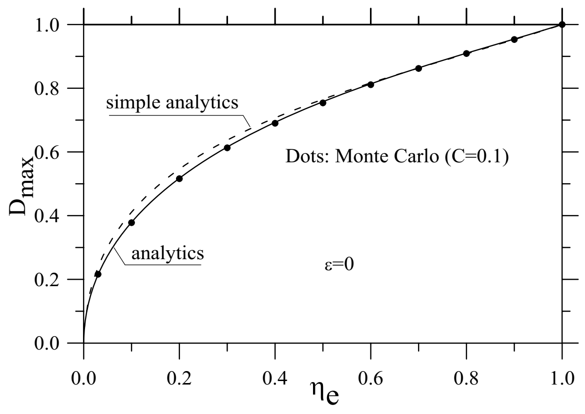


**Figure 2.** Efficiency (fidelity) $D$ of the quantum feedback operation in the case of extra dephasing (characterized by the quantum efficiency $\eta_e$ of detection), maximized over the feedback strength $F$ (maximum of $D$ is reached at $F \to \infty$). Dots show Monte Carlo results for weak coupling between the qubit and detector ($\mathcal{C} = 0.1$), solid line corresponds to Eq. (9), and dashed line shows approximate formula (8).

For analytical analysis we parameterize the qubit state as $\rho_{11} - \rho_{22} = P\cos\phi$ and $\rho_{12} = iP(\sin\phi)/2$ (we assume $\varepsilon = 0$ ), and derive equations for $P$ and $\phi$ from Eqs. (1)–(2):

$$\dot{P} = (\Delta I/S_I)(1-P^2)[(\Delta I/2)\,P\cos\phi + \xi]\cos\phi - \gamma P\sin^2\phi, \tag{5}$$

$$\dot{\phi} = 2H_{fb}/\hbar - \sin\phi\,(\Delta I/PS_I)\,[(\Delta I/2)\,P\cos\phi + \xi) - (\gamma/2)\sin 2\phi. \tag{6}$$

Translating these equations from Stratonovich into Itô form and averaging over $\phi$ we obtain

$$dP^2/dt = [(\Delta I)^2/2S_I](1-P^2)(1-P^2/2) - \gamma P^2 + (\sqrt{2}\Delta I/S_I)P(1-P^2)\,\tilde{\xi}\,, \tag{7}$$

where $\tilde{\xi}$ is the white noise with the same spectral density $S_I$ as $\xi$. In case of sufficiently strong feedback the phase $\phi$ is arbitrarily close to the desired phase $\Omega_0 t$ (mod $2\pi$), so the maximum fidelity $D_{max}$ is equal to $\langle P\rangle$. Neglecting the noise term in Eq. (7), we find a simple estimate of the feedback efficiency:

$$D_{max} \approx [1 + 1/2\eta_e - \sqrt{(1+1/2\eta_e)^2 - 2}]^{1/2}. \tag{8}$$

The rigorous analysis of Eq. (7) leads to the following result [9] (see Fig. 2):

$$D_{max} = \left(\int_0^1 P^2\,G(P^2)\,dP\right) / \left(\int_0^1 P\,G(P^2)\,dP\right), \tag{9}$$

where $G(P^2) = (1-P^2)^{-5/2}\exp\left[-(\eta_e^{-1} - 1)/(2-2P^2)\right]$.

## 5. Effect of $\varepsilon$ and $H$ deviation

In this Section we analyze what happens if the qubit parameters $\varepsilon$ and $H$ deviate from the "nominal" values $\varepsilon = 0$ and $H = H_0$ assumed by an experimentalist and used in the processor. The monitored value $\rho^m$ calculated through "incorrect" parameters $\varepsilon = 0$ and $H_0$ differs from the actual value $\rho$ governed by actual $\varepsilon$ and $H$; and because of the mistake in qubit monitoring, the feedback performance should obviously worsen. The desired evolution is still described by Eq. (3) and the controller is still given by Eq. (4).

Let us start with deviation of $\varepsilon$ (while $H = H_0$). Solid lines in Fig. 3(a) show the numerical (Monte Carlo) results for the maximized over $F$ fidelity $D_{max}(\varepsilon/H)$ for coupling $\mathcal{C} = 0.1$, 0.3, and 1. One can see that significant decrease of $D_{max}$ starts at smaller $\varepsilon/H$ for smaller coupling $\mathcal{C}$. Rescaling of the horizontal axis by $\sqrt{\mathcal{C}}$ makes the curves (dashed lines) quite close to each other. The dotted lines in Fig. 3(a) show dependence $D_{max}(\varepsilon/H)$ for the situation when the exact $\varepsilon$ is used in the processor, but the controller is still given by Eq. (4) designed for $\varepsilon = 0$ [desired

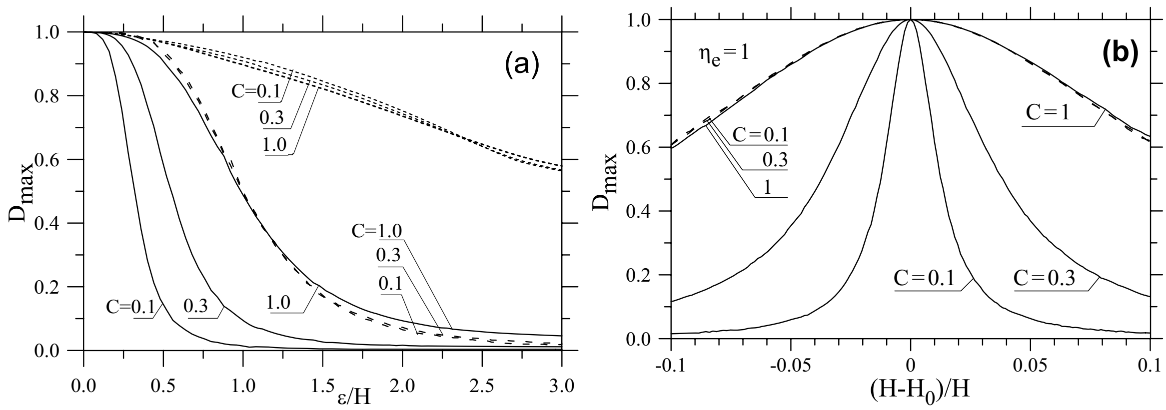


**Figure 3.** (a): Solid lines: feedback efficiency $D_{max}(\varepsilon/H)$ maximized over $F$ for coupling $\mathcal{C} = 1$, 0.3, and 0.1. Dashed lines: the same curves for $\mathcal{C} = 0.3$ and 0.1 drawn as functions of $\varepsilon/H\sqrt{\mathcal{C}}$. Dotted lines: $D_{max}(\varepsilon/H)$ for three couplings when actual $\varepsilon$ is used in the processor, while the controller (4) is still designed for $\varepsilon = 0$. (b): Solid lines: $D_{max}$ as a function of relative $H$-deviation $(H - H_0)/H$ for coupling $\mathcal{C} = 1$, 0.3, and 0.1. Dashed lines: the same curves for $\mathcal{C} = 0.3$ and 0.1 drawn as functions of $(H - H_0)/H\mathcal{C}$.

evolution is still given by Eq. (3)]. Exact monitoring of the qubit significantly improves the feedback efficiency compared with the case considered above; however, the feedback efficiency still decreases with energy asymmetry because the desired evolution (3) cannot be achieved at nonzero $\varepsilon/H$ and also because of non-optimal controller designed for $\varepsilon = 0$.

To analyze the effect of the deviation of $H$, we assume $\varepsilon = 0$. Fig. 3(b) shows the numerical results for $D_{max}(\Delta H/H)$ for several couplings. Similar to the previous case, larger deviation of $H$ can be tolerated for stronger coupling. The curves practically collapse onto one curve if $D_{max}$ is plotted as a function of $\Delta H/H\mathcal{C}$ (dashed lines). The different scaling is due to the fact that small change of $\Omega$ is linear in $H$ deviation but quadratic in $\varepsilon$. The results presented by solid and dashed lines in Fig. 3 can be crudely interpreted in the following way: $D_{max}$ decreases significantly when the Rabi frequency change due to parameter deviations ($\Delta\Omega = 2\Delta H/\hbar$ or $\Delta\Omega \approx \varepsilon^2/4H\hbar$) becomes comparable to the "measurement rate" $(\Delta I)^2/4S_I$.

The main practical conclusion of the analysis presented in this Section is that the feedback operation is robust against small unknown deviations of the qubit parameters $\varepsilon$ and $H$.

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