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TABLE OF INTEGRALS RELATED TO ERROR FUNCTION

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Preface

This book presents a table of integrals related to the error function, including indefinite and improper definite integrals. Since many tables of integrals have been published previously and, moreover, computers are widely used nowadays to find integrals numerically and analytically, a natural question is why such a new table would be useful. There are at least three reasons for that. First, to the best of our knowledge, this is the first book (except Russian versions of essentially the same book), which presents a comprehensive collection of integrals related to the error function. Most of the formulas in this book have not been presented in other tables of integrals or have been presented only for some special cases of parameters or for integration only along the real axis of the complex plane. Second, many of the integrals presented here cannot be obtained using a computer (except via an approximate numerical integration). Third, for improper integrals, this book emphasizes the necessary and sufficient conditions for the validity of the presented formulas, including the trajectory for going to infinity on the complex plane; such conditions are usually not given in computer-assisted analytical integration and often not presented in the previously published tables of integrals.

We hope that this book will be useful to researchers whose work involves the error function (e.g., via probability integrals in communication theory). It can also be useful to a broader audience.

Alexander N. Korotkov and Nikolai E. Korotkov

CONTENTS

NOTATIONS and DEFINITIONS

$\operatorname{erf}(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta \exp(-z^2)\, dz$ is the error function (probability integral).

$\operatorname{erfi}(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta \exp(z^2)\, dz = -i \operatorname{erf}(i\theta)$ is the imaginary error function.

m, n, k, l, r, q are nonnegative integers (0, 1, 2, …).

a, b, c are real numbers.

$\alpha, \beta, \gamma, \mu, \nu, \theta$ are complex numbers (can be real as well).

x is a real integration variable.

z is a complex integration variable (can be real as well).

i is the imaginary unit.

π and e have their usual meaning.

$\operatorname{Re}\theta$ and $\operatorname{Im}\theta$ are real and imaginary parts of θ, $\theta = \operatorname{Re}\theta + i \operatorname{Im}\theta$.

$\overline{\theta} = \operatorname{Re}\theta - i \operatorname{Im}\theta$ is the complex conjugate of θ.

$|\theta| = \sqrt{\operatorname{Re}^2\theta + \operatorname{Im}^2\theta}$ is the absolute value of θ.

$\arg\theta$ is the argument of a complex number θ, $-\pi < \arg\theta \le \pi$.

∞ is the infinity symbol used on the complex plane.

$-\infty$ and $+\infty$ are the infinity symbols for real numbers (or the real axis).

$\theta \to \infty(T)$ means θ goes to infinity on the complex plane along trajectory T.

$\sum_{k=m}^{n} \theta_k$ denotes summation; equals 0 if $n < m$.

$m!$ is the factorial of m (with usual convention $0! = 1! = 1$).

$E(c)$ is the integer part (floor function) of a real number c.

$\int_{\infty(T_1)}^{\infty(T_2)} \varphi(z)\, dz$ is an improper integral on the complex plane with trajectories T_1 and T_2 both going to infinity.

$\ln\theta = \ln|\theta| + i \arg\theta$ is the principal value of natural logarithm.

$\arctan\theta = \operatorname{arctg}\theta = \operatorname{atan}\theta = \frac{1}{2i} \ln \frac{1+i\theta}{1-i\theta}$ is the principal value of inverse tangent.

INTRODUCTION

Formulas in this book are numbered by part, section, subsection, and formula number. When a formula is too long, it is written in terms of abbreviated expressions, which are listed at the ends of sections.

Most of the presented formulas assume complex parameters and integration on the complex plane. However, the formulas which use letters *a*, *b*, *x* are valid only for real values of the corresponding variables (see the list of notations). In particular, integrals $\int f(x)\,dx$ assume integration only over the real axis.

The symbol "∞" denotes infinity on the complex plane, while for the real axis we use notations "$+\infty$" (positive infinity) or "$-\infty$" (negative infinity). For improper integrals on the complex plane of the type $\int_{\mu}^{\infty} f(z)\,dz$ (usually $\mu = 0$), the conditions for how the trajectory approaches infinity are listed after the formula. When both complex limits of integration are infinite, we use notation $\int_{\infty(T_1)}^{\infty(T_2)} f(z)\,dz$ and list conditions of approaching infinity for both trajectories T_1 and T_2. Note that since the integrands considered in this book are analytic functions, the results of integration do not depend on the integration contours; it is only important how these trajectories approach infinity.

The validity conditions for formulas are listed in curly brackets after them. For improper integrals, we also include in the curly brackets the conditions for approaching infinity, which are necessary and sufficient for the integral convergence.

Most integrals with complex parameters presented in this book can also be used for real values of these parameters. However, conversion of the formulas to this case is not always simple. This is why we also present integrals with real parameters (as the most practically interesting case) for convenience.

The square root $\sqrt{\theta}$ in formulas is defined with a positive real part (or positive imaginary part if the real part is zero), as consistent with our convention $-\pi < \arg\theta \le \pi$. When the left-hand side of a formula contains θ^2 but does not contain θ, we assume $\theta = \sqrt{\theta^2}$.

Multiple double-sign notations $\pm$ or $\mp$ (in formulas and validity conditions) assume that either all upper signs or all lower signs are used.

Part 1 of this book contains indefinite integrals, Part 2 contains definite (mainly improper) integrals. Appendix contains some formulas with relations between integrals, which can be used to obtain integrals not presented in this book.

Definite integrals with finite limits are presented in the Part 2 only in the case when there are no corresponding indefinite integrals. Improper integrals are presented independently of whether the corresponding indefinite integrals are presented or not.

Most of improper integrals have the form $\int_0^\infty f(z)\,dz$; however, some integrals have the form $\int_\mu^\infty f(z)\,dz$. The integrals $\int_{\infty(T_1)}^{\infty(T_2)} f(z)\,dz$ with two infinite complex limits are presented only when we are unable to obtain corresponding improper integrals with one infinite limit of integration. If an integral $\int_{-\infty}^{+\infty} f(z)\,dz$ with two real infinite limits exists, it is presented (except for odd functions).

If from an integral containing the function erf it is easy to obtain the corresponding integral containing the function erfi (using the first two formulas in Appendix), then we present only the integral with erf. However, if such a conversion is not easy (and in some other cases), we also present the integral with erfi.

The formulas in this book contain the error function $\mathrm{erf}(\theta)$ and the imaginary error function $\mathrm{erfi}(\theta)$ defined as

$$\mathrm{erf}(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta \exp(-z^2)\,dz\,, \quad \mathrm{erfi}(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta \exp(z^2)\,dz\,.$$

We do not use notation of the complementary error function $\mathrm{erfc}(\theta) = 1 - \mathrm{erf}(\theta)$.

Let us briefly discuss some properties of the functions $\mathrm{erf}(\theta)$ and $\mathrm{erfi}(\theta)$.

1. These functions are related to each other as

$$\mathrm{erfi}(\theta) = -i\,\mathrm{erf}(i\theta)\,, \quad \mathrm{erf}(\theta) = -i\,\mathrm{erfi}(i\theta)\,.$$

2. Both functions are odd,

$$\mathrm{erf}(-\theta) = -\mathrm{erf}(\theta)\,, \ \mathrm{erfi}(-\theta) = -\mathrm{erfi}(\theta)\,.$$

Obviously, $\mathrm{erf}(0) = \mathrm{erfi}(0) = 0$.

3. For complex conjugate arguments, these functions are also conjugate,

$$\mathrm{erf}(\overline{\theta}) = \overline{\mathrm{erf}(\theta)}\,, \ \mathrm{erfi}(\overline{\theta}) = \overline{\mathrm{erfi}(\theta)}\,.$$

4. For a real θ, both $\mathrm{erf}(\theta)$ and $\mathrm{erfi}(\theta)$ are real and have the same sign as θ. For an imaginary θ, both $\mathrm{erf}(\theta)$ and $\mathrm{erfi}(\theta)$ are imaginary, and their imaginary parts have the same sign as the imaginary part of θ.

5. For $|\theta| \gg 1$ (in practice, for at least $|\theta| > 3$) the error function $\mathrm{erf}(\theta)$ can be approximated by using a few terms in its asymptotic expansion, e.g.,

$$\mathrm{erf}(\theta) \approx \mathrm{sign}[\mathrm{Re}(\theta)] - \frac{\exp(-\theta^2)}{\sqrt{\pi}\,\theta} \left(1 - \frac{1}{2\theta^2} + \frac{3}{(2\theta^2)^2} - \frac{3\cdot 5}{(2\theta^2)^3}\right),$$

where $\mathrm{sign}[\mathrm{Re}(\theta)]$ is 1 for $\mathrm{Re}(\theta) \ge 0$ and -1 for $\mathrm{Re}(\theta) < 0$. The discontinuity of this approximation at $\mathrm{Re}(\theta) = 0$ is irrelevant because in this case the approximation becomes applicable only at exponentially large $|\mathrm{erf}(\theta)|$. The addition of more terms into this expansion requires larger $|\theta|$ for accuracy improvement. Numerical calculations show that the absolute value of error in using the formula above is always less than three times the absolute value of the first neglected term of the expansion, i.e., the term $-\exp(-\theta^2)/(\sqrt{\pi}\,\theta)\cdot 3\cdot 5\cdot 7/(2\theta^2)^4$.

As mentioned above, all improper integrals in this book are given with the necessary and sufficient conditions of their convergence. These conditions have been obtained using in particular the following properties of $\operatorname{erf}(\theta)$ for $\theta\to\infty$.

The error function $\operatorname{erf}(\theta)$ has a limit at $\theta\to\infty$ (i.e., converges) if and only if

$$\lim_{\theta\to\infty}\left[\operatorname{Re}(\theta^2)+\ln|\theta|\right]=+\infty\,;$$

in this case $\lim\limits_{\theta\to\infty}\operatorname{erf}(\theta)=\pm 1$ for $\lim\limits_{\theta\to\infty}\operatorname{Re}(\theta)=\pm\infty$.

For $\theta\to\infty$, there is an inequality

$$\frac{\left|\pm 1-\operatorname{erf}(\theta)\right|}{\exp[-\operatorname{Re}(\theta^2)]}<\frac{N}{|\theta|}\qquad\text{for}\qquad \pm\operatorname{Re}(\theta)>0\,,$$

where N is some positive number (it is possible to use $N=1.4/\sqrt{\pi}$ for any θ). As a consequence, for trajectories, for which $\operatorname{Re}(\theta^2)+\ln|\theta|\to-\infty$ or at least $\operatorname{Re}(\theta^2)+\ln|\theta|$ remains bounded from above as $\theta\to\infty$, there is another similar inequality:

$$\frac{\left|\operatorname{erf}(\theta)\right|}{\exp[-\operatorname{Re}(\theta^2)]}<\frac{1}{\exp[-\operatorname{Re}(\theta^2)]}+\frac{N}{|\theta|}<\frac{M}{|\theta|}\,,$$

where M is also some positive number (which depends on the trajectory).

The asymptotic properties of the error function $\operatorname{erf}(\theta)$ at $\theta\to\infty$ and their use to find the necessary and sufficient conditions for convergence of the improper integrals presented in this book, are discussed in detail in our Russian-language book:

Коротков Н.Е., Коротков А.Н, Интегралы, связанные с интегралом вероятностей. Под ред. В.И. Борисова. – Воронеж: изд. ОАО «Концерн «Созвездие», 2012. – 276 с. – ISBN 978-5-900777-18-4.

The methods to derive and check the presented integrals are described in another book, also published in Russia:

Коротков Н.Е., Интегралы для приложений интеграла вероятностей. Под ред. В.И. Борисова. – Воронеж: изд. ФГУП «ВНИИС», 2002. – 800 с. – ISBN 5-900777-10-3.

PART 1

INDEFINITE INTEGRALS

1.1. Integrals of the form $\int z^n \exp\left[\mp(\alpha z+\beta)^2\right] dz$

1.1.1.

1. $\int \exp\left[-(\alpha z+\beta)^2\right] dz = \dfrac{\sqrt{\pi}}{2\alpha} \operatorname{erf}(\alpha z+\beta).$

2. $\int \exp\left[(\alpha z+\beta)^2\right] dz = \dfrac{\sqrt{\pi}}{2\alpha} \operatorname{erfi}(\alpha z+\beta).$

1.1.2.

1. $\int z \exp\left[-(\alpha z+\beta)^2\right] dz = -\dfrac{1}{2\alpha^2}\left\{\exp\left[-(\alpha z+\beta)^2\right] + \sqrt{\pi}\,\beta \operatorname{erf}(\alpha z+\beta)\right\}.$

2. $\int z \exp\left[(\alpha z+\beta)^2\right] dz = \dfrac{1}{2\alpha^2}\left\{\exp\left[(\alpha z+\beta)^2\right] - \sqrt{\pi}\,\beta \operatorname{erfi}(\alpha z+\beta)\right\}.$

1.1.3.

1. $\int z^2 \exp\left[-(\alpha z+\beta)^2\right] dz =$

$$= \frac{1}{2\alpha^3}\left\{(\beta-\alpha z)\exp\left[-(\alpha z+\beta)^2\right] + \frac{\sqrt{\pi}\left(2\beta^2+1\right)}{2} \operatorname{erf}(\alpha z+\beta)\right\}.$$

2. $\int z^2 \exp\left[(\alpha z+\beta)^2\right] dz =$

$$= \frac{1}{2\alpha^3}\left\{(\alpha z-\beta)\exp\left[(\alpha z+\beta)^2\right] + \frac{\sqrt{\pi}\left(2\beta^2-1\right)}{2} \operatorname{erfi}(\alpha z+\beta)\right\}.$$

1.1.4.

1. $\int z^n \exp\left[-(\alpha z+\beta)^2\right] dz = \dfrac{\sqrt{\pi}}{2\alpha^{n+1}} \operatorname{erf}(\alpha z+\beta) \sum_{k=0}^{E(n/2)} \dfrac{n!\,(-\beta)^{n-2k}}{4^k\, k!\,(n-2k)!} -$

$$-\frac{1}{2\alpha^{n+1}}\exp\left[-(\alpha z+\beta)^2\right]\left[\sum_{k=1}^{E(n/2)}\frac{n!(-\beta)^{n-2k}}{k!(n-2k)!}\sum_{l=1}^{k}\frac{l!(\alpha z+\beta)^{2l-1}}{4^{k-l}(2l)!}+\right.$$

$$\left.+\sum_{k=1}^{n-E(n/2)}\frac{n!(k-1)!(-\beta)^{n+1-2k}}{(2k-1)!(n+1-2k)!}\sum_{l=1}^{k}\frac{(\alpha z+\beta)^{2l-2}}{(l-1)!}\right].$$

2. $$\int z^{2m}\exp\left(-\alpha^2z^2\right)dz=\frac{(2m)!}{m!}\left[\frac{\sqrt{\pi}\,\mathrm{erf}(\alpha z)}{(2\alpha)^{2m+1}}-\frac{\exp\left(-\alpha^2z^2\right)}{\alpha}\sum_{k=1}^{m}\frac{k!\,z^{2k-1}}{(2k)!(2\alpha)^{2m+1-2k}}\right].$$

3. $$\int z^{2m+1}\exp\left(-\alpha^2z^2\right)dz=-\frac{m!}{2\alpha^2}\exp\left(-\alpha^2z^2\right)\sum_{k=0}^{m}\frac{z^{2k}}{k!\left(\alpha^2\right)^{m-k}}.$$

4. $$\int z^n\exp\left[(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{2\alpha^{n+1}}\mathrm{erfi}(\alpha z+\beta)\sum_{k=0}^{E(n/2)}\frac{n!(-\beta)^{n-2k}}{(-4)^k\,k!(n-2k)!}+$$

$$+\frac{(-1)^n}{2\alpha^{n+1}}\exp\left[(\alpha z+\beta)^2\right]\left[\sum_{k=1}^{E(n/2)}\frac{n!\,\beta^{n-2k}}{k!(n-2k)!}\sum_{l=1}^{k}\frac{l!(\alpha z+\beta)^{2l-1}}{(-4)^{k-l}(2l)!}-\right.$$

$$\left.-\sum_{k=1}^{n-E(n/2)}\frac{n!(k-1)!\,\beta^{n+1-2k}}{(2k-1)!(n+1-2k)!}\sum_{l=1}^{k}\frac{(\alpha z+\beta)^{2l-2}}{(-1)^{k-l}(l-1)!}\right].$$

5. $$\int z^{2m}\exp\left(\alpha^2z^2\right)dz=\frac{(2m)!}{m!}\left[\frac{\sqrt{\pi}\,\mathrm{erfi}(\alpha z)}{2\alpha\left(-4\alpha^2\right)^m}-\exp\left(\alpha^2z^2\right)\times\right.$$

$$\left.\times\sum_{k=1}^{m}\frac{k!\,z^{2k-1}}{2^{2m+1-2k}(2k)!\left(-\alpha^2\right)^{m+1-k}}\right].$$

6. $$\int z^{2m+1}\exp\left(\alpha^2z^2\right)dz=\frac{m!}{2\alpha^2}\exp\left(\alpha^2z^2\right)\sum_{k=0}^{m}\frac{z^{2k}}{k!\left(-\alpha^2\right)^{m-k}}.$$

1.2. Integrals of the form $\int z^n \exp\left(\mp\alpha^2 z^2 + \beta z + \gamma\right) dz$

1.2.1.

1. $\int \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}}{2\alpha} \exp\left(\frac{\beta^2}{4\alpha^2} + \gamma\right) \operatorname{erf}\left(\alpha z - \frac{\beta}{2\alpha}\right).$

2. $\int \exp\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}}{2\alpha} \exp\left(\gamma - \frac{\beta^2}{4\alpha^2}\right) \operatorname{erfi}\left(\alpha z + \frac{\beta}{2\alpha}\right).$

1.2.2.

1. $$\int z \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}\,\beta}{4\alpha^3} \exp\left(\frac{\beta^2}{4\alpha^2} + \gamma\right) \operatorname{erf}\left(\alpha z - \frac{\beta}{2\alpha}\right) - \frac{1}{2\alpha^2} \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right).$$

2. $$\int z \exp\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{1}{2\alpha^2} \exp\left(\alpha^2 z^2 + \beta z + \gamma\right) - \frac{\sqrt{\pi}\,\beta}{4\alpha^3} \exp\left(\gamma - \frac{\beta^2}{4\alpha^2}\right) \times \operatorname{erfi}\left(\alpha z + \frac{\beta}{2\alpha}\right).$$

1.2.3.

1. $$\int z^2 \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}\left(2\alpha^2 + \beta^2\right)}{8\alpha^5} \exp\left(\frac{\beta^2}{4\alpha^2} + \gamma\right) \operatorname{erf}\left(\alpha z - \frac{\beta}{2\alpha}\right) - \frac{2\alpha^2 z + \beta}{4\alpha^4} \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right).$$

2. $$\int z^2 \exp\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}\left(\beta^2 - 2\alpha^2\right)}{8\alpha^5} \exp\left(\gamma - \frac{\beta^2}{4\alpha^2}\right) \operatorname{erfi}\left(\alpha z + \frac{\beta}{2\alpha}\right) + \frac{2\alpha^2 z - \beta}{4\alpha^4} \exp\left(\alpha^2 z^2 + \beta z + \gamma\right).$$

1.2.4.

1. $$\int z^n \exp\left(-\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{\sqrt{\pi}}{2^{n+1}\alpha} \exp\left(\frac{\beta^2}{4\alpha^2} + \gamma\right) \operatorname{erf}\left(\alpha z - \frac{\beta}{2\alpha}\right) \times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{n!\beta^{n-2k}}{k!\,(n-2k)!\,\left(\alpha^2\right)^{n-k}}-\frac{1}{2^n}\exp\left(-\alpha^2z^2+\beta z+\gamma\right)\left[\sum_{k=1}^{E(n/2)}\frac{n!\,\beta^{n-2k}}{k!(n-2k)!}\times\right.$$

$$\left.\times\sum_{l=1}^{k}\frac{l!\left(2\alpha^2z-\beta\right)^{2l-1}}{(2l)!\left(\alpha^2\right)^{n-k+l}}+\sum_{k=1}^{n-E(n/2)}\frac{n!\,(k-1)!\,\beta^{n+1-2k}}{(2k-1)!\,(n+1-2k)!}\sum_{l=1}^{k}\frac{4^{k-l}\left(2\alpha^2z-\beta\right)^{2l-2}}{(l-1)!\left(\alpha^2\right)^{n-k+l}}\right].$$

2. $\int z^n\exp\left(\alpha^2z^2+\beta z+\gamma\right)\,dz=\frac{\sqrt{\pi}}{2^{n+1}\alpha}\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\mathrm{erfi}\left(\alpha z+\frac{\beta}{2\alpha}\right)\times$

$$\times\sum_{k=0}^{E(n/2)}\frac{n!\,\beta^{n-2k}}{k!\,(n-2k)!\left(-\alpha^2\right)^{n-k}}+\frac{1}{2^n}\exp\left(\alpha^2z^2+\beta z+\gamma\right)\left[\sum_{k=1}^{E(n/2)}\frac{n!\,\beta^{n-2k}}{k!\,(n-2k)!}\times\right.$$

$$\left.\times\sum_{l=1}^{k}\frac{l!\left(2\alpha^2z+\beta\right)^{2l-1}}{(2l)!\left(-\alpha^2\right)^{n-k+l}}-\sum_{k=1}^{n-E(n/2)}\frac{n!(k-1)!\beta^{n+1-2k}}{(2k-1)!(n+1-2k)!}\sum_{l=1}^{k}\frac{4^{k-l}\left(2\alpha^2z+\beta\right)^{2l-2}}{(l-1)!\left(-\alpha^2\right)^{n-k+l}}\right].$$

1.2.5. $\int z^n\exp(\beta z+\gamma)\,dz=\frac{n!}{\beta}\exp(\beta z+\gamma)\sum_{k=0}^{n}\frac{z^k}{k!(-\beta)^{n-k}}$.

1.3. Integrals of the form $\int \mathrm{erf}^n\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz$

1.3.1. $\int\mathrm{erf}\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz=\frac{\sqrt{\pi}}{4\alpha}\mathrm{erf}^2\left(\alpha z+\beta\right)$.

1.3.2. $\int\mathrm{erf}^2\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz=\frac{\sqrt{\pi}}{6\alpha}\mathrm{erf}^3\left(\alpha z+\beta\right)$.

1.3.3. $\int\mathrm{erf}^n\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz=\frac{\sqrt{\pi}}{\left(2n+2\right)\alpha}\mathrm{erf}^{n+1}\left(\alpha z+\beta\right)$.

1.4. Integrals of the form $\int z^n\,\mathrm{erf}\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz$

1.4.1. $\int z\,\mathrm{erf}\left(\alpha z+\beta\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]\,dz=\frac{1}{4\alpha^2}\left\{\sqrt{2}\,\mathrm{erf}\left[\sqrt{2}\left(\alpha z+\beta\right)\right]-\right.$

$$-\sqrt{\pi}\,\beta\,\mathrm{erf}^2(\alpha z+\beta)\Big\}-\frac{1}{2\alpha^2}\,\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right].$$

1.4.2. $\int z^2\,\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\beta-\alpha z}{2\alpha^3}\,\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]+$

$$+\frac{\sqrt{\pi}\left(2\beta^2+1\right)}{8\alpha^3}\,\mathrm{erf}^2(\alpha z+\beta)-\frac{\beta}{\sqrt{2}\alpha^3}\,\mathrm{erf}\left[\sqrt{2}(\alpha z+\beta)\right]-\frac{1}{4\sqrt{\pi}\alpha^3}\times$$

$$\times\exp\left[-2(\alpha z+\beta)^2\right].$$

1.4.3.

1. $\int z^n\,\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{4\alpha^{n+1}}\,\mathrm{erf}^2(\alpha z+\beta)\times$

$$\times\sum_{k=0}^{E(n/2)}\frac{n!(-\beta)^{n-2k}}{4^k\,k!(n-2k)!}-\frac{n!}{\alpha^{n+1}}\sum_{k=1}^{E(n/2)}\frac{(-\beta)^{n-2k}}{4^k\,k!(n-2k)!}\Bigg\{\mathrm{erf}(\alpha z+\beta)\times$$

$$\times\exp\left[-(\alpha z+\beta)^2\right]\sum_{l=0}^{k-1}\frac{4^l\,l!(\alpha z+\beta)^{2l+1}}{(2l+1)!}+\frac{1}{2\sqrt{\pi}}\exp\left[-2(\alpha z+\beta)^2\right]\sum_{l=0}^{k-1}\frac{(l!)^2}{(2l+1)!}\times$$

$$\times\sum_{r=0}^{l}\frac{2^{l+r}(\alpha z+\beta)^{2r}}{r!}\Bigg\}+\frac{n!}{2\alpha^{n+1}}\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!(-\beta)^{n+1-2k}}{(2k-1)!(n+1-2k)!}\Bigg\{\frac{1}{\sqrt{2}}\times$$

$$\times\mathrm{erf}\left[\sqrt{2}(\alpha z+\beta)\right]\sum_{l=0}^{k-1}\frac{(2l)!}{8^l(l!)^2}-\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]\sum_{l=0}^{k-1}\frac{(\alpha z+\beta)^{2l}}{l!}-$$

$$-\frac{2}{\sqrt{\pi}}\exp\left[-2(\alpha z+\beta)^2\right]\sum_{l=1}^{k-1}\frac{(2l)!}{(l!)^2}\sum_{r=0}^{l-1}\frac{r!(\alpha z+\beta)^{2r+1}}{8^{l-r}(2r+1)!}\Bigg\}.$$

2. $\int z^{2m}\,\mathrm{erf}(\alpha z)\exp\left(-\alpha^2z^2\right)dz=\frac{(2m)!}{m!}\Bigg[\frac{\sqrt{\pi}\,\alpha}{\left(4\alpha^2\right)^{m+1}}\,\mathrm{erf}^2(\alpha z)-$

$$-\mathrm{erf}(\alpha z)\exp\left(-\alpha^2z^2\right)\sum_{k=0}^{m-1}\frac{k!\,z^{2k+1}}{(2k+1)!\left(4\alpha^2\right)^{m-k}}-\frac{1}{2\sqrt{\pi}\,\alpha}\exp\left(-2\alpha^2z^2\right)\times$$

$$\times\sum_{k=0}^{m-1}\frac{(k!)^2}{2^{m-k}(2k+1)!}\sum_{l=0}^{k}\frac{z^{2l}}{l!\left(2\alpha^2\right)^{m-l}}\Bigg].$$

3. $\int z^{2m+1}\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2\right)dz=\frac{m!}{2\alpha^{2m+2}}\left[\frac{1}{\sqrt{2}}\operatorname{erf}\left(\sqrt{2}\,\alpha z\right)\sum_{k=0}^{m}\frac{(2k)!}{8^k(k!)^2}-\right.$

$$-\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2\right)\sum_{k=0}^{m}\frac{(\alpha z)^{2k}}{k!}-\frac{2}{\sqrt{\pi}}\exp\left(-2\alpha^2 z^2\right)\sum_{k=1}^{m}\frac{(2k)!}{(k!)^2}\times$$

$$\left.\times\sum_{l=0}^{k-1}\frac{l!\,(\alpha z)^{2l+1}}{8^{k-l}(2l+1)!}\right].$$

1.5. Integrals of the form $\int z^n \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz$

1.5.1. $\int \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz=\frac{1}{\beta_1}\left[\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)-\right.$

$$\left.-\exp\left(\frac{\beta_1{}^2-4\alpha\beta\beta_1}{4\alpha^2}+\gamma\right)\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1}{2\alpha}\right)\right].$$

1.5.2. $\int z\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz=\frac{\beta_1 z-1}{\beta_1^2}\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)+$

$$+\left(\frac{1}{\beta_1^2}+\frac{\beta}{\alpha\beta_1}-\frac{1}{2\alpha^2}\right)\exp\left(\frac{\beta_1^2-4\alpha\beta\beta_1}{4\alpha^2}+\gamma\right)\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1}{2\alpha}\right)+\frac{1}{\sqrt{\pi}\,\alpha\beta_1}\times$$

$$\times\exp\left[-(\alpha z+\beta)^2+\beta_1 z+\gamma\right].$$

1.5.3. $\int z^n\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz=\frac{n!}{\beta_1}\left\{\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\sum_{k=0}^{n}\frac{z^k}{k!(-\beta_1)^{n-k}}-\right.$

$$-\exp\left(\frac{\beta_1^2-4\alpha\beta\beta_1}{4\alpha^2}+\gamma\right)\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1}{2\alpha}\right)\sum_{k=0}^{n}\frac{1}{2^k(-\beta_1)^{n-k}}\times$$

$$\times\sum_{l=0}^{E(k/2)}\frac{(\beta_1-2\alpha\beta)^{k-2l}}{l!(k-2l)!\left(\alpha^2\right)^{k-l}}+\frac{2\alpha}{\sqrt{\pi}}\exp\left[-(\alpha z+\beta)^2+\beta_1 z+\gamma\right]\sum_{k=1}^{n}\frac{1}{2^k(-\beta_1)^{n-k}}\times$$

$$\times\left[\sum_{l=1}^{E(k/2)}\frac{(\beta_1-2\alpha\beta)^{k-2l}}{l!(k-2l)!}\sum_{r=1}^{l}\frac{r!\left(2\alpha^2 z+2\alpha\beta-\beta_1\right)^{2r-1}}{(2r)!\left(\alpha^2\right)^{k-l+r}}+\right.$$

$$+\sum_{l=1}^{k-E(k/2)} \frac{(l-1)!\,(\beta_1-2\alpha\beta)^{k+1-2l}}{(2l-1)!\,(k+1-2l)!} \sum_{r=1}^{l} \frac{4^{l-r}\left(2\alpha^2 z+2\alpha\beta-\beta_1\right)^{2r-2}}{(r-1)!\left(\alpha^2\right)^{k-l+r}} \Bigg]\Bigg\}.$$

1.6. Integrals of the form $\int z^{2m+1}\,\mathrm{erf}\left(\alpha z+\beta\right)\exp\left(\mp\alpha^2 z^2+\gamma\right)\,dz$

1.6.1.

1. $$\int z\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha^2 z^2+\gamma\right)dz=\frac{1}{2\sqrt{2}\,\alpha^2}\exp\left(\gamma-\frac{\beta^2}{2}\right)\mathrm{erf}\left(\sqrt{2}\,\alpha z+\frac{\beta}{\sqrt{2}}\right)-$$
$$-\frac{1}{2\alpha^2}\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha^2 z^2+\gamma\right).$$

2. $$\int z\,\mathrm{erf}\left(\alpha z\right)\exp\left(-\alpha^2 z^2+\gamma\right)\,dz=\frac{\exp(\gamma)}{2\alpha^2}\left[\frac{\mathrm{erf}\left(\sqrt{2}\alpha z\right)}{\sqrt{2}}-\mathrm{erf}\left(\alpha z\right)\exp\left(-\alpha^2 z^2\right)\right].$$

3. $$\int z\,\mathrm{erf}\left(\alpha z+\beta\right)\exp\left(\alpha^2 z^2+\gamma\right)\,dz=\frac{1}{2\alpha^2}\left[\mathrm{erf}\left(\alpha z+\beta\right)\exp\left(\alpha^2 z^2+\gamma\right)+\right.$$
$$\left.+\frac{1}{\sqrt{\pi}\,\beta}\exp\left(\gamma-\beta^2-2\alpha\beta z\right)\right].$$

4. $$\int z\,\mathrm{erf}\left(\alpha z\right)\exp\left(\alpha^2 z^2+\gamma\right)\,dz=\frac{\exp(\gamma)}{\alpha}\left[\frac{1}{2\alpha}\,\mathrm{erf}\left(\alpha z\right)\exp\left(\alpha^2 z^2\right)-\frac{z}{\sqrt{\pi}}\right].$$

1.6.2.

1. $$\int z^{2m+1}\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha^2 z^2+\gamma\right)dz=-\frac{m!}{2}\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha^2 z^2+\gamma\right)\times$$
$$\times\sum_{k=0}^{m}\frac{z^{2k}}{k!\left(\alpha^2\right)^{m+1-k}}+\frac{m!}{2\left(\alpha^2\right)^{m+1}}\sum_{k=0}^{m}\frac{(2k)!}{k!}\left\{\frac{1}{\sqrt{2}}\exp\left(\gamma-\frac{\beta^2}{2}\right)\mathrm{erf}\left(\sqrt{2}\,\alpha z+\frac{\beta}{\sqrt{2}}\right)\times\right.$$
$$\times\sum_{l=0}^{k}\frac{\beta^{2k-2l}}{2^{2k+l}\,l!\,(2k-2l)!}+\frac{1}{\sqrt{\pi}}\exp\left(\gamma-\beta^2-2\alpha\beta z-2\alpha^2 z^2\right)\sum_{l=1}^{k}\frac{\beta^{2k-2l}}{(2k-2l)!}\times$$
$$\left.\times\sum_{r=1}^{l}\frac{(2\alpha z+\beta)^{2r-2}}{2^{2k-l+r}}\left[\frac{(l-1)!\,\beta}{(2k+1-2l)\,(2l-1)!\,(r-1)!}-\frac{r!\,(2\alpha z+\beta)}{4^{l-r}\,l!\,(2r)!}\right]\right\}.$$

2. $\int z^{2m+1}\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2+\gamma\right)dz=\dfrac{m!\exp(\gamma)}{\left(\alpha^2\right)^{m+1}}\sum_{k=0}^{m}\dfrac{1}{k!}\times$

$$\times\left\{\frac{(2k)!}{8^k\,k!}\left[\frac{\operatorname{erf}\left(\sqrt{2}\,\alpha z\right)}{\sqrt{8}}-\frac{\exp\left(-2\alpha^2 z^2\right)}{4\sqrt{\pi}}\sum_{l=1}^{k}\frac{8^l\,l!(\alpha z)^{2l-1}}{(2l)!}\right]-\right.$$

$$\left.-\frac{(\alpha z)^{2k}}{2}\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2\right)\right\}.$$

3. $\int z^{2m+1}\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha^2 z^2+\gamma\right)dz=$

$$=\frac{m!}{2\alpha^2}\left[\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha^2 z^2+\gamma\right)\sum_{k=0}^{m}\frac{z^{2k}}{k!\left(-\alpha^2\right)^{m-k}}+\right.$$

$$\left.+\frac{\exp\left(\gamma-\beta^2-2\alpha\beta z\right)}{\sqrt{\pi}\,\beta}\sum_{k=0}^{m}\frac{(2k)!}{k!\left(-\alpha^2\right)^{m-k}}\sum_{l=0}^{2k}\frac{z^l}{l!(2\alpha\beta)^{2k-l}}\right].$$

4. $\int z^{2m+1}\operatorname{erf}(\alpha z)\exp\left(\alpha^2 z^2+\gamma\right)dz=\dfrac{m!\exp(\gamma)}{\alpha}\sum_{k=0}^{m}\dfrac{z^{2k}}{k!\left(-\alpha^2\right)^{m-k}}\times$

$$\times\left[\frac{\operatorname{erf}(\alpha z)}{2\alpha}\exp\left(\alpha^2 z^2\right)-\frac{z}{(2k+1)\sqrt{\pi}}\right].$$

1.7. Integrals of the form $\int z^{2m+1}\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha_1 z^2+\gamma\right)dz$

1.7.1.

1. $\int z\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha_1 z^2+\gamma\right)dz=\dfrac{1}{2\alpha_1}\left[\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha_1 z^2+\gamma\right)-\right.$

$$\left.-\alpha\exp\left(\frac{\alpha_1\beta^2}{\alpha^2-\alpha_1}+\gamma\right)V(\beta)\right].$$

2. $\int z\operatorname{erf}(\alpha z)\exp\left(\alpha_1 z^2+\gamma\right)dz=\dfrac{\operatorname{erf}(\alpha z)}{2\alpha_1}\exp\left(\alpha_1 z^2+\gamma\right)-\dfrac{\alpha\exp(\gamma)}{2\alpha_1}V(0)$

$\{\alpha_1\neq\alpha^2\}$.

1.7.2.

1. $\int z^{2m+1} \operatorname{erf}(\alpha z+\beta) \exp\left(\alpha_1 z^2+\gamma\right) dz = \frac{m!}{2} \operatorname{erf}(\alpha z+\beta) \exp\left(\alpha_1 z^2+\gamma\right) \sum_{k=0}^{m} \frac{(-1)^{m-k} z^{2k}}{k!\, \alpha_1^{m+1-k}} +$

$$+\frac{m!\,\alpha}{2} \sum_{k=0}^{m} \frac{(2k)!}{k!\,(-\alpha_1)^{m+1-k}} \left\{ \exp\left(\frac{\alpha_1 \beta^2}{\alpha^2-\alpha_1}+\gamma\right) V(\beta) \times \right.$$

$$\times \sum_{l=0}^{k} \frac{(\alpha\beta)^{2k-2l}}{4^l\, l!\,(2k-2l)!\left(\alpha^2-\alpha_1\right)^{2k-l}} + \frac{1}{\sqrt{\pi}} \exp\left[-(\alpha z+\beta)^2+\alpha_1 z^2+\gamma\right] \times$$

$$\times \left[\sum_{l=1}^{k} \frac{(l-1)!\,(\alpha\beta)^{2k+1-2l}}{(2l-1)!\,(2k+1-2l)!} \sum_{r=1}^{l} \frac{\left(\alpha^2 z-\alpha_1 z+\alpha\beta\right)^{2r-2}}{(r-1)!\left(\alpha^2-\alpha_1\right)^{2k-l+r}} - \sum_{l=1}^{k} \frac{(\alpha\beta)^{2k-2l}}{l!\,(2k-2l)!} \times \right.$$

$$\left.\left. \times \sum_{r=1}^{l} \frac{r!\left(\alpha^2 z-\alpha_1 z+\alpha\beta\right)^{2r-1}}{4^{l-r}(2r)!\left(\alpha^2-\alpha_1\right)^{2k-l+r}} \right] \right\}.$$

2. $\int z^{2m+1} \operatorname{erf}(\alpha z) \exp\left(\alpha_1 z^2+\gamma\right) dz = \frac{m!}{2} \operatorname{erf}(\alpha z) \exp\left(\alpha_1 z^2+\gamma\right) \sum_{k=0}^{m} \frac{(-1)^{m-k} z^{2k}}{k!\, \alpha_1^{m+1-k}} +$

$$+\frac{m!\,\alpha \exp(\gamma)}{2} \sum_{k=0}^{m} \frac{(2k)!}{(k!)^2 (-\alpha_1)^{m+1-k}} \left\{ \frac{1}{4^k\left(\alpha^2-\alpha_1\right)^k} V(0) - \right.$$

$$\left. -\frac{\exp\left[\left(\alpha_1-\alpha^2\right) z^2\right]}{\sqrt{\pi}} \sum_{l=1}^{k} \frac{l!\, z^{2l-1}}{4^{k-l}\,(2l)!\left(\alpha^2-\alpha_1\right)^{k+1-l}} \right\}.$$

Introduced notation:

$$V(\beta) = \frac{1}{\sqrt{\alpha^2-\alpha_1}} \operatorname{erf}\left(\sqrt{\alpha^2-\alpha_1}\, z + \frac{\alpha\beta}{\sqrt{\alpha^2-\alpha_1}}\right) =$$

$$= \frac{1}{\sqrt{\alpha_1-\alpha^2}} \operatorname{erfi}\left(\sqrt{\alpha_1-\alpha^2}\, z - \frac{\alpha\beta}{\sqrt{\alpha_1-\alpha^2}}\right).$$

1.8. Integrals of the form $\int z^n \operatorname{erf}^2(\alpha z)\exp\left(\mp\alpha^2 z^2\right) dz$

1.8.1. $$\int z\operatorname{erf}^2(\alpha z)\exp\left(\alpha^2 z^2\right) dz = \frac{\operatorname{erf}^2(\alpha z)}{2\alpha^2}\exp\left(\alpha^2 z^2\right) - \frac{2}{\pi\alpha^2}\times$$
$$\times\exp\left(-\alpha^2 z^2\right) - \frac{2z}{\sqrt{\pi}\,\alpha}\operatorname{erf}(\alpha z).$$

1.8.2. $$\int z^2\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2 z^2\right) dz = \frac{\sqrt{\pi}\operatorname{erf}^3(\alpha z)}{12\alpha^3} + \frac{\operatorname{erf}\left(\sqrt{3}\alpha z\right)}{2\sqrt{3\pi}\,\alpha^3} - \frac{\operatorname{erf}(\alpha z)}{2\sqrt{\pi}\,\alpha^3}\times$$
$$\times\exp\left(-2\alpha^2 z^2\right) - \frac{z\operatorname{erf}^2(\alpha z)}{2\alpha^2}\exp\left(-\alpha^2 z^2\right).$$

1.8.3. $$\int z^{2m}\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2 z^2\right) dz = \frac{(2m)!}{m!}\left\{\frac{\sqrt{\pi}\operatorname{erf}^3(\alpha z)}{3(2\alpha)^{2m+1}} -\right.$$
$$-\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2 z^2\right)\sum_{k=0}^{m-1}\frac{k!\,z^{2k+1}}{(2k+1)!\left(4\alpha^2\right)^{m-k}} + \frac{1}{\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!}\times$$
$$\times\left[\frac{\operatorname{erf}\left(\sqrt{3}\,\alpha z\right)}{\sqrt{3}\alpha^{2m+1}}\sum_{l=0}^{k}\frac{(2l)!}{2^{2m-k+l}3^l(l!)^2} - \operatorname{erf}(\alpha z)\exp\left(-2\alpha^2 z^2\right)\times\right.$$
$$\left.\times\sum_{l=0}^{k}\frac{z^{2l}}{2^{2m-k-l}\,l!\,\alpha^{2m+1-2l}}\right] - \frac{\exp\left(-3\alpha^2 z^2\right)}{\pi}\sum_{k=1}^{m-1}\frac{(k!)^2}{(2k+1)!}\sum_{l=1}^{k}\frac{(2l)!}{(l!)^2}\times$$
$$\left.\times\sum_{r=1}^{l}\frac{r!\,z^{2r-1}}{2^{2m-k+l-2r}3^{l+1-r}(2r)!\left(\alpha^2\right)^{m+1-r}}\right\}.$$

1.8.4. $$\int z^{2m+1}\operatorname{erf}^2(\alpha z)\exp\left(\alpha^2 z^2\right) dz = \frac{\operatorname{erf}^2(\alpha z)}{2\alpha^2}\exp\left(\alpha^2 z^2\right)\sum_{k=0}^{m}\frac{m!\,z^{2k}}{k!\left(-\alpha^2\right)^{m-k}} -$$
$$-\frac{2}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(-1)^{m-k}m!}{2k+1}\left[\frac{z^{2k+1}\operatorname{erf}(\alpha z)}{k!\,\alpha^{2m+1-2k}} + \frac{\exp\left(-\alpha^2 z^2\right)}{\sqrt{\pi}}\sum_{l=0}^{k}\frac{z^{2l}}{l!\left(\alpha^2\right)^{m+1-l}}\right].$$

1.9. Integrals of the form $\int z^n \operatorname{erf}(\alpha z+\beta)\,dz$

1.9.1. $\int \operatorname{erf}(\alpha z+\beta)\,dz=\left(z+\frac{\beta}{\alpha}\right)\operatorname{erf}(\alpha z+\beta)+\frac{1}{\sqrt{\pi}\,\alpha}\exp\left[-(\alpha z+\beta)^2\right].$

1.9.2. $\int z\operatorname{erf}(\alpha z+\beta)\,dz=\left(\frac{z^2}{2}-\frac{2\beta^2+1}{4\alpha^2}\right)\operatorname{erf}(\alpha z+\beta)+\frac{\alpha z-\beta}{2\sqrt{\pi}\alpha^2}\exp\left[-(\alpha z+\beta)^2\right].$

1.9.3. $\int z^2\operatorname{erf}(\alpha z+\beta)\,dz=\left(\frac{z^3}{3}+\frac{2\beta^3+3\beta}{6\alpha^3}\right)\operatorname{erf}(\alpha z+\beta)+\frac{\alpha^2z^2-\alpha\beta z+\beta^2+1}{3\sqrt{\pi}\,\alpha^3}\times$

$\times\exp\left[-(\alpha z+\beta)^2\right].$

1.9.4.

1. $\int z^n\operatorname{erf}(\alpha z+\beta)\,dz=\operatorname{erf}(\alpha z+\beta)\left[\frac{z^{n+1}}{n+1}-\frac{n!}{\alpha^{n+1}}\sum_{k=0}^{n-E(n/2)}\frac{(-\beta)^{n+1-2k}}{4^k\,k!\,(n+1-2k)!}\right]+$

$+\frac{n!}{\sqrt{\pi}\,\alpha^{n+1}}\exp\left[-(\alpha z+\beta)^2\right]\left[\sum_{k=0}^{E(n/2)}\frac{k!\,(-\beta)^{n-2k}}{(2k+1)!\,(n-2k)!}\sum_{l=0}^{k}\frac{(\alpha z+\beta)^{2l}}{l!}+\right.$

$\left.+\sum_{k=1}^{n-E(n/2)}\frac{(-\beta)^{n+1-2k}}{k!\,(n+1-2k)!}\sum_{l=1}^{k}\frac{l!\,(\alpha z+\beta)^{2l-1}}{4^{k-l}\,(2l)!}\right].$

2. $\int z^{2m}\operatorname{erf}(\alpha z)\,dz=\frac{z^{2m+1}}{2m+1}\operatorname{erf}(\alpha z)+\frac{m!}{(2m+1)\sqrt{\pi}}\exp\left(-\alpha^2z^2\right)\sum_{k=0}^{m}\frac{z^{2k}}{k!\,\alpha^{2m+1-2k}}.$

3. $\int z^{2m+1}\operatorname{erf}(\alpha z)\,dz=\operatorname{erf}(\alpha z)\left[\frac{z^{2m+2}}{2m+2}-\frac{(2m+1)!}{(m+1)!\left(4\alpha^2\right)^{m+1}}\right]+\frac{(2m+1)!}{(m+1)!\sqrt{\pi}\,\alpha}\times$

$\times\exp\left(-\alpha^2z^2\right)\sum_{k=0}^{m}\frac{(k+1)!\,z^{2k+1}}{(2k+2)!\left(4\alpha^2\right)^{m-k}}.$

1.10. Integrals of the form $\int z^n \operatorname{erf}^2(\alpha z+\beta)\,dz$

1.10.1. $\int\operatorname{erf}^2(\alpha z+\beta)\,dz=\left(z+\frac{\beta}{\alpha}\right)\operatorname{erf}^2(\alpha z+\beta)+\frac{2}{\sqrt{\pi}\,\alpha}\operatorname{erf}(\alpha z+\beta)\times$

$\times\exp\left[-(\alpha z+\beta)^2\right]-\frac{\sqrt{2}}{\sqrt{\pi}\,\alpha}\operatorname{erf}\left[\sqrt{2}(\alpha z+\beta)\right].$

1.10.2. $$\int z\,\mathrm{erf}^2(\alpha z+\beta)\,dz=\left(\frac{z^2}{2}-\frac{2\beta^2+1}{4\alpha^2}\right)\mathrm{erf}^2(\alpha z+\beta)+\frac{\alpha z-\beta}{\sqrt{\pi}\,\alpha^2}\mathrm{erf}(\alpha z+\beta)\times$$
$$\times\exp\left[-(\alpha z+\beta)^2\right]+\frac{\sqrt{2}\beta}{\sqrt{\pi}\alpha^2}\mathrm{erf}\left[\sqrt{2}(\alpha z+\beta)\right]+\frac{1}{2\pi\alpha^2}\exp\left[-2(\alpha z+\beta)^2\right].$$

1.10.3. $$\int z^2\,\mathrm{erf}^2(\alpha z+\beta)\,dz=\left(\frac{z^3}{3}+\frac{2\beta^3+3\beta}{6\alpha^3}\right)\mathrm{erf}^2(\alpha z+\beta)+\frac{\alpha z-2\beta}{3\pi\alpha^3}\times$$
$$\times\exp\left[-2(\alpha z+\beta)^2\right]+\frac{2\left(\alpha^2z^2-\alpha\beta z+\beta^2+1\right)}{3\sqrt{\pi}\,\alpha^3}\exp\left[-(\alpha z+\beta)^2\right]\mathrm{erf}(\alpha z+\beta)-$$
$$-\frac{\sqrt{2}\left(12\beta^2+5\right)}{12\sqrt{\pi}\,\alpha^3}\mathrm{erf}\left[\sqrt{2}(\alpha z+\beta)\right].$$

1.10.4.

1. $$\int z^n\,\mathrm{erf}^2(\alpha z+\beta)dz=\mathrm{erf}^2(\alpha z+\beta)\left[\frac{z^{n+1}}{n+1}-\frac{n!}{\alpha^{n+1}}\sum_{k=0}^{n-E(n/2)}\frac{(-\beta)^{n+1-2k}}{4^k\,k!\,(n+1-2k)!}\right]+$$
$$+\frac{n!}{\sqrt{\pi}\,\alpha^{n+1}}\sum_{k=0}^{E(n/2)}\frac{k!\,(-\beta)^{n-2k}}{(2k+1)!\,(n-2k)!}\left\{2\,\mathrm{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]\times\right.$$
$$\times\sum_{l=0}^{k}\frac{(\alpha z+\beta)^{2l}}{l!}-\sqrt{2}\,\mathrm{erf}\left[\sqrt{2}(\alpha z+\beta)\right]\sum_{l=0}^{k}\frac{(2l)!}{8^l\,(l!)^2}+\frac{4}{\sqrt{\pi}}\exp\left[-2(\alpha z+\beta)^2\right]\times$$
$$\left.\times\sum_{l=1}^{k}\frac{(2l)!}{(l!)^2}\sum_{r=0}^{l-1}\frac{r!\,(\alpha z+\beta)^{2r+1}}{8^{l-r}\,(2r+1)!}\right\}+\frac{n!}{\sqrt{\pi}\,\alpha^{n+1}}\sum_{k=1}^{n-E(n/2)}\frac{(-\beta)^{n+1-2k}}{k!\,(n+1-2k)!}\times$$
$$\times\left\{\mathrm{erf}\,(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]\sum_{l=1}^{k}\frac{(l-1)!\,(\alpha z+\beta)^{2l-1}}{4^{k-l}\,(2l-1)!}+\right.$$
$$\left.+\frac{2}{\sqrt{\pi}}\exp\left[-2(\alpha z+\beta)^2\right]\sum_{l=0}^{k-1}\frac{(l!)^2}{(2l+1)!}\sum_{r=0}^{l}\frac{(\alpha z+\beta)^{2r}}{2^{2k-l-r}\,r!}\right\}.$$

2. $$\int z^{2m}\,\mathrm{erf}^2(\alpha z)\,dz=\frac{z^{2m+1}}{2m+1}\mathrm{erf}^2(\alpha z)+\frac{m!}{(2m+1)\sqrt{\pi}\,\alpha^{2m+1}}\left[2\,\mathrm{erf}\,(\alpha z)\times\right.$$
$$\times\exp\left(-\alpha^2z^2\right)\sum_{k=0}^{m}\frac{(\alpha z)^{2k}}{k!}-\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\alpha z\right)\sum_{k=0}^{m}\frac{(2k)!}{8^k\,(k!)^2}+\frac{1}{2\sqrt{\pi}}\times$$
$$\left.\times\exp\left(-2\alpha^2z^2\right)\sum_{k=1}^{m}\frac{(2k)!}{(k!)^2}\sum_{l=1}^{k}\frac{(l-1)!\,(\alpha z)^{2l-1}}{8^{k-l}\,(2l-1)!}\right].$$

3. $\int z^{2m+1}\operatorname{erf}^2(\alpha z)dz = \operatorname{erf}^2(\alpha z)\left[\frac{z^{2m+2}}{2m+2} - \frac{(2m+1)!}{(m+1)!\left(4\alpha^2\right)^{m+1}}\right] +$

$$+\frac{(2m+1)!}{(m+1)!\sqrt{\pi}\,\alpha^{2m+1}}\left[z\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2\right)\sum_{k=0}^{m}\frac{k!(\alpha z)^{2k}}{4^{m-k}(2k+1)!} +\right.$$

$$\left.+\frac{1}{2\sqrt{\pi}\,\alpha}\exp\left(-2\alpha^2 z^2\right)\sum_{k=0}^{m}\frac{(k!)^2}{(2k+1)!}\sum_{l=0}^{k}\frac{(\alpha z)^{2l}}{2^{2m-k-l}\,l!}\right].$$

1.11. Integrals of the form $\int z^{2m}\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z)\,dz$

1.11.1.

1. $\int \operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z)\,dz = z\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z) + \frac{1}{\sqrt{\pi}}\left[\frac{\operatorname{erf}(\alpha z)}{\alpha_1}\exp\left(-\alpha_1^2 z^2\right) +\right.$

$$\left.+\frac{\operatorname{erf}(\alpha_1 z)}{\alpha}\exp\left(-\alpha^2 z^2\right) - \frac{\sqrt{\alpha^2+\alpha_1^2}}{\alpha\alpha_1}\operatorname{erf}\left(\sqrt{\alpha^2+\alpha_1^2}\,z\right)\right].$$

2. $\int \operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z)dz = z\operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z) + \frac{1}{\sqrt{\pi}\,\alpha}\times$

$$\times\left[\operatorname{erfi}(\alpha z)\exp\left(-\alpha^2 z^2\right) - \operatorname{erf}(\alpha z)\exp\left(\alpha^2 z^2\right)\right].$$

1.11.2.

1. $\int z^2\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z)\,dz = \frac{z^3}{3}\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z) + \frac{z}{3\pi\,\alpha\alpha_1}\times$

$$\times\exp\left[-\left(\alpha^2+\alpha_1^2\right)z^2\right] + \frac{1}{3\sqrt{\pi}}\left[\frac{\alpha_1^2 z^2+1}{\alpha_1^3}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1^2 z^2\right) + \frac{\alpha^2 z^2+1}{\alpha^3}\times\right.$$

$$\left.\times\operatorname{erf}(\alpha_1 z)\exp\left(-\alpha^2 z^2\right) - \frac{2\alpha^4+2\alpha_1^4+(\alpha\alpha_1)^2}{2\sqrt{\alpha^2+\alpha_1^2}\,(\alpha\alpha_1)^3}\operatorname{erf}\left(\sqrt{\alpha^2+\alpha_1^2}\,z\right)\right].$$

2. $\int z^2\operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z)\,dz = \frac{z^3}{3}\operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z) + \frac{1}{3\sqrt{\pi}\,\alpha}\times$

$$\times\left[\left(\frac{1}{\alpha^2}+z^2\right)\operatorname{erfi}(\alpha z)\exp\left(-\alpha^2 z^2\right) + \left(\frac{1}{\alpha^2}-z^2\right)\operatorname{erf}(\alpha z)\times\right.$$

$$\times\exp\left(\alpha^2 z^2\right)-\frac{4z}{\sqrt{\pi}\,\alpha}\Bigg].$$

1.11.3.

1. $$\int z^{2m}\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z)\,dz=\frac{z^{2m+1}}{2m+1}\operatorname{erf}(\alpha z)\operatorname{erf}(\alpha_1 z)+\frac{1}{(2m+1)\sqrt{\pi}}\sum_{k=0}^{m}\frac{m!}{k!}\times$$

$$\times\left\{z^{2k}\left[\frac{\operatorname{erf}(\alpha z)}{\alpha_1^{2m+1-2k}}\exp\left(-\alpha_1^2 z^2\right)+\frac{\operatorname{erf}(\alpha_1 z)}{\alpha^{2m+1-2k}}\exp\left(-\alpha^2 z^2\right)\right]-\frac{(2k)!}{4^k\,k!\left(\alpha^2+\alpha_1^2\right)^k}\times\right.$$

$$\times\left(\frac{\alpha}{\alpha_1^{2m+1-2k}}+\frac{\alpha_1}{\alpha^{2m+1-2k}}\right)\left[\frac{\operatorname{erf}\left(\sqrt{\alpha^2+\alpha_1^2}\,z\right)}{\sqrt{\alpha^2+\alpha_1^2}}-\frac{1}{\sqrt{\pi}}\exp\left(-\alpha^2 z^2-\alpha_1^2 z^2\right)\times\right.$$

$$\left.\left.\times\sum_{l=1}^{k}\frac{4^l\,l!\left(\alpha^2+\alpha_1^2\right)^{l-1}z^{2l-1}}{(2l)!}\right]\right\}$$

$$\{\ \alpha^2+\alpha_1^2\neq 0\ \text{ for }\ m>0\ \}.$$

2. $$\int z^{2m}\operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z)\,dz=\frac{1}{2m+1}\left\{\operatorname{erf}(\alpha z)\operatorname{erfi}(\alpha z)\,z^{2m+1}+\frac{m!}{\sqrt{\pi}\,\alpha}\times\right.$$

$$\times\sum_{k=0}^{m}\frac{z^{2k}}{k!\left(\alpha^2\right)^{m-k}}\left[\frac{(-1)^{m-k}-1}{2k+1}\cdot\frac{2\alpha z}{\sqrt{\pi}}+(-1)^{m+1-k}\operatorname{erf}(\alpha z)\exp\left(\alpha^2 z^2\right)+\right.$$

$$\left.\left.+\operatorname{erfi}(\alpha z)\exp\left(-\alpha^2 z^2\right)\right]\right\}.$$

1.12. Integrals of the form $\int z^{2m+1}\operatorname{erf}^3(\alpha z)\,dz$

1.12.1. $$\int z\operatorname{erf}^3(\alpha z)\,dz=\left(\frac{z^2}{2}-\frac{1}{4\alpha^2}\right)\operatorname{erf}^3(\alpha z)+\frac{3z\operatorname{erf}^2(\alpha z)}{2\sqrt{\pi}\,\alpha}\exp\left(-\alpha^2 z^2\right)+$$

$$+\frac{3\operatorname{erf}(\alpha z)}{2\pi\alpha^2}\exp\left(-2\alpha^2 z^2\right)-\frac{\sqrt{3}\operatorname{erf}\left(\sqrt{3}\alpha z\right)}{2\pi\alpha^2}\ .$$

1.12.2. $\int z^{2m+1}\operatorname{erf}^3(\alpha z)\,dz=\operatorname{erf}^3(\alpha z)\left[\frac{z^{2m+2}}{2m+2}-\frac{(2m+1)!}{(m+1)!\left(4\alpha^2\right)^{m+1}}\right]+\frac{(2m+1)!}{(m+1)!\sqrt{\pi}}\times$

$$\times\left\{3\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2 z^2\right)\sum_{k=0}^{m}\frac{k!\,z^{2k+1}}{(2k+1)!(2\alpha)^{2m+1-2k}}+\frac{1}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(k!)^2}{(2k+1)!}\times\right.$$

$$\times\left[3\operatorname{erf}(\alpha z)\exp\left(-2\alpha^2 z^2\right)\sum_{l=0}^{k}\frac{z^{2l}}{2^{2m+1-k-l}\,l!\left(\alpha^2\right)^{m+1-l}}-\right.$$

$$\left.-\frac{\sqrt{3}\operatorname{erf}\left(\sqrt{3}\alpha z\right)}{\left(\alpha^2\right)^{m+1}}\sum_{l=0}^{k}\frac{(2l)!}{2^{2m+1-k+l}3^l(l!)^2}\right]+\frac{\exp\left(-3\alpha^2 z^2\right)}{2\pi\alpha}\times$$

$$\left.\times\sum_{k=1}^{m}\frac{(k!)^2}{(2k+1)!}\sum_{l=1}^{k}\frac{(2l)!}{(l!)^2}\sum_{r=1}^{l}\frac{r!\,z^{2r-1}}{2^{2m-k+l-2r}3^{l-r}(2r)!\left(\alpha^2\right)^{m+1-r}}\right\}.$$

1.13. Integrals of the forms $\int z^n \sin^m\left(\alpha^2 z^2+\beta z+\gamma\right)dz$, $\int z^n \sinh^m\left(\alpha^2 z^2+\beta z+\gamma\right)dz$, $\int z^n \cos^m\left(\alpha^2 z^2+\beta z+\gamma\right)dz$, $\int z^n \cosh^m\left(\alpha^2 z^2+\beta z+\gamma\right)dz$

1.13.1.

1. $\int\sin\left(a^2x^2+bx+\gamma\right)dx=\frac{\sqrt{2\pi}}{4a}\left\{\left[\sin\left(\gamma-\frac{b^2}{4a^2}\right)+\cos\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Re}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b}{2a}\right)\right]+\right.$

$$\left.+\left[\sin\left(\gamma-\frac{b^2}{4a^2}\right)-\cos\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b}{2a}\right)\right]\right\}.$$

2. $\int\sin\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(1+i)\exp\left[i\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\times\right.$

$$\left.\times\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]+(1-i)\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\operatorname{erf}\left[\frac{1-i}{\sqrt{2}}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]\right\}.$$

3. $\int\sinh\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{4\alpha}\left[\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\operatorname{erfi}\left(\alpha z+\frac{\beta}{2\alpha}\right)-\right.$

$$-\exp\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\operatorname{erf}\left(\alpha z+\frac{\beta}{2\alpha}\right)\Bigg].$$

1.13.2.

1. $$\int\cos\left(a^2x^2+bx+\gamma\right)dx=\frac{\sqrt{2\pi}}{4a}\left\{\left[\cos\left(\gamma-\frac{b^2}{4a^2}\right)-\sin\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$

$$\times\operatorname{Re}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b}{2a}\right)\right]+\left[\sin\left(\gamma-\frac{b^2}{4a^2}\right)+\cos\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b}{2a}\right)\right]\Bigg\}.$$

2. $$\int\cos\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(1-i)\exp\left[i\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]+\right.$$

$$+(1+i)\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\operatorname{erf}\left[\frac{1-i}{\sqrt{2}}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]\Bigg\}.$$

3. $$\int\cosh\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{4\alpha}\left[\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\operatorname{erfi}\left(\alpha z+\frac{\beta}{2\alpha}\right)+\right.$$

$$+\exp\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\operatorname{erf}\left(\alpha z+\frac{\beta}{2\alpha}\right)\Bigg].$$

1.13.3.

1. $$\int x^n\sin\left(a^2x^2+bx+\gamma\right)dx=\frac{n!}{2^n}\left[\sin\left(\gamma-\frac{b^2}{4a^2}\right)\operatorname{Re}V_1^{(13)}(n,1,a,b,x)-\right.$$

$$-\cos\left(\gamma-\frac{b^2}{4a^2}\right)\operatorname{Im}V_1^{(13)}(n,1,a,b,x)-\sin\left(a^2x^2+bx+\gamma\right)\operatorname{Re}V_2^{(13)}(n,1,a,b,x)+$$

$$+\cos\left(a^2x^2+bx+\gamma\right)\operatorname{Im}V_2^{(13)}(n,1,a,b,x)\Bigg].$$

2. $$\int z^n\sin\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{n!}{2^{n+1}}i\left\{\exp\left[i\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_1^{(13)}(n,1,\alpha,\beta,z)-\right.$$

$$-\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_1^{(13)}(n,-1,\alpha,\beta,z)+\exp\left[i\left(\alpha^2z^2+\beta z+\gamma\right)\right]\times$$

$$\times V_2^{(13)}(n,-1,\alpha,\beta,z)-\exp\left[-i(\alpha^2z^2+\beta z+\gamma)\right]V_2^{(13)}(n,1,\alpha,\beta,z)\Bigg\}.$$

$$3. \int z^n \sinh\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{n!}{2^{n+1}}\left[\exp\left(\gamma - \frac{\beta^2}{4\alpha^2}\right) V_3^{(13)}(n,1,\beta) + \right.$$

$$\left. + V_4^{(13)}(n,1,\beta) - V_4^{(13)}(n,-1,\beta) - \exp\left(\frac{\beta^2}{4\alpha^2} - \gamma\right) V_5^{(13)}(n,1,\beta)\right].$$

1.13.4.

$$1. \int x^n \cos\left(a^2 x^2 + bx + \gamma\right) dx = \frac{n!}{2^n}\left[\sin\left(\gamma - \frac{b^2}{4a^2}\right) \operatorname{Im} V_1^{(13)}(n,1,a,b,x) + \right.$$

$$+ \cos\left(\gamma - \frac{b^2}{4a^2}\right) \operatorname{Re} V_1^{(13)}(n,1,a,b,x) - \sin\left(a^2 x^2 + bx + \gamma\right) \operatorname{Im} V_2^{(13)}(n,1,a,b,x) -$$

$$\left. - \cos\left(a^2 x^2 + bx + \gamma\right) \operatorname{Re} V_2^{(13)}(n,1,a,b,x)\right].$$

$$2. \int z^n \cos\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{n!}{2^{n+1}}\left\{\exp\left[i\left(\frac{\beta^2}{4\alpha^2} - \gamma\right)\right] V_1^{(13)}(n,1,\alpha,\beta,z) + \right.$$

$$+ \exp\left[i\left(\gamma - \frac{\beta^2}{4\alpha^2}\right)\right] V_1^{(13)}(n,-1,\alpha,\beta,z) - \exp\left[i(\alpha^2 z^2 + \beta z + \gamma)\right] V_2^{(13)}(n,-1,\alpha,\beta,z) -$$

$$\left. - \exp\left[-i(\alpha^2 z^2 + \beta z + \gamma)\right] V_2^{(13)}(n,1,\alpha,\beta,z)\right\}.$$

$$3. \int z^n \cosh\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{n!}{2^{n+1}}\left[\exp\left(\gamma - \frac{\beta^2}{4\alpha^2}\right) V_3^{(13)}(n,1,\beta) + \right.$$

$$\left. + V_4^{(13)}(n,1,\beta) + V_4^{(13)}(n,-1,\beta) + \exp\left(\frac{\beta^2}{4\alpha^2} - \gamma\right) V_5^{(13)}(n,1,\beta)\right].$$

1.13.5.

$$1. \int x^n \sin^{2m}\left(a^2 x^2 + bx + \gamma\right) dx = \frac{(2m)!\, x^{n+1}}{4^m (n+1)(m!)^2} + \frac{n!(2m)!}{2^{n-1+2m}} \sum_{k=1}^{m} \frac{(-1)^k}{(m-k)!(m+k)!} \times$$

$$\times \left\{\sin\left[2k\left(\gamma - \frac{b^2}{4a^2}\right)\right] \operatorname{Im} V_1^{(13)}(n,2k,a,b,x) + \cos\left[2k\left(\gamma - \frac{b^2}{4a^2}\right)\right] \times \right.$$

$$\times \operatorname{Re} V_1^{(13)}(n,2k,a,b,x) - \sin\left[2k(a^2 x^2 + bx + \gamma)\right] \operatorname{Im} V_2^{(13)}(n,2k,a,b,x) -$$

$$-\cos\left[2k(a^2x^2+bx+\gamma)\right]\operatorname{Re}V_2^{(13)}(n,2k,a,b,x)\Bigg\}.$$

2. $$\int x^n \sin^{2m+1}\left(a^2x^2+bx+\gamma\right)dx = \frac{n!(2m+1)!}{2^{n+2m}}\sum_{k=0}^{m}\frac{(-1)^k}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\sin\left[(2k+1)\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Re}V_1^{(13)}(n,2k+1,a,b,x)-\cos\left[(2k+1)\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$

$$\times\operatorname{Im}V_1^{(13)}(n,2k+1,a,b,x)-\sin[(2k+1)(a^2x^2+bx+\gamma)]\operatorname{Re}V_2^{(13)}(n,2k+1,a,b,x)+$$

$$\left.+\cos[(2k+1)(a^2x^2+bx+\gamma)]\ \operatorname{Im}V_2^{(13)}(n,2k+1,a,b,x)\right\}.$$

3. $$\int z^n \sin^{2m}\left(\alpha^2z^2+\beta z+\gamma\right)dz = \frac{(2m)!z^{n+1}}{4^m(n+1)(m!)^2}+\frac{n!(2m)!}{2^{n+2m}}\sum_{k=1}^{m}\frac{(-1)^k}{(m-k)!(m+k)!}\times$$

$$\times\left\{\exp\left[2ki\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_1^{(13)}(n,2k,\alpha,\beta,z)+\exp\left[2ki\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\times V_1^{(13)}(n,-2k,\alpha,\beta,z)-\exp\left[2ki\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,-2k,\alpha,\beta,z)-$$

$$\left.-\exp\left[-2ki\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,2k,\alpha,\beta,z)\right\}.$$

4. $$\int z^n \sin^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz = \frac{n!(2m+1)!i}{2^{n+1+2m}}\sum_{k=0}^{m}\frac{(-1)^k}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\exp\left[i(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_1^{(13)}(n,2k+1,\alpha,\beta,z)-\exp\left[i(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\times V_1^{(13)}(n,-2k-1,\alpha,\beta,z)+\exp\left[i(2k+1)\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,-2k-1,\alpha,\beta,z)-$$

$$\left.-\exp\left[-i(2k+1)\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,2k+1,\alpha,\beta,z)\right\}.$$

5. $$\int z^n\sinh^{2m}\left(\alpha^2z^2+\beta z+\gamma\right)dz = \frac{(2m)!z^{n+1}}{(-4)^m(n+1)(m!)^2}+\frac{n!(2m)!}{2^{n+2m}}\times$$

$$\times\sum_{k=1}^{m}\frac{(-1)^{m-k}}{(m-k)!(m+k)!}\left\{\exp\left[2k\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_3^{(13)}(n,2k,\beta)+V_4^{(13)}(n,2k,\beta)+\right.$$

$$\left.+V_4^{(13)}(n,-2k,\beta)+\exp\left[2k\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_5^{(13)}(n,2k,\beta)\right\}.$$

6. $$\int z^n\sinh^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{n!(2m+1)!}{2^{n+1+2m}}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\exp\left[(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_3^{(13)}(n,2k+1,\beta)+V_4^{(13)}(n,2k+1,\beta)-\right.$$

$$\left.-V_4^{(13)}(n,-2k-1,\beta)-\exp\left[(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_5^{(13)}(n,2k+1,\beta)\right\}.$$

1.13.6.

1. $$\int x^n\cos^{2m}\left(a^2x^2+bx+\gamma\right)dx=\frac{(2m)!x^{n+1}}{4^m(n+1)(m!)^2}+\frac{n!(2m)!}{2^{n-1+2m}}\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!}\times$$

$$\times\left\{\sin\left[2k\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}V_1^{(13)}(n,2k,a,b,x)+\cos\left[2k\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$

$$\times\operatorname{Re}V_1^{(13)}(n,2k,a,b,x)-\sin\left[2k\left(a^2x^2+bx+\gamma\right)\right]\operatorname{Im}V_2^{(13)}(n,2k,a,b,x)-$$

$$\left.-\cos\left[2k\left(a^2x^2+bx+\gamma\right)\right]\operatorname{Re}V_2^{(13)}(n,2k,a,b,x)\right\}.$$

2. $$\int x^n\cos^{2m+1}\left(a^2x^2+bx+\gamma\right)dx=\frac{n!(2m+1)!}{2^{n+2m}}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\sin\left[(2k+1)\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}V_1^{(13)}(n,2k+1,a,b,x)+\cos\left[(2k+1)\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$

$$\times\operatorname{Re}V_1^{(13)}(n,2k+1,a,b,x)-\sin\left[(2k+1)\left(a^2x^2+bx+\gamma\right)\right]\operatorname{Im}V_2^{(13)}(n,2k+1,a,b,x)-$$

$$\left.-\cos\left[(2k+1)\left(a^2x^2+bx+\gamma\right)\right]\operatorname{Re}V_2^{(13)}(n,2k+1,a,b,x)\right\}.$$

3. $$\int z^n\cos^{2m}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{(2m)!z^{n+1}}{4^m(n+1)(m!)^2}+\frac{n!(2m)!}{2^{n+2m}}\times$$

$$\times\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!}\left\{\exp\left[2ki\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_1^{(13)}(n,2k,\alpha,\beta,z)+\right.$$

$$+\exp\left[2ki\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_1^{(13)}(n,-2k,\alpha,\beta,z)-\exp\left[2ki\left(\alpha^2z^2+\beta z+\gamma\right)\right]\times$$

$$\left.\times V_2^{(13)}(n,-2k,\alpha,\beta,z)-\exp\left[-2ki\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,2k,\alpha,\beta,z)\right\}.$$

4. $$\int z^n\cos^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{n!(2m+1)!}{2^{n+1+2m}}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\exp\left[i(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_1^{(13)}(n,2k+1,\alpha,\beta,z)+\exp\left[i(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\times V_1^{(13)}(n,-2k-1,\alpha,\beta,z)-\exp\left[i(2k+1)\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,-2k-1,\alpha,\beta,z)-$$

$$\left.-\exp\left[-i(2k+1)\left(\alpha^2z^2+\beta z+\gamma\right)\right]V_2^{(13)}(n,2k+1,\alpha,\beta,z)\right\}.$$

5. $$\int z^n\cosh^{2m}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{(2m)!z^{n+1}}{4^m(n+1)(m!)^2}+\frac{n!(2m)!}{2^{n+2m}}\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!}\times$$

$$\times\left\{\exp\left[2k\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_3^{(13)}(n,2k,\beta)+V_4^{(13)}(n,2k,\beta)+V_4^{(13)}(n,-2k,\beta)+\right.$$

$$\left.+\exp\left[2k\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_5^{(13)}(n,2k,\beta)\right\}.$$

6. $$\int z^n\cosh^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{n!(2m+1)!}{2^{n+1+2m}}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!}\times$$

$$\times\left\{\exp\left[(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]V_3^{(13)}(n,2k+1,\beta)+V_4^{(13)}(n,2k+1,\beta)+\right.$$

$$\left.+V_4^{(13)}(n,-2k-1,\beta)+\exp\left[(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]V_5^{(13)}(n,2k+1,\beta)\right\}.$$

1.13.7.

1. $$\int z^n\sin^{2m}\left(\beta z+\gamma\right)dz=\frac{(2m)!z^{n+1}}{4^m(n+1)(m!)^2}+\frac{n!\,(2m)!}{2^{2m-1}}\sum_{k=1}^{m}\frac{(-1)^k}{(m-k)!(m+k)!}V_6^{(13)}(n,2k)$$

$\{\beta\neq0\}$.

2. $\int z^n \sin^{2m+1}(\beta z+\gamma)dz = \frac{n!(2m+1)!}{4^m}\sum_{k=0}^{m}\frac{(-1)^{k+1}}{(m-k)!(m+1+k)!}V_7^{(13)}(n,\,2k+1)$

$\{\,\beta \neq 0\,\}.$

3. $\int z^n \sinh^{2m}(\beta z+\gamma)dz = \frac{(2m)!\,z^{n+1}}{(-4)^m(n+1)(m!)^2} + \frac{n!(2m)!}{2^{2m+1}\beta}\sum_{k=1}^{m}\frac{(-1)^{m-k}}{(m-k)!(m+k)!k}\times$

$\times\left[V_8^{(13)}(n,2k)-V_8^{(13)}(n,-2k)\right].$

4. $\int z^n \sinh^{2m+1}(\beta z+\gamma)dz = \frac{n!(2m+1)!}{2^{2m+1}\beta}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{(m-k)!(m+1+k)!(2k+1)}\times$

$\times\left[V_8^{(13)}(n,2k+1)+V_8^{(13)}(n,-2k-1)\right].$

1.13.8.

1. $\int z^n \cos^{2m}(\beta z+\gamma)dz = \frac{(2m)!z^{n+1}}{4^m(n+1)(m!)^2} + \frac{n!(2m)!}{2^{2m-1}}\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!}\;V_6^{(13)}(n,2k)$

$\{\beta \neq 0\}.$

2. $\int z^n \cos^{2m+1}(\beta z+\gamma)dz = \frac{n!(2m+1)!}{4^m}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!}V_6^{(13)}(n,\,2k+1)$

$\{\beta \neq 0\}\;.$

3. $\int z^n \cosh^{2m}(\beta z+\gamma)dz = \frac{(2m)!\,z^{n+1}}{4^m(n+1)(m!)^2} + \frac{n!(2m)!}{2^{2m+1}\beta}\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!k}\times$

$\times\left[V_8^{(13)}(n,2k)-V_8^{(13)}(n,-2k)\right].$

4. $\int z^n \cosh^{2m+1}(\beta z+\gamma)dz = \frac{n!(2m+1)!}{2^{2m+1}\beta}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!(2k+1)}\times$

$\times\left[V_8^{(13)}(n,2k+1)-V_8^{(13)}(n,-2k-1)\right].$

Introduced notations:

1) $$V_1^{(13)}(n,\pm s,\alpha,\beta,z)=\frac{\sqrt{\pi}(1\mp i)}{2\sqrt{2s\alpha}}\operatorname{erf}\left[\sqrt{\frac{s}{2}}(1\pm i)\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]\times$$

$$\times\sum_{l=0}^{E(n/2)}\frac{(\pm\beta)^{n-2l}}{l!(n-2l)!(\mp\alpha^2)^{n-l}}\left(\frac{i}{s}\right)^l \quad (s>0),$$

$$V_1^{(13)}(2n_1,\pm s,\alpha,0,z)=\frac{\sqrt{\pi}(1\mp i)}{n_1!\sqrt{8s}\alpha^{2n_1+1}}\left(\mp\frac{i}{s}\right)^{n_1}\operatorname{erf}\left(\frac{1\pm i}{\sqrt{2}}\sqrt{s}\,\alpha z\right) \quad (s>0),$$

$$V_1^{(13)}(2n_1+1,s,\alpha,0,z)=V_1^{(13)}(2n_1+1,-s,\alpha,0,z)=0;$$

2) $$V_2^{(13)}(n,s,\alpha,\beta,z)=\sum_{l=1}^{E(n/2)}\frac{(-\beta)^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}\frac{r!\left(2\alpha^2 z+\beta\right)^{2r-1}}{(2r)!\alpha^{2n-2l+2r}}\left(-\frac{i}{s}\right)^{l+1-r}+$$

$$+\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!(-\beta)^{n+1-2l}}{(2l-1)!(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}\left(2\alpha^2 z+\beta\right)^{2r-2}}{(r-1)!\alpha^{2n-2l+2r}}\left(-\frac{i}{s}\right)^{l+1-r},$$

$$V_2^{(13)}(2n_1,s,\alpha,0,z)=\frac{1}{n_1!}\sum_{l=1}^{n_1}\frac{l!\,(2z)^{2l-1}}{(2l)!}\left(-\frac{i}{s\alpha^2}\right)^{n_1+1-l},$$

$$V_2^{(13)}(2n_1+1,s,\alpha,0,z)=\frac{4^{n_1}\,n_1!}{(2n_1+1)!}\sum_{l=0}^{n_1}\frac{z^{2l}}{l!}\left(-\frac{i}{s\alpha^2}\right)^{n_1+1-l};$$

3) $$V_3^{(13)}(n,s,\beta)=\frac{\sqrt{\pi}}{2\sqrt{s\alpha}}\operatorname{erfi}\left[\sqrt{s}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]\sum_{l=0}^{E(n/2)}\frac{(-1)^{n-l}\beta^{n-2l}}{l!(n-2l)!s^l\alpha^{2n-2l}},$$

$$V_3^{(13)}(2n_1,s,0)=\frac{\sqrt{\pi}}{n_1!(-s)^{n_1}\alpha^{2n_1+1}}\cdot\frac{\operatorname{erfi}(\sqrt{s}\alpha z)}{2\sqrt{s}}, \qquad V_3^{(13)}(2n_1+1,s,0)=0;$$

4) $$V_4^{(13)}(n,s,\beta)=\exp\left[s\left(\alpha^2 z^2+\beta z+\gamma\right)\right]\times$$

$$\times\left[\sum_{l=1}^{E(n/2)}\frac{\beta^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}\frac{r!(2\alpha^2 z+\beta)^{2r-1}}{(2r)!s^{l+1-r}(-\alpha^2)^{n-l+r}}-\right.$$

$$\left.-\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!\beta^{n+1-2l}}{(2l-1)!(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}(2\alpha^2 z+\beta)^{2r-2}}{(r-1)!s^{l+1-r}(-\alpha^2)^{n-l+r}}\right],$$

$$V_4^{(13)}(2n_1,s,0)=\frac{1}{n_1!}\exp\left[s(\alpha^2 z^2+\gamma)\right]\sum_{l=1}^{n_1}\frac{(-1)^{n_1-l}\,l!(2z)^{2l-1}}{(2l)!s^{n_1+1-l}\alpha^{2n_1+2-2l}},$$

$$V_4^{(13)}(2n_1+1,s,0)=-\frac{4^{n_1}n_1!}{(2n_1+1)!}\exp\left[s\left(\alpha^2z^2+\gamma\right)\right]\sum_{l=0}^{n_1}\frac{z^{2l}}{l!(-s\alpha^2)^{n_1+1-l}};$$

5) $$V_5^{(13)}(n,s,\beta)=\frac{\sqrt{\pi}}{2\sqrt{s}\alpha}\operatorname{erf}\left[\sqrt{s}\left(\alpha z+\frac{\beta}{2\alpha}\right)\right]\sum_{l=0}^{E(n/2)}\frac{(-\beta)^{n-2l}}{l!(n-2l)!s^l\alpha^{2n-2l}},$$

$$V_5^{(13)}(2n_1,s,0)=\frac{\sqrt{\pi}}{n_1!s^{n_1}\alpha^{2n_1+1}}\cdot\frac{\operatorname{erf}(\sqrt{s}\,\alpha z)}{2\sqrt{s}},\qquad V_5^{(13)}(2n_1+1,s,0)=0;$$

6) $$V_6^{(13)}(n,s)=\sin\left[s(\beta z+\gamma)\right]\sum_{l=0}^{E(n/2)}\frac{(-1)^l z^{n-2l}}{(n-2l)!(s\beta)^{2l+1}}-\cos\left[s(\beta z+\gamma)\right]\times$$

$$\times\sum_{l=1}^{n-E(n/2)}\frac{(-1)^l z^{n+1-2l}}{(n+1-2l)!(s\beta)^{2l}},\quad V_6^{(13)}(0,s)=\frac{1}{s\beta}\sin\left[s(\beta z+\gamma)\right];$$

7) $$V_7^{(13)}(n,s)=\cos\left[s(\beta z+\gamma)\right]\sum_{l=0}^{E(n/2)}\frac{(-1)^l z^{n-2l}}{(n-2l)!(s\beta)^{2l+1}}+\sin\left[s(\beta z+\gamma)\right]\times$$

$$\times\sum_{l=1}^{n-E(n/2)}\frac{(-1)^l z^{n+1-2l}}{(n+1-2l)!(s\beta)^{2l}},\quad V_7^{(13)}(0,s)=\frac{1}{s\beta}\cos\left[s(\beta z+\gamma)\right];$$

8) $$V_8^{(13)}(n,s)=\exp\left[s(\beta z+\gamma)\right]\sum_{l=0}^{n}\frac{z^l}{l!(-s\beta)^{n-l}}.$$

1.14. Integrals of the form $\int z^n \sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp(\beta_1 z)\,dz$

1.14.1.

1. $$\int\sin\left(a^2x^2+bx+\gamma\right)\exp(b_1x)\,dx=\frac{\sqrt{2\pi}}{4a}\exp\left(-\frac{bb_1}{2a^2}\right)\times$$

$$\times\left\{\left[\sin\left(\gamma-\frac{b^2-b_1^2}{4a^2}\right)+\cos\left(\gamma-\frac{b^2-b_1^2}{4a^2}\right)\right]\operatorname{Re}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b+ib_1}{2a}\right)\right]+\right.$$

$$\left.+\left[\sin\left(\gamma-\frac{b^2-b_1^2}{4a^2}\right)-\cos\left(\gamma-\frac{b^2-b_1^2}{4a^2}\right)\right]\operatorname{Im}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b+ib_1}{2a}\right)\right]\right\}.$$

2. $$\int\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp(\beta_1 z)\,dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(i+1)\exp\left[\frac{(\beta_1-i\beta)^2}{4i\alpha^2}-i\gamma\right]\times\right.$$

$$\times \operatorname{erf}\left[\frac{i+1}{\sqrt{2}}\left(\alpha z+\frac{\beta+i\beta_1}{2\alpha}\right)\right]+(1-i)\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\operatorname{erf}\left[\frac{1-i}{\sqrt{2}}\left(\alpha z+\frac{\beta-i\beta_1}{2\alpha}\right)\right]\Bigg\}.$$

1.14.2.

$$1. \int x\sin\left(a^2x^2+bx+\gamma\right)\exp\,(b_1x)\,dx=\frac{\sqrt{2\pi}}{8a^3}\exp\left(-\frac{bb_1}{2a^2}\right)\times$$

$$\times\left\{\left[(b_1+b)\sin\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)+(b_1-b)\cos\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\right]\operatorname{Re}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b+ib_1}{2a}\right)\right]+\right.$$

$$\left.+\left[(b-b_1)\sin\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)+(b+b_1)\cos\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\right]\operatorname{Im}\operatorname{erf}\left[\frac{1+i}{\sqrt{2}}\left(ax+\frac{b+ib_1}{2a}\right)\right]\right\}-$$

$$-\frac{1}{2a^2}\cos\left(a^2x^2+bx+\gamma\right)\exp(b_1x).$$

$$2. \int z\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp\,(\beta_1 z)\;dz=\frac{\sqrt{2\pi}}{16\alpha^3}\left\{(1-i)(\beta_1-i\beta)\exp\left[\frac{(\beta_1-i\beta)^2}{4i\alpha^2}-i\gamma\right]\times\right.$$

$$\times\operatorname{erf}\left[\frac{i+1}{\sqrt{2}}\left(\alpha z+\frac{\beta+i\beta_1}{2\alpha}\right)\right]+(1+i)\;(\beta_1+i\beta)\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\times$$

$$\left.\times\operatorname{erf}\left[\frac{1-i}{\sqrt{2}}\left(\alpha z+\frac{\beta-i\beta_1}{2\alpha}\right)\right]\right\}-\frac{1}{2\alpha^2}\cos\left(\alpha^2z^2+\beta z+\gamma\right)\exp(\beta_1 z).$$

1.14.3.

$$1. \int x^n\sin\left(a^2x^2+bx+\gamma\right)\exp(b_1x)dx=\frac{n!}{2^n}\left\{\exp\,(b_1x)\left[\cos\left(a^2x^2+bx+\gamma\right)\times\right.\right.$$

$$\left.\times\operatorname{Im}V_1^{(14)}(n,1,a,b,b_1,x)-\sin\left(a^2x^2+bx+\gamma\right)\operatorname{Re}V_1^{(14)}(n,1,a,b,b_1,x)\right]-\exp\left(-\frac{bb_1}{2a^2}\right)\times$$

$$\times\left[\sin\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\operatorname{Re}\,V_2^{(14)}(n,1,a,b,b_1,x)+\right.$$

$$\left.\left.+\cos\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\operatorname{Im}V_2^{(14)}\,(n,1,a,b,b_1,x)\right]\right\}.$$

2. $$\int z^n \sin\left(\alpha^2 z^2+\beta z+\gamma\right)\exp(\beta_1 z)\,dz=\frac{n!i}{2^{n+1}}\left\{\exp\left[\beta_1 z+i\left(\alpha^2 z^2+\beta z+\gamma\right)\right]\times\right.$$

$$\times V_1^{(14)}(n,-1,\alpha,\beta,\beta_1,z)-\exp\left[\beta_1 z-i\left(\alpha^2 z^2+\beta z+\gamma\right)\right]V_1^{(14)}(n,1,\alpha,\beta,\beta_1,z)+$$

$$+\exp\left[\frac{(\beta_1-i\beta)^2}{4i\alpha^2}-i\gamma\right]V_2^{(14)}(n,1,\alpha,\beta,\beta_1,z)-\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\times$$

$$\left.\times V_2^{(14)}(n,-1,\alpha,\beta,\beta_1,z)\right\}.$$

1.14.4.

1. $$\int z^n \sin(\beta z+\gamma)\exp(\beta_1 z)\,dz=n!\exp(\beta_1 z)\sum_{l=1}^{n+1}\frac{l!z^{n+1-l}}{(n+1-l)!(\beta^2+\beta_1^2)^l}\times$$

$$\times\left[\sin(\beta z+\gamma)\sum_{r=0}^{E(l/2)}\frac{(-1)^{l+1-r}\beta^{2r}\beta_1^{l-2r}}{(2r)!(l-2r)!}+\cos(\beta z+\gamma)\sum_{r=1}^{l-E(l/2)}\frac{(-1)^{l+1-r}\beta^{2r-1}\beta_1^{l+1-2r}}{(2r-1)!(l+1-2r)!}\right]=$$

$$=\frac{n!}{2i}\sum_{k=0}^{n}\frac{(-1)^{n-k}z^k}{k!}\left[\frac{\exp(\beta_1 z+i\beta z+i\gamma)}{(\beta_1+i\beta)^{n+1-k}}-\frac{\exp(\beta_1 z-i\beta z-i\gamma)}{(\beta_1-i\beta)^{n+1-k}}\right].$$

2. $$\int z^n \sin(\beta z+\gamma)\exp(\pm i\beta z)dz=\pm\frac{iz^{n+1}}{2n+2}\exp(\mp i\gamma)-$$

$$-\frac{n!}{2}\exp\left[\pm i(2\beta z+\gamma)\right]\sum_{l=0}^{n}\frac{(\pm i)^{n-l}z^l}{l!(2\beta)^{n+1-l}}.$$

Introduced notations:

1) $$V_1^{(14)}(n,s,\alpha,\beta,\beta_1,z)=\sum_{l=1}^{E(n/2)}\frac{(\beta_1-is\beta)^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}\frac{r!\left[is(2\alpha^2 z+\beta)-\beta_1\right]^{2r-1}}{(2r)!(is\alpha^2)^{n-l+r}}+$$

$$+\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!(\beta_1-is\beta)^{n+1-2l}}{(2l-1)!\,(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}\left[is(2\alpha^2 z+\beta)-\beta_1\right]^{2r-2}}{(r-1)!(is\alpha^2)^{n-l+r}},$$

$$V_1^{(14)}(2n_1,s,\alpha,\beta,is\beta,z)=\frac{1}{n_1!}\sum_{l=1}^{n_1}\frac{l!(2z)^{2l-1}}{(2l)!\,(is\alpha^2)^{n_1+1-l}},$$

$$V_1^{(14)}(2n_1+1,s,\alpha,\beta,is\beta,z)=\frac{n_1!4^{n_1}}{(2n_1+1)!}\sum_{l=0}^{n_1}\frac{z^{2l}}{l!(is\alpha^2)^{n_1+1-l}};$$

2) $$V_2^{(14)}(n,s,\alpha,\beta,\beta_1,z)=\frac{\sqrt{2\pi}(1-is)}{4\alpha}\operatorname{erf}\left[\frac{1+is}{\sqrt{2}}\left(\alpha z+\frac{\beta+is\beta_1}{2\alpha}\right)\right]\times$$

$$\times\sum_{l=0}^{E(n/2)}\frac{(\beta_1-is\beta)^{n-2l}}{l!(n-2l)!(is\alpha^2)^{n-l}},$$

$$V_2^{(14)}(2n_1,s,\alpha,\beta,is\beta,z)=\frac{\sqrt{2\pi}(1-is)}{n_1!4\alpha(is\alpha^2)^{n_1}}\operatorname{erf}\left(\frac{1+is}{\sqrt{2}}\alpha z\right),$$

$$V_2^{(14)}(2n_1+1,s,\alpha,\beta,is\beta,z)=0.$$

1.15. Integrals of the form $\int z^n\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\beta_1 z+\gamma\right)dz$

1.15.1.

1. $$\int\exp\left(-a^2x^2+bx\right)\sin\left(b_1x+\gamma\right)dx=\frac{\sqrt{\pi}}{2a}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\left[\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\times\right.$$

$$\left.\times\operatorname{Re}\operatorname{erf}\left(ax-\frac{b+ib_1}{2a}\right)+\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\operatorname{Im}\operatorname{erf}\left(ax-\frac{b+ib_1}{2a}\right)\right].$$

2. $$\int\exp\left(a^2x^2+bx\right)\sin\left(b_1x+\gamma\right)dx=\frac{\sqrt{\pi}}{2a}\exp\left(\frac{b_1^2-b^2}{4a^2}\right)\left[\cos\left(\frac{bb_1}{2a^2}-\gamma\right)\times\right.$$

$$\left.\times\operatorname{Im}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)-\sin\left(\frac{bb_1}{2a^2}-\gamma\right)\operatorname{Re}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)\right].$$

3. $$\int\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\beta_1z+\gamma\right)dz=\frac{\sqrt{\pi}\,i}{4\alpha}\left\{\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\times\right.$$

$$\left.\times\operatorname{erf}\left(\alpha z-\frac{\beta-i\beta_1}{2\alpha}\right)-\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]\operatorname{erf}\left(\alpha z-\frac{\beta+i\beta_1}{2\alpha}\right)\right\}.$$

1.15.2.

1. $$\int x\exp\left(-a^2x^2+bx\right)\sin\left(b_1x+\gamma\right)dx=\frac{\sqrt{\pi}}{4a^3}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\times$$

$$\times\left\{\left[b\sin\left(\frac{bb_1}{2a^2}+\gamma\right)+b_1\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\right]\mathrm{Re\,erf}\left(ax-\frac{b+ib_1}{2a}\right)+\right.$$

$$\left.+\left[b\cos\left(\frac{bb_1}{2a^2}+\gamma\right)-b_1\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\right]\mathrm{Im\,erf}\left(ax-\frac{b+ib_1}{2a}\right)\right\}-$$

$$-\frac{\sin\,(b_1x+\gamma)}{2a^2}\exp\left(-a^2x^2+bx\right).$$

2. $$\int x\exp\left(a^2x^2+bx\right)\sin\left(b_1x+\gamma\right)\,dx=\frac{\sqrt{\pi}}{4a^3}\exp\left(\frac{b_1^2-b^2}{4a^2}\right)\times$$

$$\times\left\{\left[b\sin\left(\frac{bb_1}{2a^2}-\gamma\right)-b_1\cos\left(\frac{bb_1}{2a^2}-\gamma\right)\right]\mathrm{Re\,erfi}\left(ax+\frac{b+ib_1}{2a}\right)-\right.$$

$$\left.-\left[b_1\sin\left(\frac{bb_1}{2a^2}-\gamma\right)+b\cos\left(\frac{bb_1}{2a^2}-\gamma\right)\right]\mathrm{Im\,erfi}\left(ax+\frac{b+ib_1}{2a}\right)\right\}+$$

$$+\frac{\sin\,(b_1x+\gamma)}{2a^2}\exp\left(a^2x^2+bx\right).$$

3. $$\int z\exp\left(-\alpha^2z^2+\beta z\right)\sin\,(\beta_1z+\gamma)dz=\frac{\sqrt{\pi}\,i}{8\alpha^3}\left\{(\beta-i\beta_1)\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\times\right.$$

$$\left.\times\mathrm{erf}\left(\alpha z-\frac{\beta-i\,\beta_1}{2\alpha}\right)-(\beta+i\beta_1)\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]\mathrm{erf}\left(\alpha z-\frac{\beta+i\beta_1}{2\alpha}\right)\right\}-$$

$$-\frac{\sin(\beta_1z+\gamma)}{2\alpha^2}\exp\left(-\alpha^2z^2+\beta z\right).$$

1.15.3.

1. $$\int x^n\exp\left(-a^2x^2+bx\right)\sin\,(b_1x+\gamma)dx=\frac{n!\sqrt{\pi}}{2^{n+1}a}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\left\{\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\times\right.$$

$$\times\left[\mathrm{Re\,erf}\left(ax-\frac{b+ib_1}{2a}\right)\mathrm{Re}\,V_1^{(15)}(n,a^2,b,b_1)-\mathrm{Im\,erf}\left(ax-\frac{b+ib_1}{2a}\right)\mathrm{Im}\,V_1^{(15)}(n,a^2,b,b_1)\right]+$$

$$+\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\left[\mathrm{Re\,erf}\left(ax-\frac{b+ib_1}{2a}\right)\mathrm{Im}V_1^{(15)}(n,a^2,b,b_1)+\mathrm{Im\,erf}\left(ax-\frac{b+ib_1}{2a}\right)\times\right.$$

$$\left.\left.\times\mathrm{Re}V_1^{(15)}(n,a^2,b,b_1)\right]\right\}-\frac{n!}{2^n}\exp\left(-a^2x^2+bx\right)\times$$

$$\times\left[\sin\,(b_1x+\gamma)\mathrm{Re}V_2^{(15)}\,(n,a^2,b,b_1,x)+\cos(b_1x+\gamma)\,\mathrm{Im}V_2^{(15)}\,(n,a^2,b,b_1,x)\right].$$

2. $$\int x^n \exp\left(a^2x^2+bx\right)\sin\left(b_1x+\gamma\right)dx=\frac{n!\sqrt{\pi}}{2^{n+1}a}\exp\left(\frac{b_1^2-b^2}{4a^2}\right)\left\{\sin\left(\frac{bb_1}{2a^2}-\gamma\right)\times\right.$$

$$\times\left[\operatorname{Im}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)\operatorname{Im}V_1^{(15)}\left(n,-a^2,b,b_1\right)-\operatorname{Re}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)\times\right.$$

$$\left.\times\operatorname{Re}V_1^{(15)}\left(n,-a^2,b,b_1\right)\right]+\cos\left(\frac{bb_1}{2a^2}-\gamma\right)\left[\operatorname{Re}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)\operatorname{Im}V_1^{(15)}\left(n,-a^2,b,b_1\right)+\right.$$

$$\left.\left.+\operatorname{Im}\operatorname{erfi}\left(ax+\frac{b+ib_1}{2a}\right)\operatorname{Re}V_1^{(15)}\left(n,-a^2,b,b_1\right)\right]\right\}-\frac{n!}{2^n}\exp\left(a^2x^2+bx\right)\times$$

$$\times\left[\sin\left(b_1x+\gamma\right)\operatorname{Re}V_2^{(15)}(n,-a^2,b,b_1,x)+\cos(b_1x+\gamma)\operatorname{Im}V_2^{(15)}(n,-a^2,b,b_1,x)\right].$$

3. $$\int z^n \exp\left(-\alpha^2z^2+\beta z\right)\sin(\beta_1 z+\gamma)dz=\frac{\sqrt{\pi}\,n!}{2^{n+2}i\alpha}\left\{\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]\operatorname{erf}\left(\alpha z-\frac{\beta+i\beta_1}{2\alpha}\right)\times\right.$$

$$\left.\times V_1^{(15)}(n,\alpha^2,\beta,\beta_1)-\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\operatorname{erf}\left(\alpha z-\frac{\beta-i\beta_1}{2\alpha}\right)V_1^{(15)}(n,\alpha^2,\beta,-\beta_1)\right\}+$$

$$+\frac{n!}{2^{n+1}i}\exp\left(-\alpha^2z^2+\beta z\right)\left[\exp(-i\beta_1z-i\gamma)\,V_2^{(15)}(n,\alpha^2,\beta,-\beta_1,z)-\right.$$

$$\left.-\exp(i\beta_1z+i\gamma)\,V_2^{(15)}(n,\alpha^2,\beta,\beta_1,z)\right].$$

Introduced notations:

1) $$V_1^{(15)}(n,\alpha^2,\beta,\beta_1)=\sum_{l=0}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2l}}{l!(n-2l)!\left(\alpha^2\right)^{n-l}},$$

$$V_1^{(15)}(2n_1,\alpha^2,\beta,i\beta)=\frac{1}{n_1!\left(\alpha^2\right)^{n_1}},\quad V_1^{(15)}(2n_1+1,\alpha^2,\beta,i\beta)=0;$$

2) $$V_2^{(15)}(n,\alpha^2,\beta,\beta_1,z)=\sum_{l=1}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2l}}{l!\,(n-2l)!}\sum_{r=1}^{l}\frac{r!\,(2\alpha^2z-\beta-i\beta_1)^{2r-1}}{(2r)!\,(\alpha^2)^{n-l+r}}+$$

$$+\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!\,(\beta+i\beta_1)^{n+1-2l}}{(2l-1)!\,(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}(2\alpha^2z-\beta-i\beta_1)^{2r-2}}{(r-1)!(\alpha^2)^{n-l+r}},$$

$$V_2^{(15)}(2n_1,\alpha^2,\beta,i\beta,z)=\frac{1}{n_1!}\sum_{l=1}^{n_1}\frac{l!(2z)^{2l-1}}{(2l)!\,(\alpha^2)^{n_1+1-l}},$$

$$V_2^{(15)}(2n_1+1,\alpha^2,\beta,i\beta,z)=\frac{n_1!4^{n_1}}{(2n_1+1)!}\sum_{l=0}^{n_1}\frac{z^{2l}}{l!(\alpha^2)^{n_1+1-l}}.$$

1.16. Integrals of the form $\int z^n \exp\left(-\alpha z^2+\beta z\right)\sin\left(\alpha_1 z^2+\beta_1 z+\gamma\right) dz$

1.16.1.

1. $$\int \exp\left(-ax^2+bx\right)\sin\left(a_1x^2+b_1x+\gamma\right)dx=\frac{\sqrt{\pi}}{2}\exp\left(\frac{ab^2-ab_1^2-2a_1bb_1}{4a^2+4a_1^2}\right)\times$$
$$\times\left\{\sin\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\mathrm{Re}\left[\frac{1}{\sqrt{a+ia_1}}\operatorname{erf}\left(\sqrt{a+ia_1}\;x-\frac{b-ib_1}{2\sqrt{a+ia_1}}\right)\right]-\right.$$
$$\left.-\cos\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\mathrm{Im}\left[\frac{1}{\sqrt{a+ia_1}}\operatorname{erf}\left(\sqrt{a+ia_1}\;x-\frac{b-ib_1}{2\sqrt{a+ia_1}}\right)\right]\right\}$$
$$\{|a|+|a_1|>0\}.$$

2. $$\int \exp\left(-\alpha z^2+\beta z\right)\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)dz=\frac{\sqrt{\pi}\,i}{4}\times$$
$$\times\left\{\frac{1}{\sqrt{\alpha+i\alpha_1}}\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha+4i\alpha_1}-i\gamma\right]\operatorname{erf}\left(\sqrt{\alpha+i\alpha_1}\;z-\frac{\beta-i\beta_1}{2\sqrt{\alpha+i\alpha_1}}\right)-\frac{1}{\sqrt{\alpha-i\alpha_1}}\times\right.$$
$$\left.\times\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha-4i\alpha_1}+i\gamma\right]\operatorname{erf}\left(\sqrt{\alpha-i\alpha_1}\;z-\frac{\beta+i\beta_1}{2\sqrt{\alpha-i\alpha_1}}\right)\right\}\quad\{\alpha_1^2\neq-\alpha^2\}.$$

3. $$\int \exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2+\beta_1z+\gamma\right)dz=\pm\frac{\sqrt{\pi}\,i}{4\sqrt{2\alpha}}\exp\left[\frac{(\beta\mp i\beta_1)^2}{8\alpha}\mp i\gamma\right]\times$$
$$\times\operatorname{erf}\left(\sqrt{2\alpha}z-\frac{\beta\mp i\beta_1}{2\sqrt{2\alpha}}\right)-\frac{1}{2\beta_1\mp2i\beta}\exp[\beta z\pm i(\beta_1z+\gamma)]\quad\{\beta_1\neq\pm i\beta\}.$$

4. $$\int \exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2\pm i\beta z+\gamma\right)dz=\pm\frac{\sqrt{\pi}\,i}{4\sqrt{2\alpha}}\exp\left(\frac{\beta^2}{2\alpha}\mp i\gamma\right)\times$$
$$\times\operatorname{erf}\left(\sqrt{2\alpha}z-\frac{\beta}{\sqrt{2\alpha}}\right)\mp\frac{iz}{2}\exp(\pm i\gamma).$$

1.16.2.

1. $$\int x\exp\left(-ax^2+bx\right)\sin\left(a_1x^2+b_1x+\gamma\right)dx=\frac{\sqrt{\pi}}{4}\exp\left(\frac{ab^2-ab_1^2-2a_1bb_1}{4a^2+4a_1^2}\right)\times$$

$$\times\left\{\sin\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\mathrm{Re}\left[\frac{b-ib_1}{(a+ia_1)\sqrt{a+ia_1}}\mathrm{erf}\left(\sqrt{a+ia_1}\;x-\frac{b-ib_1}{2\sqrt{a+ia_1}}\right)\right]-\right.$$

$$\left.-\cos\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\mathrm{Im}\left[\frac{b-ib_1}{(a+ia_1)\sqrt{a+ia_1}}\mathrm{erf}\left(\sqrt{a+ia_1}\;x-\frac{b-ib_1}{2\sqrt{a+ia_1}}\right)\right]\right\}-$$

$$-\frac{\exp\left(-ax^2+bx\right)}{2a^2+2a_1^2}\left[a\sin\left(a_1x^2+b_1x+\gamma\right)+a_1\cos\left(a_1x^2+b_1x+\gamma\right)\right]$$

$$\{|a|+|a_1|>0\}.$$

2. $$\int z\exp\left(-\alpha z^2+\beta z\right)\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)dz=\frac{\sqrt{\pi}\,i}{8}\left\{\frac{\beta-i\beta_1}{(\alpha+i\alpha_1)\sqrt{\alpha+i\alpha_1}}\times\right.$$

$$\times\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha+4i\alpha_1}-i\gamma\right]\mathrm{erf}\left(\sqrt{\alpha+i\alpha_1}\;z-\frac{\beta-i\beta_1}{2\sqrt{\alpha+i\alpha_1}}\right)-\frac{\beta+i\beta_1}{(\alpha-i\alpha_1)\sqrt{\alpha-i\alpha_1}}\times$$

$$\left.\times\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha-4i\alpha_1}+i\gamma\right]\mathrm{erf}\left(\sqrt{\alpha-i\alpha_1}\;z-\frac{\beta+i\beta_1}{2\sqrt{\alpha-i\alpha_1}}\right)\right\}-\frac{\exp\left(-\alpha z^2+\beta z\right)}{2\alpha^2+2\alpha_1^2}\times$$

$$\times\left[\alpha\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)+\alpha_1\cos\left(\alpha_1z^2+\beta_1z+\gamma\right)\right]\quad\{\alpha_1^2\neq-\alpha^2\}.$$

3. $$\int z\exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2+\beta_1z+\gamma\right)dz=\frac{\sqrt{\pi}\,(\beta_1\pm i\beta)}{16\alpha\sqrt{2\alpha}}\times$$

$$\times\exp\left[\frac{(\beta\mp i\beta_1)^2}{8\alpha}\mp i\gamma\right]\mathrm{erf}\left(\sqrt{2\alpha}z-\frac{\beta\mp i\beta_1}{2\sqrt{2\alpha}}\right)+\frac{(\beta_1\mp i\beta)z\pm i}{2(\beta\pm i\beta_1)^2}\exp\left[\beta z\pm i(\beta_1z+\gamma)\right]\mp$$

$$\mp\frac{i}{8\alpha}\exp\left[-2\alpha z^2+\beta z\mp i(\beta_1z+\gamma)\right]\qquad\{\beta_1\neq\pm i\beta\}.$$

4. $$\int z\exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2\pm i\beta z+\gamma\right)dz=\pm\frac{\sqrt{\pi}\,i\beta}{8\alpha\sqrt{2\alpha}}\exp\left(\frac{\beta^2}{2\alpha}\mp i\gamma\right)\times$$

$$\times\mathrm{erf}\left(\sqrt{2\alpha}z-\frac{\beta}{\sqrt{2\alpha}}\right)\mp\frac{i}{8\alpha}\exp\left(-2\alpha z^2+2\beta z\mp i\gamma\right)\mp\frac{iz^2}{4}\exp\left(\pm i\gamma\right).$$

1.16.3.

1. $$\int x^n\exp\left(-ax^2+bx\right)\sin\left(a_1x^2+b_1x+\gamma\right)dx=\frac{n!}{2^n}\left\{\exp\left(\frac{ab^2-ab_1^2-2a_1bb_1}{4a^2+4a_1^2}\right)\times\right.$$

$$\times\left[\sin\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\mathrm{Re}\;V_1^{(16)}(n,a,a_1,b,b_1,x)+\right.$$

$$+\cos\left(\frac{a_1b^2-a_1b_1^2+2abb_1}{4a^2+4a_1^2}+\gamma\right)\operatorname{Im}V_1^{(16)}(n,a,a_1,b,b_1,x)\Bigg]-$$

$$-\exp\left(-ax^2+bx\right)\Bigg[\ \sin\left(a_1x^2+b_1x+\gamma\right)\operatorname{Re}\ V_2^{(16)}(n,a,a_1,b,b_1,x)+$$

$$+\cos\left(a_1x^2+b_1x+\gamma\right)\operatorname{Im}V_2^{(16)}(n,a,a_1,b,b_1,x)\Bigg]\Bigg\}\ \{|a|+|a_1|>0\}.$$

2. $$\int z^n\exp\left(-\alpha z^2+\beta z\right)\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)dz=\frac{n!i}{2^{n+1}}\left\{\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha+4i\alpha_1}-i\gamma\right]\times\right.$$

$$\times V_1^{(16)}(n,\alpha,-\alpha_1,\beta,-\beta_1,z)-\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha-4i\alpha_1}+i\gamma\right]V_1^{(16)}(n,\alpha,\alpha_1,\beta,\beta_1,z)+$$

$$+\exp\left[-(\alpha-i\alpha_1)z^2+(\beta+i\beta_1)z+i\gamma\right]V_2^{(16)}(n,\alpha,\alpha_1,\beta,\beta_1,z)-$$

$$\left.-\exp\left[-(\alpha+i\alpha_1)z^2+(\beta-i\beta_1)z-i\gamma\right]V_2^{(16)}(n,\alpha,-\alpha_1,\beta,-\beta_1,z)\right\}$$

$$\{\alpha_1^2\neq-\alpha^2\}.$$

3. $$\int z^n\exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2+\beta_1z+\gamma\right)dz=\pm\frac{n!i}{2^{n+1}}\left\{\exp\left[\frac{(\beta\mp i\beta_1)^2}{8\alpha}\mp i\gamma\right]\times\right.$$

$$\left.\times V_1^{(16)}(n,\alpha,i\alpha,\beta,\mp\beta_1,z)-\exp\left[-2\alpha z^2+(\beta\mp i\beta_1)z\mp i\gamma\right]V_2^{(16)}(n,\alpha,i\alpha,\beta,\mp\beta_1,z)\right\}\pm$$

$$\pm\frac{n!}{2i}\exp\left[\beta z\pm i(\beta_1z+\gamma)\right]\sum_{k=0}^{n}\frac{(-1)^{n-k}z^k}{k!(\beta\pm i\beta_1)^{n+1-k}}\qquad\{\beta_1\neq\pm i\beta\}.$$

4. $$\int z^n\exp\left(-\alpha z^2+\beta z\right)\sin\left(\mp i\alpha z^2\pm i\beta z+\gamma\right)dz=\pm\frac{n!i}{2^{n+1}}\left[\exp\left(\frac{\beta^2}{2\alpha}\mp i\gamma\right)\times\right.$$

$$\left.\times V_1^{(16)}(n,\alpha,i\alpha,\beta,-i\beta,z)-\exp\left(-2\alpha z^2+2\beta z\mp i\gamma\right)V_2^{(16)}(n,\alpha,i\alpha,\beta,-i\beta,z)\right]\mp$$

$$\mp\frac{iz^{n+1}}{2n+2}\exp\left(\pm i\gamma\right).$$

Introduced notations:

1) $$V_1^{(16)}(n,\alpha,\alpha_1,\beta,\beta_1,z)=\frac{\sqrt{\pi}}{2\sqrt{\alpha-i\alpha_1}}\operatorname{erf}\left(\sqrt{\alpha-i\alpha_1}\ z-\frac{\beta+i\beta_1}{2\sqrt{\alpha-i\alpha_1}}\right)\times$$

$$\times\sum_{l=0}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2l}}{l!(n-2l)!(\alpha-i\alpha_1)^{n-l}},$$

$$V_1^{(16)}(2n_1,\alpha,\alpha_1,\beta,i\beta,z)=\frac{\sqrt{\pi}}{n_1!2(\alpha-i\alpha)^{n_1}\sqrt{\alpha^2-i\alpha_1}}\operatorname{erf}\left(\sqrt{\alpha-i\alpha_1}\ z\right),$$

$$V_1^{(16)}(2n_1+1,\alpha,\alpha_1,\beta,i\beta,z)=0;$$

2) $$V_2^{(16)}(n,\alpha,\alpha_1,\beta,\beta_1,z)=\sum_{l=1}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}\frac{r!(2\alpha z-2i\alpha_1 z-\beta-i\beta_1)^{2r-1}}{(2r)!(\alpha-i\alpha_1)^{n-l+r}}+$$

$$+\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!(\beta+i\beta_1)^{n+1-2l}}{(2l-1)!(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}(2\alpha z-2i\alpha_1 z-\beta-i\beta_1)^{2r-2}}{(r-1)!(\alpha-i\alpha_1)^{n-l+r}},$$

$$V_2^{(16)}(2n_1,\alpha,\alpha_1,\beta,i\beta,z)=\frac{1}{n_1!}\sum_{l=1}^{n_1}\frac{l!(2z)^{2l-1}}{(2l)!(\alpha-i\alpha_1)^{n_1+1-l}},$$

$$V_2^{(16)}(2n_1+1,\alpha,\alpha_1,\beta,i\beta,z)=\frac{n_1!\,4^{n_1}}{(2n_1+1)!}\sum_{l=0}^{n_1}\frac{z^{2l}}{l!(\alpha-i\alpha_1)^{n_1+1-l}}.$$

1.17. Integrals of the form $\int z^n \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z)\sin(\beta_2 z+\gamma)\,dz$

1.17.1.

1. $$\int \operatorname{erf}(ax+b)\exp(b_1x)\sin(b_2x+\gamma)\,dx=\frac{\operatorname{erf}(ax+b)}{b_1^2+b_2^2}\exp(b_1x)\times$$

$$\times[b_1\sin(b_2x+\gamma)-b_2\cos(b_2x+\gamma)]+\frac{1}{b_1^2+b_2^2}\exp\left(\frac{b_1^2-b_2^2-4abb_1}{4a^2}\right)\times$$

$$\times\left\{\cos\left(\frac{b_1b_2-2abb_2}{2a^2}+\gamma\right)\left[b_2\operatorname{Re}\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)-\right.\right.$$

$$\left.\left.-b_1\operatorname{Im}\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)\right]-\sin\left(\frac{b_1b_2-2abb_2}{2a^2}+\gamma\right)\times\right.$$

$$\times\left[b_1 \operatorname{Re}\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)+b_2 \operatorname{Im}\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)\right]\Bigg\}.$$

2. $$\int \operatorname{erfi}(ax+b)\exp(b_1x)\sin(b_2x+\gamma)\,dx=\frac{\operatorname{erfi}(ax+b)}{b_1^2+b_2^2}\exp(b_1x)[b_1\sin(b_2x+\gamma)-$$

$$-b_2\cos(b_2x+\gamma)]+\frac{1}{b_1^2+b_2^2}\exp\left(\frac{b_2^2-b_1^2-4abb_1}{4a^2}\right)\left\{\cos\left(\frac{b_1b_2+2abb_2}{2a^2}-\gamma\right)\times\right.$$

$$\times\left[b_2 \operatorname{Re}\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)-b_1 \operatorname{Im}\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)\right]+$$

$$+\sin\left(\frac{b_1b_2+2abb_2}{2a^2}-\gamma\right)\left[b_1 \operatorname{Re}\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)+\right.$$

$$\left.\left.+b_2 \operatorname{Im}\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)\right]\right\}.$$

3. $$\int \operatorname{erf}(ax+b)\sin(b_1x+\gamma)dx=\frac{1}{b_1}\exp\left(-\frac{b_1^2}{4a^2}\right)\left[\cos\left(\frac{bb_1}{a}-\gamma\right)\operatorname{Re}\operatorname{erf}\left(ax+\frac{2ab-ib_1}{2a}\right)+\right.$$

$$\left.+\sin\left(\frac{bb_1}{a}-\gamma\right)\operatorname{Im}\operatorname{erf}\left(ax+\frac{2ab-ib_1}{2a}\right)\right]-\frac{1}{b_1}\operatorname{erf}(ax+b)\cos(b_1x+\gamma).$$

4. $$\int \operatorname{erfi}(ax+b)\sin(b_1x+\gamma)\,dx=\frac{1}{b_1}\exp\left(\frac{b_1^2}{4a^2}\right)\left[\cos\left(\frac{bb_1}{a}-\gamma\right)\operatorname{Re}\operatorname{erfi}\left(ax+\frac{2ab+ib_1}{2a}\right)+\right.$$

$$\left.+\sin\left(\frac{bb_1}{a}-\gamma\right)\operatorname{Im}\operatorname{erfi}\left(ax+\frac{2ab+ib_1}{2a}\right)\right]-\frac{1}{b_1}\operatorname{erfi}(ax+b)\cos(b_1x+\gamma).$$

5. $$\int \operatorname{erf}(\alpha z+\beta)\exp(\beta_1z)\sin(\beta_2z+\gamma)\,dz=\frac{1}{2}\operatorname{erf}(\alpha z+\beta)\left\{\frac{1}{i\beta_1-\beta_2}\times\right.$$

$$\left.\times\exp[\beta_1z+i(\beta_2z+\gamma)]-\frac{1}{i\beta_1+\beta_2}\exp[\beta_1z-i(\beta_2z+\gamma)]\right\}+$$

$$+\frac{1}{2}\left\{\frac{1}{i\beta_1+\beta_2}\exp\left[\frac{(\beta_1-i\beta_2)(\beta_1-4\alpha\beta-i\beta_2)}{4\alpha^2}-i\gamma\right]\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1-i\beta_2}{2\alpha}\right)-\right.$$

$$-\frac{1}{i\beta_1-\beta_2}\exp\left[\frac{(\beta_1+i\beta_2)(\beta_1-4\alpha\beta+i\beta_2)}{4\alpha^2}+i\gamma\right]\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1+i\beta_2}{2\alpha}\right)\Bigg\}$$

$$\{\ \beta_1^2+\beta_2^2\neq 0\ \}.$$

6. $$\int \operatorname{erf}(\alpha z+\beta)\sin(\beta_1 z+\gamma)dz=\frac{1}{2\beta_1}\left[\exp\left(\frac{4i\alpha\beta\beta_1-\beta_1^2}{4\alpha^2}-i\gamma\right)\operatorname{erf}\left(\alpha z+\beta+\frac{i\beta_1}{2\alpha}\right)+\right.$$

$$\left.+\exp\left(i\gamma-\frac{\beta_1^2+4i\alpha\beta\beta_1}{4\alpha^2}\right)\operatorname{erf}\left(\alpha z+\beta-\frac{i\beta_1}{2\alpha}\right)\right]-\frac{1}{\beta_1}\operatorname{erf}(\alpha z+\beta)\cos(\beta_1 z+\gamma).$$

1.17.2. $$\int \operatorname{erf}(\alpha z+\beta)\exp(\pm i\beta_1 z)\sin(\beta_1 z+\gamma)\ dz=\pm\frac{i\exp(\mp i\gamma)}{2}\left\{\left(z+\frac{\beta}{\alpha}\right)\operatorname{erf}(\alpha z+\beta)+\right.$$

$$\left.+\frac{1}{\sqrt{\pi}\,\alpha}\exp\left[-(\alpha z+\beta)^2\right]\right\}-\frac{1}{4\beta_1}\Bigg\{\operatorname{erf}(\alpha z+\beta)\exp[\pm i(2\beta_1 z+\gamma)]-$$

$$-\exp\left(-\frac{\beta_1^2\pm 2i\alpha\beta\beta_1}{\alpha^2}\ \pm\ i\gamma\right)\operatorname{erf}\left(\alpha z+\beta\ \mp\ \frac{i\beta_1}{\alpha}\right)\Bigg\}.$$

1.17.3.

1. $$\int x^n\operatorname{erf}(ax+b)\exp(b_1x)\sin(b_2x+\gamma)dx=\exp\left(\frac{b_1^2-b_2^2-4abb_1}{4a^2}\right)\times$$

$$\times\left\{\sin\left(\frac{b_1b_2-2abb_2}{2a^2}+\gamma\right)\operatorname{Re}\left[\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)V_1^{(17)}(a,b,b_1,b_2)\right]+\right.$$

$$\left.+\cos\left(\frac{b_1b_2-2abb_2}{2a^2}+\gamma\right)\operatorname{Im}\left[\operatorname{erf}\left(ax+b-\frac{b_1+ib_2}{2a}\right)V_1^{(17)}(a,b,b_1,b_2)\right]\right\}-\exp(b_1x)\times$$

$$\times\left\{\sin(b_2x+\gamma)\left[\operatorname{erf}(ax+b)\operatorname{Re}V_2^{(17)}(b_1,b_2,x)+\frac{2a}{\sqrt{\pi}}\operatorname{Re}V_3^{(17)}(a,b,b_1,b_2,x)\right]+\right.$$

$$\left.+\cos(b_2x+\gamma)\left[\operatorname{erf}(ax+b)\operatorname{Im}V_2^{(17)}(b_1,b_2,x)+\frac{2a}{\sqrt{\pi}}\operatorname{Im}V_3^{(17)}(a,b,b_1,b_2,x)\right]\right\}$$

$$\{|b_1|+|b_2|>0\}.$$

2. $$\int x^n\operatorname{erfi}(ax+b)\exp(b_1x)\sin(b_2x+\gamma)\ dx=\exp\left(\frac{b_2^2-b_1^2-4abb_1}{4a^2}\right)\times$$

$$\times\left\{\cos\left(\frac{2abb_2+b_1b_2}{2a^2}-\gamma\right)\operatorname{Im}\left[\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)V_1^{(17)}(ia,ib,b_1,b_2)\right]-\right.$$

$$\left.-\sin\left(\frac{2abb_2+b_1b_2}{2a^2}-\gamma\right)\operatorname{Re}\left[\operatorname{erfi}\left(ax+b+\frac{b_1+ib_2}{2a}\right)V_1^{(17)}(ia,ib,b_1,b_2)\right]\right\}-\exp(b_1x)\times$$

$$\times\left\{\sin(b_2x+\gamma)\left[\operatorname{erfi}(ax+b)\operatorname{Re}V_2^{(17)}(b_1,b_2,x)+\frac{2a}{\sqrt{\pi}}\operatorname{Re}V_3^{(17)}(ia,ib,b_1,b_2,x)\right]+\right.$$

$$\left.+\cos(b_2x+\gamma)\left[\operatorname{erfi}(ax+b)\operatorname{Im}V_2^{(17)}(b_1,b_2,x)+\frac{2a}{\sqrt{\pi}}\operatorname{Im}V_3^{(17)}(ia,ib,b_1,b_2,x)\right]\right\}$$

$$\{|b_1|+|b_2|>0\}.$$

3. $$\int z^n\operatorname{erf}(\alpha z+\beta)\exp(\beta_1z)\sin(\beta_2z+\gamma)\,dz=\frac{1}{2i}\exp\left(\frac{\beta_1^2-\beta_2^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\left\{\exp\left[i\left(\frac{\beta_1\beta_2-2\alpha\beta\beta_2}{2\alpha^2}+\gamma\right)\right]\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1+i\beta_2}{2\alpha}\right)V_1^{(17)}(\alpha,\beta,\beta_1,\beta_2)-\right.$$

$$\left.-\exp\left[i\left(\frac{2\alpha\beta\beta_2-\beta_1\beta_2}{2\alpha^2}-\gamma\right)\right]\operatorname{erf}\left(\alpha z+\beta-\frac{\beta_1-i\beta_2}{2\alpha}\right)V_1^{(17)}(\alpha,\beta,\beta_1,-\beta_2)\right\}+$$

$$+\frac{\exp(\beta_1z)}{2i}\left\{\exp\left[-i(\beta_2z+\gamma)\right]\left[\operatorname{erf}(\alpha z+\beta)V_2^{(17)}(\beta_1,-\beta_2,z)+\frac{2\alpha}{\sqrt{\pi}}V_3^{(17)}(\alpha,\beta,\beta_1,-\beta_2,z)\right]-\right.$$

$$\left.-\exp\left[i(\beta_2z+\gamma)\right]\left[\operatorname{erf}(\alpha z+\beta)V_2^{(17)}(\beta_1,\beta_2,z)+\frac{2\alpha}{\sqrt{\pi}}V_3^{(17)}(\alpha,\beta,\beta_1,\beta_2,z)\right]\right\}$$

$$\{\beta_1^2+\beta_2^2\neq 0\}.$$

1.17.4. $$\int z^n\operatorname{erf}(\alpha z+\beta)\exp(\pm i\beta_1z)\sin(\beta_1z+\gamma)dz=\pm\frac{1}{2i}\left\{\exp\left(-\frac{\beta_1^2\pm 2i\alpha\beta\beta_1}{\alpha^2}\pm i\gamma\right)\times\right.$$

$$\times\operatorname{erf}\left(\alpha z+\beta\mp\frac{i\beta_1}{\alpha}\right)V_1^{(17)}(\alpha,\beta,\pm i\beta_1,\pm\beta_1)-\exp\left[\pm i(2\beta_1z+\gamma)\right]\times$$

$$\left.\times\left[\operatorname{erf}(\alpha z+\beta)V_2^{(17)}(\pm i\beta_1,\pm\beta_1,z)+\frac{2\alpha}{\sqrt{\pi}}V_3^{(17)}(\alpha,\beta,\pm i\beta_1,\pm\beta_1,z)\right]\right\}\pm$$

$$\pm\frac{i}{2}\exp(\mp i\gamma)\left\{\operatorname{erf}(\alpha z+\beta)\left[\frac{z^{n+1}}{n+1}+\frac{(-1)^n n!}{\alpha^{n+1}}\sum_{k=0}^{n-E(n/2)}\frac{\beta^{n+1-2k}}{4^k\,k!(n+1-2k)!}\right]+\right.$$

$$+\frac{(-1)^n n!}{\sqrt{\pi}\alpha^{n+1}}\exp\left[-(\alpha z+\beta)^2\right]\left[\sum_{k=0}^{E(n/2)}\frac{k!\beta^{n-2k}}{(2k+1)!(n-2k)!}\sum_{l=0}^{k}\frac{(\alpha z+\beta)^{2l}}{l!}-\right.$$

$$-\sum_{k=1}^{n-E(n/2)} \frac{\beta^{n+1-2k}}{k!(n+1-2k)!} \sum_{l=1}^{k} \frac{l!\,(\alpha z+\beta)^{2l-1}}{4^{k-l}\,(2l)!} \Bigg]\Bigg\} \qquad \{\beta_1 \neq 0\}.$$

Introduced notations:

1) $$V_1^{(17)}(\alpha,\beta,\beta_1,\beta_2) = \sum_{k=0}^{n} \frac{n!}{2^k} \left(-\frac{1}{\beta_1+i\beta_2}\right)^{n+1-k} \sum_{l=0}^{E(k/2)} \frac{(\beta_1-2\alpha\beta+i\beta_2)^{k-2l}}{l!(k-2l)!\alpha^{2k-2l}},$$

$$V_1^{(17)}(\alpha,\beta,\beta_1,i\beta_1-2i\alpha\beta) = \frac{n!}{(2\alpha)^{n+1}} \sum_{k=0}^{E(n/2)} \frac{1}{k!(-\beta)^{n+1-2k}};$$

2) $$V_2^{(17)}(\beta_1,\beta_2,z) = \sum_{k=0}^{n} \frac{n!z^k}{k!} \left(-\frac{1}{\beta_1+i\beta_2}\right)^{n+1-k};$$

3) $$V_3^{(17)}(\alpha,\beta,\beta_1,\beta_2,z) = \exp\left[-(\alpha z+\beta)^2\right] \sum_{k=1}^{n} \frac{n!}{2^k} \left(-\frac{1}{\beta_1+i\beta_2}\right)^{n+1-k} \times$$

$$\times\left[\sum_{l=1}^{E(k/2)} \frac{(\beta_1-2\alpha\beta+i\beta_2)^{k-2l}}{l!(k-2l)!} \sum_{r=1}^{l} \frac{r!(2\alpha^2 z+2\alpha\beta-\beta_1-i\beta_2)^{2r-1}}{(2r)!\alpha^{2k-2l+2r}} + \right.$$

$$\left. + \sum_{l=1}^{k-E(k/2)} \frac{(l-1)!(\beta_1-2\alpha\beta+i\beta_2)^{k+1-2l}}{(2l-1)!(k+1-2l)!} \sum_{r=1}^{l} \frac{4^{l-r}(2\alpha^2 z+2\alpha\beta-\beta_1-i\beta_2)^{2r-2}}{(r-1)!\alpha^{2k-2l+2r}}\right],$$

$$V_3^{(17)}(\alpha,\beta,\beta_1,i\beta_1-2i\alpha\beta,z) = \exp\left[-(\alpha z+\beta)^2\right] \times$$

$$\times\left[\sum_{k=1}^{E(n/2)} \frac{n!}{k!(-\beta)^{n+1-2k}} \sum_{l=0}^{k-1} \frac{l!z^{2l+1}}{(2l+1)!(2\alpha)^{n+1-2l}} + \right.$$

$$\left. + \sum_{k=1}^{n-E(n/2)} \frac{n!k!}{(2k)!(-2\beta)^{n+2-2k}} \sum_{l=0}^{k-1} \frac{z^{2l}}{l!\alpha^{n+2-2l}}\right].$$

1.18. Integrals of the form $\int z^{2n+1}\mathrm{erf}(\alpha z+\beta)\exp\left(\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz$

1.18.1.

1. $$\int x\ \mathrm{erf}(ax+b)\exp\left(a_1x^2\right)\sin\left(a_2x^2+\gamma\right)dx = \frac{\mathrm{erf}\ (ax+b)\exp(a_1x^2)}{2a_1^2+2a_2^2} \times$$

$$\times\left[a_1\sin\left(a_2x^2+\gamma\right)-a_2\cos\left(a_2x^2+\gamma\right)\right]-\frac{a}{2a_1^2+2a_2^2}\exp\left[\frac{b^2\left(a^2a_1-a_1^2-a_2^2\right)}{a^4+a_1^2+a_2^2-2a^2a_1}\right]\times$$

$$\times\left\{\sin\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}+\gamma\right)\times\right.$$

$$\times\mathrm{Re}\left[\frac{a_1-ia_2}{\sqrt{a^2-a_1-ia_2}}\operatorname{erf}\left(\sqrt{a^2-a_1-ia_2}\;x+\frac{ab}{\sqrt{a^2-a_1-ia_2}}\right)\right]+$$

$$+\cos\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}+\gamma\right)\times$$

$$\left.\times\mathrm{Im}\left[\frac{a_1-ia_2}{\sqrt{a^2-a_1-ia_2}}\operatorname{erf}\left(\sqrt{a^2-a_1-ia_2}\;x+\frac{ab}{\sqrt{a^2-a_1-ia_2}}\right)\right]\right\}$$

$$\{|a_1|+|a_2|>0,\quad \left|a^2-a_1\right|+|a_2|>0\}.$$

2. $$\int x\operatorname{erfi}(ax+b)\exp\left(a_1x^2\right)\sin\left(a_2x^2+\gamma\right)dx=\frac{\operatorname{erfi}(ax+b)\exp\left(a_1x^2\right)}{2a_1^2+2a_2^2}\times$$

$$\times\left[a_1\sin\left(a_2x^2+\gamma\right)-a_2\cos\left(a_2x^2+\gamma\right)\right]-\frac{a}{2a_1^2+2a_2^2}\exp\left[\frac{b^2\left(a^2a_1+a_1^2+a_2^2\right)}{a^4+a_1^2+a_2^2+2a^2a_1}\right]\times$$

$$\times\left\{\mathrm{Re}\left[\frac{a_1-i\,a_2}{\sqrt{a^2+a_1+i\,a_2}}\operatorname{erfi}\left(\sqrt{a^2+a_1+i\,a_2}\;x+\frac{a\,b}{\sqrt{a^2+a_1+i\,a_2}}\right)\right]\times\right.$$

$$\times\sin\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}+\gamma\right)+\cos\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}+\gamma\right)\times$$

$$\left.\times\mathrm{Im}\left[\frac{a_1-i\,a_2}{\sqrt{a^2+a_1+i\,a_2}}\operatorname{erfi}\left(\sqrt{a^2+a_1+i\,a_2}\;x+\frac{a\,b}{\sqrt{a^2+a_1+i\,a_2}}\right)\right]\right\}$$

$$\{|a_1|+|a_2|>0,\quad \left|a^2+a_1\right|+|a_2|>0\}.$$

3. $$\int x\operatorname{erf}(ax+b)\sin\left(a_1x^2+\gamma\right)dx=\frac{a}{2a_1}\exp\left(-\frac{a_1^2b^2}{a^4+a_1^2}\right)\left\{\cos\left(\frac{a^2a_1b^2}{a^4+a_1^2}+\gamma\right)\times\right.$$

$$\times\mathrm{Re}\left[\frac{1}{\sqrt{a^2-ia_1}}\operatorname{erf}\left(\sqrt{a^2-ia_1}\;x+\frac{ab}{\sqrt{a^2-ia_1}}\right)\right]-\sin\left(\frac{a^2a_1b^2}{a^4+a_1^2}+\gamma\right)\times$$

$$\times \operatorname{Im}\left[\frac{1}{\sqrt{a^2-ia_1}}\operatorname{erf}\left(\sqrt{a^2-ia_1}\,x+\frac{ab}{\sqrt{a^2-ia_1}}\right)\right]\Bigg\}-\frac{\operatorname{erf}\,(ax+b)}{2a_1}\cos\left(a_1x^2+\gamma\right)$$

$$\{|a_1|>0\}.$$

4. $$\int x\ \operatorname{erfi}\,(ax+b)\sin\left(a_1x^2+\gamma\right)dx=\frac{a}{2a_1}\exp\left(\frac{a_1^2b^2}{a^4+a_1^2}\right)\left\{\cos\left(\frac{a^2a_1b^2}{a^4+a_1^2}+\gamma\right)\times\right.$$

$$\times \operatorname{Re}\left[\frac{1}{\sqrt{a^2+ia_1}}\operatorname{erfi}\left(\sqrt{a^2+ia_1}\,x+\frac{ab}{\sqrt{a^2+ia_1}}\right)\right]-\sin\left(\frac{a^2a_1b^2}{a^4+a_1^2}+\gamma\right)\times$$

$$\left.\times \operatorname{Im}\left[\frac{1}{\sqrt{a^2+ia_1}}\operatorname{erfi}\left(\sqrt{a^2+ia_1}\,x+\frac{ab}{\sqrt{a^2+ia_1}}\right)\right]\right\}-\frac{\operatorname{erfi}\,(ax+b)}{2a_1}\cos\left(a_1x^2+\gamma\right)$$

$$\{|a_1|>0\}.$$

5. $$\int z\ \operatorname{erf}\,(\alpha z+\beta)\exp\left(\alpha_1z^2\right)\sin\,(\alpha_2z^2+\gamma)\ dz=\frac{\operatorname{erf}\,(\alpha z+\beta)\exp\left(\alpha_1z^2\right)}{2\alpha_1^2+2\alpha_2^2}\times$$

$$\times\left[\alpha_1\sin\left(\alpha_2z^2+\gamma\right)-\alpha_2\cos\left(\alpha_2z^2+\gamma\right)\right]+\frac{\alpha}{4(\alpha_2+i\alpha_1)\sqrt{\alpha^2-\alpha_1+i\alpha_2}}\times$$

$$\times\exp\left[\frac{\beta^2(\alpha_1-i\alpha_2)}{\alpha^2-\alpha_1+i\alpha_2}-i\gamma\right]\operatorname{erf}\left(\sqrt{\alpha^2-\alpha_1+i\alpha_2}\ z+\frac{\alpha\beta}{\sqrt{\alpha^2-\alpha_1+i\alpha_2}}\right)+$$

$$+\frac{\alpha}{4(\alpha_2-i\alpha_1)\sqrt{\alpha^2-\alpha_1-i\alpha_2}}\exp\left[\frac{\beta^2(\alpha_1+i\alpha_2)}{\alpha^2-\alpha_1-i\alpha_2}+i\gamma\right]\times$$

$$\times\operatorname{erf}\left(\sqrt{\alpha^2-\alpha_1-i\alpha_2}\ z+\frac{\alpha\beta}{\sqrt{\alpha^2-\alpha_1-i\alpha_2}}\right)$$

$$\{\alpha_2^2\neq-\alpha_1^2,\quad (\alpha^2-\alpha_1)^2\neq-\alpha_2^2\}.$$

6. $$\int z\operatorname{erf}\,(\alpha z+\beta)\sin\left(\alpha_1z^2+\gamma\right)dz=\frac{\alpha}{4\alpha_1}\left[\frac{1}{\sqrt{\alpha^2+i\alpha_1}}\exp\left(\frac{\alpha_1\beta^2}{i\,\alpha^2-\alpha_1}-i\,\gamma\right)\times\right.$$

$$\times\operatorname{erf}\left(\sqrt{\alpha^2+i\alpha_1}\ z+\frac{\alpha\beta}{\sqrt{\alpha^2+i\alpha_1}}\right)+\frac{1}{\sqrt{\alpha^2-i\alpha_1}}\exp\left(i\,\gamma-\frac{\alpha_1\beta^2}{\alpha_1+i\alpha^2}\right)\times$$

$$\left.\times\operatorname{erf}\left(\sqrt{\alpha^2-i\alpha_1}\ z+\frac{\alpha\beta}{\sqrt{\alpha^2-i\alpha_1}}\right)\right]-\frac{\operatorname{erf}\,(\alpha z+\beta)}{2\alpha_1}\cos\left(\alpha_1z^2+\gamma\right)\qquad\{\alpha_1^2\neq-\alpha^4\}.$$

1.18.2.

$$1.\ \int z\,\mathrm{erf}\,[(1\pm i)\alpha z+\beta]\sin\left(2\alpha^2z^2+\gamma\right)dz=\frac{1}{8\alpha^2}\left\{\frac{1}{\sqrt{2}}\exp\left(-\frac{\beta^2}{2}\mp i\gamma\right)\times\right.$$

$$\times\,\mathrm{erf}\left[(1\pm i)\sqrt{2}\ \alpha z+\frac{\beta}{\sqrt{2}}\right]-\frac{1}{\sqrt{\pi}\,\beta}\exp\left[-2(1\pm i)\alpha\beta z-\beta^2\pm i\gamma\right]-$$

$$-\exp\left[\pm i\left(2\alpha^2z^2+\gamma\right)\right]\mathrm{erf}\,[(1\pm i)\alpha z+\beta]-\exp\left[\mp i\left(2\alpha^2z^2+\gamma\right)\right]\times$$

$$\left.\times\,\mathrm{erf}\,[(1\pm i)\alpha z+\beta]\right\}.$$

$$2.\ \int z\,\mathrm{erf}\,[(1\pm i)\alpha z]\sin\left(2\alpha^2z^2+\gamma\right)dz=\frac{(1\pm i)\exp(\pm i\gamma)}{4\sqrt{\pi}\alpha}z+\frac{\exp(\mp i\gamma)}{8\sqrt{2}\alpha^2}\times$$

$$\times\,\mathrm{erf}\left[(1\pm i)\sqrt{2}\,\alpha z\right]-\frac{1}{8\alpha^2}\mathrm{erf}\,[(1\pm i)\alpha z]\times$$

$$\times\left\{\exp\left[\pm i\left(2\alpha^2z^2+\gamma\right)\right]+\exp\left[\mp i\left(2\alpha^2z^2+\gamma\right)\right]\right\}.$$

$$3.\ \int z\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(\pm i\alpha_1z^2\right)\sin\left(\alpha_1z^2+\gamma\right)dz=\frac{1}{8\alpha_1}\left\{\frac{\alpha}{\sqrt{\alpha^2\mp 2i\alpha_1}}\times\right.$$

$$\times\exp\left[\pm i\left(\frac{2\alpha_1\beta^2}{\alpha^2\mp 2i\alpha_1}+\gamma\right)\right]\mathrm{erf}\left(\sqrt{\alpha^2\mp 2i\alpha_1}\ z+\frac{\alpha\beta}{\sqrt{\alpha^2\mp 2i\alpha_1}}\right)-$$

$$\left.-\exp\left[\pm i\left(2\alpha_1z^2+\gamma\right)\right]\mathrm{erf}\,(\alpha z+\beta)\right\}\pm\frac{i}{4\alpha^2}\left\{\frac{2\alpha^2z^2-2\beta^2-1}{2}\times\right.$$

$$\left.\times\exp(\mp i\gamma)\,\mathrm{erf}\,(\alpha z+\beta)+\frac{\alpha z-\beta}{\sqrt{\pi}}\exp\left[\mp i\gamma-(\alpha z+\beta)^2\right]\right\}.$$

$$4.\ \int z\,\mathrm{erf}\,[(1\pm i)\alpha z+\beta]\exp\left(\pm i\alpha^2z^2\right)\sin\left(\alpha^2z^2+\gamma\right)dz=\frac{(1\pm i)\alpha z-\beta}{8\sqrt{\pi}\alpha^2}\times$$

$$\times\exp\left\{\mp i\gamma-[(1\pm i)\alpha z+\beta]^2\right\}-\frac{\exp(\pm i\gamma)}{8\alpha^2}\times$$

$$\times\left\{\operatorname{erf}[(1\pm i)\alpha z+\beta]\exp\left(\pm 2i\alpha^2 z^2\right)+\frac{1}{\sqrt{\pi}\,\beta}\exp\left[-2\alpha\beta z(1\pm i)-\beta^2\right]\right\}-$$

$$-\frac{\exp(\mp i\gamma)}{16\alpha^2}\left(1+2\beta^2\mp 4i\alpha^2 z^2\right)\operatorname{erf}\ [(1\pm i)\alpha z+\beta].$$

5. $$\int z\operatorname{erf}[(1\pm i)\alpha z]\exp\left(\pm i\alpha^2 z^2\right)\sin\left(\alpha^2 z^2+\gamma\right)dz=\frac{(1\pm i)z}{4\sqrt{\pi}\alpha}\times$$

$$\times\left\{\exp(\pm i\gamma)+\frac{1}{2}\exp\left[\mp i\left(2\alpha^2 z^2+\gamma\right)\right]\right\}-\frac{1}{8\alpha^2}\operatorname{erf}[(1\pm i)\alpha z]\times$$

$$\times\left\{\exp\left[\pm i\left(2\alpha^2 z^2+\gamma\right)\right]+\frac{\exp(\mp i\gamma)}{2}\left(1\mp 4i\alpha^2 z^2\right)\right\}.$$

6. $$\int z\operatorname{erf}(\alpha z+\beta)\exp\left[\left(\alpha^2\pm i\alpha_1\right)z^2\right]\sin\left(\alpha_1 z^2+\gamma\right)dz=\pm\frac{i\exp(\mp i\gamma)}{4\alpha^2}\times$$

$$\times\left[\frac{1}{\sqrt{\pi}\ \beta}\exp\left(-2\alpha\beta z-\beta^2\right)+\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha^2 z^2\right)\right]-$$

$$-\frac{\operatorname{erf}(\alpha z+\beta)}{4\left(2\alpha_1\mp i\alpha^2\right)}\exp\left[\left(\alpha^2\pm 2i\alpha_1\right)z^2\pm i\gamma\right]-\frac{(1\mp i)\alpha}{8\sqrt{\alpha_1}\left(\alpha^2\pm 2i\alpha_1\right)}\times$$

$$\times\exp\left[\pm i\left(\beta^2\frac{\alpha^2\pm 2i\alpha_1}{2\alpha_1}+\gamma\right)\right]\operatorname{erf}\left[(1\mp i)\sqrt{\alpha_1}\ z+\frac{\alpha\beta}{(1\mp i)\sqrt{\alpha_1}}\right].$$

7. $$\int z\operatorname{erf}(\alpha z)\exp\left[\left(\alpha^2\pm i\alpha_1\right)z^2\right]\sin\left(\alpha_1 z^2+\gamma\right)dz=\pm\frac{i\exp(\mp i\gamma)}{2\alpha}\times$$

$$\times\left[\frac{\operatorname{erf}(\alpha z)}{2\alpha}\exp\left(\alpha^2 z^2\right)-\frac{z}{\sqrt{\pi}}\right]-\frac{\exp(\pm i\gamma)}{4}\times$$

$$\times\left\{\frac{\operatorname{erf}(\alpha z)}{2\alpha_1\mp i\alpha^2}\exp\left[\left(\alpha^2\pm 2i\alpha_1\right)z^2\right]+\frac{(1\mp i)\alpha}{2\sqrt{\alpha_1}\left(\alpha^2\pm 2i\alpha_1\right)}\operatorname{erf}\left[(1\mp i)\sqrt{\alpha_1}z\right]\right\}.$$

1.18.3.

1. $$\int x^{2n+1}\operatorname{erf}\ (ax+b)\exp\left(a_1 x^2\right)\sin\left(a_2 x^2+\gamma\right)dx=\frac{n!a}{2}\exp\left[\frac{b^2\,(a^2 a_1-a_1^2-a_2^2)}{a^4+a_1^2+a_2^2-2a^2 a_1}\right]\times$$

$$\times\left[\sin\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}+\gamma\right)\operatorname{Re}V_1^{(18)}(a,a_1,a_2,b,x)+\right.$$

$$+\cos\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}+\gamma\right)\times$$

$$\left.\times\operatorname{Im}V_1^{(18)}(a,a_1,a_2,b,x)\right]-\frac{n!}{2}\exp\left(a_1x^2\right)\left\{\sin\left(a_2x^2+\gamma\right)\left[\operatorname{erf}(ax+b)\times\right.\right.$$

$$\left.\times\operatorname{Re}V_2^{(18)}(a_1,a_2,x)+\frac{a}{\sqrt{\pi}}\operatorname{Re}V_3^{(18)}(a,a_1,a_2,b,x)\right]+\cos\left(a_2x^2+\gamma\right)\times$$

$$\left.\times\left[\operatorname{erf}(ax+b)\operatorname{Im}V_2^{(18)}(a_1,a_2,x)+\frac{a}{\sqrt{\pi}}\operatorname{Im}V_3^{(18)}(a,a_1,a_2,b,x)\right]\right\}$$

$$\{|a_1|+|a_2|>0,\quad \left|a^2-a_1\right|+|a_2|>0\}.$$

2. $$\int x^{2n+1}\operatorname{erfi}(ax+b)\exp\left(a_1x^2\right)\sin\left(a_2x^2+\gamma\right)dx=\frac{n!a}{2}\exp\left[\frac{b^2(a^2a_1+a_1^2+a_2^2)}{a^4+a_1^2+a_2^2+2a^2a_1}\right]\times$$

$$\times\left[\sin\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}+\gamma\right)\operatorname{Re}V_1^{(18)}(ia,a_1,a_2,ib,x)+\right.$$

$$+\cos\left(\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}+\gamma\right)\times$$

$$\left.\times\operatorname{Im}V_1^{(18)}(ia,a_1,a_2,ib,x)\right]-\frac{n!}{2}\exp\left(a_1x^2\right)\left\{\sin\left(a_2x^2+\gamma\right)\left[\operatorname{erfi}(ax+b)\times\right.\right.$$

$$\left.\times\operatorname{Re}V_2^{(18)}\left(a_1,a_2,x\right)+\frac{a}{\sqrt{\pi}}\operatorname{Re}V_3^{(18)}\left(ia,a_1,a_2,ib,x\right)\right]+\cos\left(a_2x^2+\gamma\right)\times$$

$$\left.\times\left[\operatorname{erfi}\left(ax+b\right)\operatorname{Im}V_2^{(18)}(a_1,a_2,x)+\frac{a}{\sqrt{\pi}}\operatorname{Im}V_3^{(18)}(ia,a_1,a_2,ib,x)\right]\right\}$$

$$\{|a_1|+|a_2|>0,\quad \left|a^2+a_1\right|+|a_2|>0\}.$$

3. $$\int z^{2n+1}\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha_1z^2\right)\sin\left(\alpha_2z^2+\gamma\right)dz=\frac{n!i\alpha}{4}\left\{\exp\left[\frac{\beta^2(\alpha_1-i\alpha_2)}{\alpha^2-\alpha_1+i\alpha_2}-i\gamma\right]\times\right.$$

$$\left.\times V_1^{(18)}(\alpha,\alpha_1,-\alpha_2,\beta,z)-\exp\left[\frac{\beta^2(\alpha_1+i\alpha_2)}{\alpha^2-\alpha_1-i\alpha_2}+i\gamma\right]V_1^{(18)}(\alpha,\alpha_1,\alpha_2,\beta,z)\right\}+\frac{n!i}{4}\times$$

$$\times\left\{\exp\left[\alpha_1 z^2 + i(\alpha_2 z^2 + \gamma)\right]\left[\mathrm{erf}\ (\alpha z + \beta)\, V_2^{(18)}(\alpha_1, \alpha_2, z) + \frac{\alpha}{\sqrt{\pi}} V_3^{(18)}(\alpha, \alpha_1, \alpha_2, \beta, z)\right] -\right.$$

$$\left. - \exp\left[\alpha_1 z^2 - i(\alpha_2 z^2 + \gamma)\right]\left[\mathrm{erf}\ (\alpha z + \beta) V_2^{(18)}(\alpha_1, -\alpha_2, z) + \frac{\alpha}{\sqrt{\pi}} V_3^{(18)}(\alpha, \alpha_1, -\alpha_2, \beta, z)\right]\right\}$$

$$\{\alpha_2^2 \neq -\alpha_1^2, \quad \alpha_2^2 \neq -(\alpha^2 - \alpha_1)^2\}.$$

1.18.4.

1. $$\int z^{2n+1} \mathrm{erf}\ [(1 \pm i)\alpha z + \beta] \sin\left(2\alpha^2 z^2 + \gamma\right) dz = \pm \frac{n!\, i}{4}\left[(1 \pm i)\alpha \exp\left(-\frac{\beta^2}{2} \mp i\ \gamma\right) \times\right.$$

$$\left. \times V_1^{(18)}\left(\frac{2\alpha}{1 \mp i}, 0, \mp 2\alpha^2, \beta, z\right) - \exp(\pm i\gamma)\, V_4^{(18)}\left(\frac{2\alpha}{1 \mp i}, \beta\right)\right] \mp \frac{n! i}{4} \exp\left[\mp i\ (2\alpha^2 z^2 + \gamma)\right] \times$$

$$\times\left\{\mathrm{erf}\,[\,(1 \pm i)\alpha z + \beta]\, V_2^{(18)}(0, \mp 2\alpha^2, z) + \frac{(1 \pm i)\alpha}{\sqrt{\pi}} V_3^{(18)}\left(\frac{2\alpha}{1 \mp i}, 0, \mp 2\alpha^2, \beta, z\right)\right\}$$

$$\{\alpha \neq 0\}.$$

2. $$\int z^{2n+1} \mathrm{erf}\,(\alpha z + \beta) \exp\left(\pm i\alpha_1 z^2\right) \sin\left(\alpha_1 z^2 + \gamma\right) dz = \pm \frac{n!}{4i}\left\{\alpha \exp\left(\pm i\gamma - \frac{2\alpha_1 \beta^2}{2\alpha_1 \pm i\alpha^2}\right) \times\right.$$

$$\times V_1^{(18)}(\alpha, \pm i\alpha_1, \pm\alpha_1, \beta, z) - \exp\left[\pm i(2\alpha_1 z^2 + \gamma)\right]\left[\mathrm{erf}\,(\alpha z + \beta) V_2^{(18)}(\pm i\alpha_1, \pm\alpha_1, z) +\right.$$

$$\left.\left. + \frac{\alpha}{\sqrt{\pi}} V_3^{(18)}(\alpha, \pm i\,\alpha_1, \pm\alpha_1, \beta, z)\right]\right\} \pm \frac{i}{2} \exp(\mp i\ \gamma) V_5^{(18)}(\alpha, \beta)$$

$$\{\alpha \neq 0, \alpha_1 \neq 0, \alpha_1 \neq \mp \frac{i\alpha^2}{2}\}.$$

3. $$\int z^{2n+1} \mathrm{erf}\ [(1 \pm i)\alpha z + \beta] \exp\left(\pm i\alpha^2 z^2\right) \sin\left(\alpha^2 z^2 + \gamma\right) dz = \pm \frac{n!}{4i} \exp\ (\pm i\ \gamma) \times$$

$$\times V_4^{(18)}\left(\frac{2\alpha}{1 \mp i}, \beta\right) \pm \frac{i}{2} \exp(\mp i\gamma) V_5^{(18)}\left(\frac{2\alpha}{1 \mp i}, \beta\right) \qquad \{\alpha \neq 0\}.$$

4. $$\int z^{2n+1} \mathrm{erf}\,(\alpha z + \beta) \exp\left[\left(\alpha^2 \pm i\alpha_1) z^2\right)\right] \sin\left(\alpha_1 z^2 + \gamma\right)\ dz =$$

$$= \pm \frac{n!\alpha}{4i} \exp\left[\pm i\left(\beta^2 \frac{\alpha^2 \pm 2i\alpha_1}{2\alpha_1} + \gamma\right)\right] V_1^{(18)}(\alpha, \alpha^2 \pm i\alpha_1, \pm\alpha_1, \beta, z) \pm$$

$$\pm \frac{n!i}{4}\left\{\exp\left[\alpha^2 z^2 \pm i(2\alpha_1 z^2 + \gamma)\right]\times\right.$$

$$\times\left[\operatorname{erf}(\alpha z + \beta)\, V_2^{(18)}(\alpha^2 \pm i\alpha_1, \pm\alpha_1, z) + \frac{\alpha}{\sqrt{\pi}} V_3^{(18)}(\alpha, \alpha^2 \pm i\alpha_1, \pm\alpha_1, \beta, z)\right] +$$

$$\left. + \exp(\mp i\gamma)\, V_4^{(18)}(\alpha, \beta)\right\} \qquad \{\alpha \neq 0, \alpha_1 \neq \pm\frac{i\alpha^2}{2}\}.$$

Introduced notations:

1) $$V_1^{(18)}(\alpha, \alpha_1, \alpha_2, \beta, z) = \frac{1}{\sqrt{\alpha^2 - \alpha_1 - i\alpha_2}} \operatorname{erf}\left(\sqrt{\alpha^2 - \alpha_1 - i\alpha_2}\; z + \frac{\alpha\beta}{\sqrt{\alpha^2 - \alpha_1 - i\alpha_2}}\right)\times$$

$$\times \sum_{k=0}^{n} \frac{(2k)!}{k!}\left(-\frac{1}{\alpha_1 + i\alpha_2}\right)^{n+1-k} \sum_{l=0}^{k} \frac{(\alpha\beta)^{2k-2l}}{4^l\, l!(2k-2l)!\left(\alpha^2 - \alpha_1 - i\alpha_2\right)^{2k-l}} =$$

$$= \frac{1}{\sqrt{\alpha_1 - \alpha^2 + i\alpha_2}} \operatorname{erfi}\left(\sqrt{\alpha_1 - \alpha^2 + i\alpha_2}\; z - \frac{\alpha\beta}{\sqrt{\alpha_1 - \alpha^2 + i\alpha_2}}\right)\times$$

$$\times \sum_{k=0}^{n} \frac{(2k)!}{k!}\left(-\frac{1}{\alpha_1 + i\alpha_2}\right)^{n+1-k} \sum_{l=0}^{k} \frac{(\alpha\beta)^{2k-2l}}{(-4)^l\, l!(2k-2l)!\left(\alpha_1 - \alpha^2 + i\alpha_2\right)^{2k-l}},$$

$$V_1^{(18)}(\alpha, \alpha_1, \alpha_2, 0, z) = \frac{1}{\sqrt{\alpha^2 - \alpha_1 - i\alpha_2}} \operatorname{erf}\left(\sqrt{\alpha^2 - \alpha_1 - i\alpha_2}\; z\right)\times$$

$$\times \sum_{k=0}^{n} \frac{(2k)!}{4^k (k!)^2 \left(\alpha^2 - \alpha_1 - i\alpha_2\right)^k}\times$$

$$\times\left(-\frac{1}{\alpha_1 + i\alpha_2}\right)^{n+1-k} = \frac{1}{\sqrt{\alpha_1 - \alpha^2 + i\alpha_2}} \operatorname{erfi}\left(\sqrt{\alpha_1 - \alpha^2 + i\alpha_2}\; z\right)\times$$

$$\times \sum_{k=0}^{n} \frac{(2k)!}{(-4)^k (k!)^2 \left(\alpha_1 - \alpha^2 + i\alpha_2\right)^k}\left(-\frac{1}{\alpha_1 + i\alpha_2}\right)^{n+1-k};$$

2) $$V_2^{(18)}(\alpha_1, \alpha_2, z) = \sum_{k=0}^{n} \frac{z^{2k}}{k!}\left(-\frac{1}{\alpha_1 + i\alpha_2}\right)^{n+1-k};$$

3) $$V_3^{(18)}(\alpha,\alpha_1,\alpha_2,\beta,z)=\exp\left[-(\alpha z+\beta)^2\right]\sum_{k=1}^{n}\frac{(2k)!}{k!}\left(-\frac{1}{\alpha_1+i\alpha_2}\right)^{n+1-k}\times$$

$$\times\left[\sum_{l=1}^{k}\frac{(\alpha\beta)^{2k-2l}}{l!(2k-2l)!}\sum_{r=1}^{l}\frac{r!\left(\alpha^2 z+\alpha\beta-\alpha_1 z-i\alpha_2 z\right)^{2r-1}}{4^{l-r}(2r)!\left(\alpha^2-\alpha_1-i\alpha_2\right)^{2k-l+r}}-\right.$$

$$\left.-\sum_{l=1}^{k}\frac{(l-1)!(\alpha\beta)^{2k+1-2l}}{(2l-1)!(2k+1-2l)!}\sum_{r=1}^{l}\frac{\left(\alpha^2 z+\alpha\beta-\alpha_1 z-i\alpha_2 z)\right)^{2r-2}}{(r-1)!\left(\alpha^2-\alpha_1-i\alpha_2\right)^{2k-l+r}}\right],$$

$$V_3^{(18)}(\alpha,\alpha_1,\alpha_2,0,z)=\exp\left(-\alpha^2 z^2\right)\sum_{k=1}^{n}\frac{(2k)!}{(k!)^2}\left(-\frac{1}{\alpha_1+i\alpha_2}\right)^{n+1-k}\times$$

$$\times\sum_{l=1}^{k}\frac{l!z^{2l-1}}{4^{k-l}(2l)!\left(\alpha^2-\alpha_1-i\alpha_2\right)^{k+1-l}};$$

4) $$V_4^{(18)}(\alpha,\beta)=\frac{1}{\alpha^2}\left[\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha^2 z^2\right)\sum_{k=0}^{n}\frac{z^{2k}}{k!\left(-\alpha^2\right)^{n-k}}+\right.$$

$$\left.+\frac{\exp\left(-2\alpha\beta z-\beta^2\right)}{\sqrt{\pi}\beta}\sum_{k=0}^{n}\frac{(2k)!}{k!(-\alpha^2)^{n-k}}\sum_{l=0}^{2k}\frac{z^l}{l!(2\alpha\beta)^{2k-l}}\right],$$

$$V_4^{(18)}(\alpha,0)=\frac{1}{\alpha}\sum_{k=0}^{n}\frac{z^{2k}}{k!\left(-\alpha^2\right)^{n-k}}\left[\frac{\operatorname{erf}(\alpha z)}{\alpha}\exp\left(\alpha^2 z^2\right)-\frac{2z}{(2k+1)\sqrt{\pi}}\right];$$

5) $$V_5^{(18)}(\alpha,\beta)=\operatorname{erf}(\alpha z+\beta)\left[\frac{z^{2n+2}}{2n+2}-\frac{(2n+1)!}{\alpha^{2n+2}}\sum_{k=0}^{n+1}\frac{\beta^{2n+2-2k}}{4^k k!(2n+2-2k)!}\right]+$$

$$+\frac{(2n+1)!}{\sqrt{\pi}\alpha^{2n+2}}\exp\left[-(\alpha z+\beta)^2\right]\left[\sum_{k=0}^{n}\frac{\beta^{2n-2k}}{(k+1)!(2n-2k)!}\sum_{l=0}^{k}\frac{(l+1)!(\alpha z+\beta)^{2l+1}}{4^{k-l}(2l+2)!}-\right.$$

$$\left.-\sum_{k=0}^{n}\frac{k!\beta^{2n+1-2k}}{(2k+1)!(2n+1-2k)!}\sum_{l=0}^{k}\frac{(\alpha z+\beta)^{2l}}{l!}\right],$$

$$V_5^{(18)}(\alpha,0)=\operatorname{erf}(\alpha z)\left[\frac{z^{2n+2}}{2n+2}-\frac{(2n+1)!}{(n+1)!(4\alpha^2)^{n+1}}\right]+$$

$$+\frac{(2n+1)!}{(n+1)!\sqrt{\pi}\alpha}\exp\left(-\alpha^2 z^2\right)\sum_{k=0}^{n}\frac{(k+1)!z^{2k+1}}{(2k+2)!\left(4\alpha^2\right)^{n-k}}.$$

PART 2
DEFINITE INTEGRALS

2.1. Integrals of $z^n \exp\left[\mp(\alpha z+\beta)^2\right]$

2.1.1.

1. $$\int_0^{+\infty} \exp\left[-(a z+\beta)^2\right] dz = \frac{\sqrt{\pi}}{2a}\left[1-\operatorname{erf}(\beta)\right] \qquad \{ a>0 \}.$$

2. $$\int_{-\infty}^{+\infty} \exp\left[-(a\, z+\beta)^2\right] dz = \frac{\sqrt{\pi}}{a} \qquad \{ a>0 \}.$$

3. $$\int_0^{\infty} \exp\left[-(\alpha z+\beta)^2\right] dz = -\frac{\sqrt{\pi}}{2\alpha}\left[\operatorname{erf}(\beta)\mp 1\right]$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty \}.$$

4. $$\int_0^{\infty} \exp\left[(\alpha z+\beta)^2\right] dz = -\frac{\sqrt{\pi}}{2\alpha}\left[\operatorname{erfi}(\beta)\mp i\right]$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-\ln|z|\right]=-\infty,\ \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty \}.$$

2.1.2.

1. $$\int_0^{+\infty} z\exp\left[-(a z+\beta)^2\right] dz = \frac{\sqrt{\pi}\,\beta}{2a^2}\left[\operatorname{erf}(\beta)-1\right]+\frac{\exp\left(-\beta^2\right)}{2a^2} \qquad \{ a>0 \}.$$

2. $$\int_{-\infty}^{+\infty} z\exp\left[-(a z+\beta)^2\right] dz = -\frac{\sqrt{\pi}\,\beta}{a^2} \qquad \{ a>0 \}.$$

3. $$\int_0^{\infty} z\exp\left[-(\alpha z+\beta)^2\right] dz = \frac{\sqrt{\pi}\,\beta}{2\alpha^2}\left[\operatorname{erf}(\beta)\mp 1\right]+\frac{\exp\left(-\beta^2\right)}{2\alpha^2}$$

$$\{ \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty \}.$$

4. $$\int_0^{\infty} z\exp\left[(\alpha z+\beta)^2\right] dz = \frac{\sqrt{\pi}\,\beta}{2\alpha^2}\left[\operatorname{erfi}(\beta)\mp i\right]-\frac{\exp\left(\beta^2\right)}{2\alpha^2}$$

$$\{ \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)=-\infty,\ \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty \}.$$

2.1.3.

1. $$\int_0^{+\infty} z^2 \exp\left[-\left(a z+\beta\right)^2\right] dz = \frac{\sqrt{\pi}\left(2\beta^2+1\right)}{4a^3}\left[1-\operatorname{erf}(\beta)\right]-\frac{\beta\exp\left(-\beta^2\right)}{2a^3} \quad \{a>0\}.$$

2. $$\int_{-\infty}^{+\infty} z^2 \exp\left[-\left(a z+\beta\right)^2\right] dz = \frac{\sqrt{\pi}\left(2\beta^2+1\right)}{2a^3} \quad \{a>0\}.$$

3. $$\int_0^{\infty} z^2 \exp\left[-(\alpha z+\beta)^2\right] dz = -\frac{\sqrt{\pi}\left(2\beta^2+1\right)}{4\alpha^3}\left[\operatorname{erf}(\beta)\mp 1\right]-\frac{\beta\exp\left(-\beta^2\right)}{2\alpha^3}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-\ln|z|\right]=+\infty, \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

4. $$\int_0^{\infty} z^2 \exp\left[(\alpha z+\beta)^2\right] dz = \frac{\sqrt{\pi}\left(1-2\beta^2\right)}{4\alpha^3}\left[\operatorname{erf i}(\beta)\mp i\right]+\frac{\beta\exp\left(\beta^2\right)}{2\alpha^3}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=-\infty, \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\}.$$

2.1.4.

1. $$\int_0^{+\infty} z^n \exp\left[-\left(a z+\beta\right)^2\right] dz = \frac{1}{a} W_1^{(1)}(n,a,\beta)\left[1-\operatorname{erf}(\beta)\right]+W_2^{(1)}(n,a,\beta)$$

$$\{a>0\}.$$

2. $$\int_{-\infty}^{+\infty} z^n \exp\left[-\left(a z+\beta\right)^2\right] dz = \frac{2}{a} W_1^{(1)}(n,a,\beta) \quad \{a>0\}.$$

3. $$\int_0^{\infty} z^n \exp\left[-\left(\alpha z+\beta\right)^2\right] dz = W_2^{(1)}(n,\alpha,\beta)-\frac{1}{\alpha}\left[\operatorname{erf}(\beta)\mp 1\right] W_1^{(1)}(n,\alpha,\beta)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-(n-1)\ln|z|\right]=+\infty, \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

4. $$\int_0^{\infty} z^n \exp\left[\left(\alpha z+\beta\right)^2\right] dz = W_2^{(1)}(n,i\alpha,i\beta)-\frac{1}{\alpha}\left[\operatorname{erf i}(\beta)\mp i\right] W_1^{(1)}(n,i\alpha,i\beta)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+(n-1)\ln|z|\right]=-\infty, \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\}.$$

Introduced notations:

1) $$W_1^{(1)}(n,\alpha,\beta)=\frac{n!\sqrt{\pi}}{2(-\alpha)^n}\sum_{k=0}^{E(n/2)}\frac{\beta^{n-2k}}{4^k\,k!(n-2k)!},$$

$$W_1^{(1)}(2m,\alpha,0)=\frac{(2m)!\sqrt{\pi}}{2^{2m+1}\,m!\,\alpha^{2m}},\; W_1^{(1)}(2m+1,\alpha,0)=0;$$

2) $$W_2^{(1)}(n,\alpha,\beta)=\frac{n!\exp\left(-\beta^2\right)}{2(-\alpha)^{n+1}}\left[\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!}{(2k-1)!(n+1-2k)!}\times\right.$$

$$\left.\times\sum_{l=1}^{k}\frac{\beta^{n-1-2k+2l}}{(l-1)!}-\sum_{k=1}^{E(n/2)}\frac{1}{k!(n-2k)!}\sum_{l=1}^{k}\frac{l!\,\beta^{n-1-2k+2l}}{4^{k-l}(2l)!}\right],$$

$$W_2^{(1)}(2m,\alpha,0)=0,\;\; W_2^{(1)}(2m+1,\alpha,0)=\frac{m!}{2\,\alpha^{2m+2}}.$$

2.2. Integrals of $z^n\exp\left(\mp\alpha^2z^2+\beta z+\gamma\right)$

2.2.1.

1. $$\int_0^{+\infty}\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{2a}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right)\left[1+\operatorname{erf}\left(\frac{\beta}{2a}\right)\right].$$

2. $$\int_{-\infty}^{+\infty}\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{a}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right).$$

3. $$\int_0^{\infty}\exp\left(-\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{2\alpha}\exp\left(\frac{\beta^2}{4\alpha^2}+\gamma\right)\left[\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)\pm1\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2-\beta z\right)+\ln|z|\right]=+\infty,\;\;\lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty\}.$$

4. $$\int_0^{\infty}\exp\left(\alpha^2z^2+\beta z+\gamma\right)dz=-\frac{\sqrt{\pi}}{2\alpha}\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\left[\operatorname{erfi}\left(\frac{\beta}{2\alpha}\right)\mp i\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\beta z\right)-\ln|z|\right]=-\infty,\;\;\lim_{z\to\infty}\operatorname{Im}\left(\alpha z\right)=\pm\infty\}.$$

2.2.2.

1. $$\int_0^{+\infty} z\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\,\beta}{4a^3}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right)\left[1+\operatorname{erf}\left(\frac{\beta}{2a}\right)\right]+\frac{\exp(\gamma)}{2a^2}.$$

2. $$\int_{-\infty}^{+\infty} z\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\,\beta}{2a^3}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right).$$

3. $$\int_0^{\infty} z\exp\left(-\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\,\beta}{4\alpha^3}\exp\left(\frac{\beta^2}{4\alpha^2}+\gamma\right)\left[\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)\pm 1\right]+$$
$$+\frac{\exp(\gamma)}{2\alpha^2}\qquad \{\lim_{z\to\infty}\operatorname{Re}\left(\alpha^2z^2-\beta z\right)=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

4. $$\int_0^{\infty} z\exp\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\,\beta}{4\alpha^3}\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\left[\operatorname{erfi}\left(\frac{\beta}{2\alpha}\right)\mp i\right]-$$
$$-\frac{\exp(\gamma)}{2\alpha^2}\qquad \{\lim_{z\to\infty}\operatorname{Re}\left(\alpha^2z^2+\beta z\right)=-\infty,\ \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\}.$$

2.2.3.

1. $$\int_0^{+\infty} z^2\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\left(2a^2+\beta^2\right)}{8a^5}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right)\times$$
$$\times\left[1+\operatorname{erf}\left(\frac{\beta}{2a}\right)\right]+\frac{\beta}{4a^4}\exp(\gamma).$$

2. $$\int_{-\infty}^{+\infty} z^2\exp\left(-a^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\left(2a^2+\beta^2\right)}{4a^5}\exp\left(\frac{\beta^2}{4a^2}+\gamma\right).$$

3. $$\int_0^{\infty} z^2\exp\left(-\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\left(2\alpha^2+\beta^2\right)}{8\alpha^5}\exp\left(\frac{\beta^2}{4\alpha^2}+\gamma\right)\times$$
$$\times\left[\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)\pm 1\right]+\frac{\beta}{4\alpha^4}\exp(\gamma)$$
$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2-\beta z\right)-\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

4. $$\int_0^{\infty} z^2\exp\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}\left(2\alpha^2-\beta^2\right)}{8\alpha^5}\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\times$$
$$\times\left[\operatorname{erfi}\left(\frac{\beta}{2\alpha}\right)\mp i\right]+\frac{\beta}{4\alpha^4}\exp(\gamma)$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\beta z\right)+\ln|z|\right]=-\infty,\ \lim_{z\to\infty}\operatorname{Im}\left(\alpha z\right)=\pm\infty \}.$$

2.2.4.

1. $$\int_0^{+\infty} z^n \exp\left(-a^2z^2+\beta z+\gamma\right) dz=\frac{1}{a}\left[1+\operatorname{erf}\left(\frac{\beta}{2a}\right)\right]W_1^{(2)}\left(n,a^2,\beta\right)+W_2^{(2)}\left(n,a^2,\beta\right).$$

2. $$\int_{-\infty}^{+\infty} z^n \exp\left(-a^2z^2+\beta z+\gamma\right) dz=\frac{2}{a}W_1^{(2)}\left(n,a^2,\beta\right) \quad .$$

3. $$\int_0^{\infty} z^n \exp\left(-\alpha^2z^2+\beta z+\gamma\right) dz=\frac{1}{\alpha}\left[\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)\pm 1\right]W_1^{(2)}\left(n,\alpha^2,\beta\right)+$$
$$+W_2^{(2)}\left(n,\alpha^2,\beta\right)$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2-\beta z\right)-\left(n-1\right)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty \}.$$

4. $$\int_0^{\infty} z^n \exp\left(\alpha^2z^2+\beta z+\gamma\right) dz=W_2^{(2)}\left(n,-\alpha^2,\beta\right)-\frac{1}{\alpha}\left[\operatorname{erf} i\left(\frac{\beta}{2\alpha}\right)\mp i\right]\times$$
$$\times W_1^{(2)}\left(n,-\alpha^2,\beta\right)$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\beta z\right)+\left(n-1\right)\ln|z|\right]=-\infty,\ \lim_{z\to\infty}\operatorname{Im}\left(\alpha z\right)=\pm\infty \}.$$

2.2.5.

1. $$\int_0^{+\infty} z^n \exp\left(-bz+\gamma\right)dz=\left(-1\right)^n\int_{-\infty}^{0} z^n \exp\left(bz+\gamma\right)dz=\frac{n!}{b^{n+1}}\exp\left(\gamma\right) \qquad \{ b>0 \}.$$

2. $$\int_0^{\infty} z^n \exp\left(\beta z+\gamma\right)dz=\frac{n!}{\left(-\beta\right)^{n+1}}\exp\left(\gamma\right) \qquad \{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\beta z\right)+n\ln|z|\right]=-\infty \}.$$

Introduced notations:

1) $$W_1^{(2)}\left(n,\alpha^2,\beta\right)=\frac{n!\sqrt{\pi}}{2^{n+1}}\exp\left(\frac{\beta^2}{4\alpha^2}+\gamma\right)\sum_{k=0}^{E(n/2)}\frac{\beta^{n-2k}}{k!\left(n-2k\right)!\left(\alpha^2\right)^{n-k}},$$

$$W_1^{(2)}\left(2m,\alpha^2,0\right)=\frac{(2m)!\sqrt{\pi}}{2^{2m+1}m!\left(\alpha^2\right)^m}\exp(\gamma),\quad W_1^{(2)}\left(2m+1,\alpha^2,0\right)=0;$$

$$2)\ W_2^{(2)}\left(n,\alpha^2,\beta\right)=\frac{n!\exp(\gamma)}{2^n}\left[\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!}{(2k-1)!(n+1-2k)!}\times\right.$$

$$\left.\times\sum_{l=1}^{k}\frac{4^{k-l}\beta^{n-1-2k+2l}}{(l-1)!\left(\alpha^2\right)^{n-k+l}}-\sum_{k=1}^{E(n/2)}\frac{1}{k!(n-2k)!}\sum_{l=1}^{k}\frac{l!\beta^{n-1-2k+2l}}{(2l)!\left(\alpha^2\right)^{n-k+l}}\right],$$

$$W_2^{(2)}\left(2m,\alpha^2,0\right)=0,\ W_2^{(2)}\left(2m+1,\alpha^2,0\right)=\frac{m!}{2\left(\alpha^2\right)^{m+1}}\exp(\gamma).$$

2.3. Integrals of $\operatorname{erf}^n(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]$

2.3.1.

1. $$\int_0^{+\infty}\operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz=\frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}^2(\beta)\right]\qquad\{a>0\}.$$

2. $$\int_{-\infty}^{+\infty}\operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz=0\qquad\{a\neq 0\}.$$

3. $$\int_0^{\infty}\operatorname{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{4\alpha}\left[1-\operatorname{erf}^2(\beta)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty\}.$$

2.3.2.

1. $$\int_0^{+\infty}\operatorname{erf}^2(az+\beta)\exp\left[-(az+\beta)^2\right]dz=\frac{\sqrt{\pi}}{6a}\left[1-\operatorname{erf}^3(\beta)\right]\qquad\{a>0\}.$$

2. $$\int_{-\infty}^{+\infty}\operatorname{erf}^2(az+\beta)\exp\left[-(az+\beta)^2\right]dz=\frac{\sqrt{\pi}}{3a}\qquad\{a>0\}.$$

3. $$\int_0^{\infty}\operatorname{erf}^2(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz=-\frac{\sqrt{\pi}}{6\alpha}\left[\operatorname{erf}^3(\beta)\mp 1\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.3.3.

1. $\int_0^{+\infty} \operatorname{erf}^{2m}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}}{(4m+2)a}\left[1-\operatorname{erf}^{2m+1}(\beta)\right] \quad \{ a>0 \}.$

2. $\int_0^{+\infty} \operatorname{erf}^{2m+1}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}}{(4m+4)a}\left[1-\operatorname{erf}^{2m+2}(\beta)\right] \quad \{ a>0 \}.$

3. $\int_{-\infty}^{+\infty} \operatorname{erf}^{2m}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}}{(2m+1)a} \qquad \{ a>0 \}.$

4. $\int_{-\infty}^{+\infty} \operatorname{erf}^{2m+1}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = 0 \qquad \{ a\neq 0 \}.$

5. $\int_0^{\infty} \operatorname{erf}^{n}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz = \frac{\sqrt{\pi}}{(2n+2)\alpha}\left[(\pm 1)^{n+1}-\operatorname{erf}^{n+1}(\beta)\right]$

$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty \}.$

2.4. Integrals of $z^n \operatorname{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]$

2.4.1.

1. $\int_0^{+\infty} z\operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}\,\beta}{4a^2}\left[\operatorname{erf}^2(\beta)-1\right]+$

$+\frac{\operatorname{erf}(\beta)}{2a^2}\exp\left(-\beta^2\right)-\frac{\sqrt{2}}{4a^2}\left[\operatorname{erf}\left(\sqrt{2}\,\beta\right)-1\right] \qquad \{ a>0 \}.$

2. $\int_{-\infty}^{+\infty} z\operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{1}{\sqrt{2}\,a^2} \qquad \{ a>0 \}.$

3. $\int_0^{\infty} z\operatorname{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz = \frac{\sqrt{\pi}\,\beta}{4\alpha^2}\left[\operatorname{erf}^2(\beta)-1\right]+\frac{\operatorname{erf}(\beta)}{2\alpha^2}\exp\left(-\beta^2\right)-$

$-\frac{\sqrt{2}}{4\alpha^2}\left[\operatorname{erf}\left(\sqrt{2}\,\beta\right)\mp 1\right] \quad \{ \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)=+\infty, \ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty \}.$

2.4.2.

$$1.\ \int_0^{+\infty} z^2 \operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}}{4a^3}\left\{\left(\beta^2+\frac{1}{2}\right)\left[1-\operatorname{erf}^2(\beta)\right]+\right.$$

$$\left.+\frac{\exp\left(-2\beta^2\right)}{\pi}\right\}-\frac{\beta}{2a^3}\left\{\sqrt{2}\left[1-\operatorname{erf}\left(\sqrt{2}\beta\right)\right]+\operatorname{erf}(\beta)\exp\left(-\beta^2\right)\right\}\quad \{a>0\}.$$

$$2.\ \int_{-\infty}^{+\infty} z^2 \operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = -\frac{\sqrt{2}\beta}{a^3}\qquad \{a>0\}.$$

$$3.\ \int_0^{\infty} z^2 \operatorname{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz = \frac{\sqrt{\pi}}{4\alpha^3}\left\{\left(\beta^2+\frac{1}{2}\right)\left[1-\operatorname{erf}^2(\beta)\right]+\right.$$

$$\left.+\frac{\exp\left(-2\beta^2\right)}{\pi}\right\}+\frac{\beta}{2\alpha^3}\left\{\sqrt{2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]-\operatorname{erf}(\beta)\exp\left(-\beta^2\right)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.4.3.

$$1.\ \int_0^{+\infty} z^n \operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = \left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]\times$$

$$\times W_1^{(4)}(n,a,\beta)+\frac{1}{a}\left\{\left[1-\operatorname{erf}^2(\beta)\right]W_2^{(4)}(n,a,\beta)+W_3^{(4)}(n,a,\beta)\right\}\quad \{a>0\}.$$

$$2.\ \int_{-\infty}^{+\infty} z^n \operatorname{erf}(az+\beta)\exp\left[-(az+\beta)^2\right]dz = -2W_1^{(4)}(n,a,\beta)\qquad \{a>0\}.$$

$$3.\ \int_0^{\infty} z^n \operatorname{erf}(\alpha z+\beta)\exp\left[-(\alpha z+\beta)^2\right]dz = \left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]W_1^{(4)}(n,\alpha,\beta)+$$

$$+\frac{1}{\alpha}\left\{\left[1-\operatorname{erf}^2(\beta)\right]W_2^{(4)}(n,\alpha,\beta)+W_3^{(4)}(n,\alpha,\beta)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-(n-1)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

Introduced notations:

$$1) W_1^{(4)}(n,\alpha,\beta)=\frac{(-1)^n\, n!}{2\sqrt{2}\,\alpha^{n+1}}\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!\,\beta^{n+1-2k}}{(2k-1)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{(2l)!}{8^l\,(l!)^2},$$

$$W_1^{(4)}(2m,\alpha,0)=0,\ W_1^{(4)}(2m+1,\alpha,0)=-\frac{m!}{2\sqrt{2}\,\alpha^{2m+2}}\sum_{k=0}^{m}\frac{(2k)!}{8^k\,(k!)^2};$$

$$2)\ W_2^{(4)}(n,\alpha,\beta)=\frac{n!\sqrt{\pi}}{(-\alpha)^n}\sum_{k=0}^{E(n/2)}\frac{\beta^{n-2k}}{4^{k+1}\,k!(n-2k)!},$$

$$W_2^{(4)}(2m,\alpha,0)=\frac{(2m)!\sqrt{\pi}}{4^{m+1}\,m!\,\alpha^{2m}},\ W_2^{(4)}(2m+1,\alpha,0)=0;$$

$$3)\ W_3^{(4)}(n,\alpha,\beta)=\frac{n!}{(-\alpha)^n}\left\{\sum_{k=1}^{E(n/2)}\frac{\beta^{n-2k}}{4^k\,k!(n-2k)!}\left[\mathrm{erf}(\beta)\exp\left(-\beta^2\right)\times\right.\right.$$

$$\left.\times\sum_{l=0}^{k-1}\frac{4^l\,l!\,\beta^{2l+1}}{(2l+1)!}+\frac{\exp\left(-2\beta^2\right)}{2\sqrt{\pi}}\sum_{l=0}^{k-1}\frac{(l!)^2}{(2l+1)!}\sum_{r=0}^{l}\frac{2^{l+r}\,\beta^{2r}}{r!}\right]-$$

$$-\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!\,\beta^{n+1-2k}}{(2k-1)!\,(n+1-2k)!}\left[\frac{\mathrm{erf}(\beta)}{2}\exp\left(-\beta^2\right)\sum_{l=0}^{k-1}\frac{\beta^{2l}}{l!}+\right.$$

$$\left.\left.+\frac{\exp\left(-2\beta^2\right)}{\sqrt{\pi}}\sum_{l=1}^{k-1}\frac{(2l)!}{(l!)^2}\sum_{r=0}^{l-1}\frac{r!\,\beta^{2r+1}}{8^{l-r}\,(2r+1)!}\right]\right\},$$

$$W_3^{(4)}(2m,\alpha,0)=\frac{(2m)!}{m!\sqrt{\pi}\,\alpha^{2m}}\sum_{k=0}^{m-1}\frac{(k!)^2}{2^{2m+1-k}\,(2k+1)!},\ W_3^{(4)}(2m+1,\alpha,0)=0.$$

2.5. Integrals of $z^n \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)$

2.5.1.

1. $$\int_0^{+\infty} \operatorname{erf}(az+\beta)\exp(-bz+\gamma)\,dz = -\int_{-\infty}^{0} \operatorname{erf}(az-\beta)\exp(bz+\gamma)\,dz = \frac{\operatorname{erf}(\beta)}{b}\exp(\gamma)-\frac{1}{b}\times$$
$$\times\exp\left(\gamma+\frac{b^2}{4a^2}+\frac{\beta b}{a}\right)\left[\operatorname{erf}\left(\beta+\frac{b}{2a}\right)-1\right] \qquad \{a>0,\ b>0\}.$$

2. $$\int_0^{\infty} \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz = \frac{1}{\beta_1}\exp\left(\frac{\beta_1^2}{4\alpha^2}-\frac{\beta\beta_1}{\alpha}+\gamma\right)\left[\operatorname{erf}\left(\beta-\frac{\beta_1}{2\alpha}\right)\mp 1\right]-$$
$$-\frac{\operatorname{erf}(\beta)}{\beta_1}\exp(\gamma) \qquad \{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z-\beta_1 z\right)+2\ln|z|\right]=+\infty\ ,$$
$$\lim_{z\to\infty}\operatorname{Re}(\beta_1 z)=-\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.5.2.

1. $$\int_0^{+\infty} z\operatorname{erf}(az+\beta)\exp(-bz+\gamma)\,dz = \int_{-\infty}^{0} z\operatorname{erf}(az-\beta)\exp(bz+\gamma)\,dz =$$
$$=\frac{\operatorname{erf}(\beta)}{b^2}\exp(\gamma)-\frac{2a^2-b^2-2a\beta b}{2a^2b^2}\times$$
$$\times\exp\left(\gamma+\frac{b^2}{4a^2}+\frac{\beta b}{a}\right)\left[\operatorname{erf}\left(\beta+\frac{b}{2a}\right)-1\right]+\frac{\exp\left(\gamma-\beta^2\right)}{\sqrt{\pi}\,ab} \qquad \{a>0,\ b>0\}.$$

2. $$\int_0^{\infty} z\operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz = \frac{\operatorname{erf}(\beta)}{\beta_1^2}\exp(\gamma)-\frac{2\alpha^2-\beta_1^2+2\alpha\beta\beta_1}{2\alpha^2\beta_1^2}\times$$
$$\times\exp\left(\frac{\beta_1^2}{4\alpha^2}-\frac{\beta\beta_1}{\alpha}+\gamma\right)\left[\operatorname{erf}\left(\beta-\frac{\beta_1}{2\alpha}\right)\mp 1\right]-\frac{\exp\left(\gamma-\beta^2\right)}{\sqrt{\pi}\,\alpha\beta_1}$$
$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z-\beta_1 z\right)+\ln|z|\right]=+\infty\ ,$$
$$\lim_{z\to\infty}\left[\operatorname{Re}(\beta_1 z)+\ln|z|\right]=-\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.5.3.

1. $$\int_0^{+\infty} z^n \operatorname{erf}(az+\beta)\exp(-bz+\gamma)\,dz = (-1)^{n+1}\int_{-\infty}^{0} z^n \operatorname{erf}(az-\beta)\exp(bz+\gamma)\,dz =$$

$$= \frac{n!\exp(\gamma)}{b}\left\{\frac{\operatorname{erf}(\beta)}{b^n} - \left[\operatorname{erf}\left(\beta+\frac{b}{2a}\right)-1\right]\times\right.$$

$$\left.\times W_1^{(5)}(a,\beta,-b) - \frac{2a}{\sqrt{\pi}}W_2^{(5)}(a,\beta,-b)\right\} \qquad \{a>0,\ b>0\}.$$

2. $$\int_0^{\infty} z^n \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z+\gamma)\,dz = \frac{n!\exp(\gamma)}{\beta_1}\left\{\frac{(-1)^{n+1}}{\beta_1^n}\operatorname{erf}(\beta)+\right.$$

$$\left.+\left[\operatorname{erf}\left(\beta-\frac{\beta_1}{2\alpha}\right)\mp 1\right] W_1^{(5)}(\alpha,\beta,\beta_1) + \frac{2\alpha}{\sqrt{\pi}}W_2^{(5)}(\alpha,\beta,\beta_1)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + 2\alpha\beta z - \beta_1 z\right) - (n-2)\ln|z|\right] = +\infty,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}(\beta_1 z) + n\ln|z|\right] = -\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

Introduced notations:

1) $$W_1^{(5)}(\alpha,\beta,\beta_1) = \exp\left(\frac{\beta_1^2}{4\alpha^2} - \frac{\beta\beta_1}{\alpha}\right)\sum_{k=0}^{n}\frac{1}{2^k(-\beta_1)^{n-k}}\sum_{l=0}^{E(k/2)}\frac{(\beta_1-2\alpha\beta)^{k-2l}}{l!(k-2l)!\,\alpha^{2k-2l}},$$

$$W_1^{(5)}(n,\alpha,2\alpha\beta) = \frac{\exp(-\beta^2)}{(2\alpha)^n}\sum_{k=0}^{E(n/2)}\frac{1}{k!(-\beta)^{n-2k}};$$

2) $$W_2^{(5)}(\alpha,\beta,\beta_1) = \exp(-\beta^2)\sum_{k=1}^{n}\frac{1}{2^k(-\beta_1)^{n-k}}\left[\sum_{l=1}^{E(k/2)}\frac{1}{l!(k-2l)!}\times\right.$$

$$\times\sum_{r=1}^{l}\frac{r!(\beta_1-2\alpha\beta)^{k-1-2l+2r}}{(2r)!(\alpha^2)^{k-l+r}} - \sum_{l=1}^{k-E(k/2)}\frac{(l-1)!}{(2l-1)!(k+1-2l)!}\times$$

$$\left.\times\sum_{r=1}^{l}\frac{4^{l-r}(\beta_1-2\alpha\beta)^{k-1-2l+2r}}{(r-1)!(\alpha^2)^{k-l+r}}\right],$$

$$W_2^{(5)}(\alpha,\beta,2\alpha\beta)=\frac{(-1)^n}{\alpha^{n+1}}\exp\left(-\beta^2\right)\sum_{k=1}^{n-E(n/2)}\frac{k!}{(2k)!\,(2\beta)^{n+1-2k}}.$$

2.6. Integrals of $z^{2m+1}\operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1 z^2\right)$

2.6.1.

1. $$\int_0^{+\infty} z\operatorname{erf}(az+\beta)\exp\left(-a_1z^2\right)dz=\frac{\operatorname{erf}(\beta)}{2a_1}+\frac{a}{2a_1\sqrt{a^2+a_1}}\exp\left(-\frac{a_1\beta^2}{a^2+a_1}\right)\times$$
$$\times\left[1-\operatorname{erf}\left(\frac{a\beta}{\sqrt{a^2+a_1}}\right)\right]\qquad\{a\ge 0,\ a_1>0\}.$$

2. $$\int_0^{+\infty} z\operatorname{erfi}(az+\beta)\exp\left(-a_1z^2\right)dz=\frac{a}{2a_1\sqrt{a_1-a^2}}\exp\left(\frac{a_1\beta^2}{a_1-a^2}\right)\times$$
$$\times\left[1+\operatorname{erf}\left(\frac{a\beta}{\sqrt{a_1-a^2}}\right)\right]+\frac{\operatorname{erfi}(\beta)}{2a_1}\qquad\{a\ge 0,\ a_1>a^2\}.$$

3. $$\int_0^{+\infty} z\operatorname{erfi}(az+b)\exp\left(-a^2z^2\right)dz=\int_{-\infty}^{0} z\operatorname{erfi}(az-b)\exp\left(-a^2z^2\right)dz=$$
$$=\frac{\operatorname{erfi}(b)}{2a^2}-\frac{\exp\left(b^2\right)}{2\sqrt{\pi}a^2b}\qquad\{a>0,\ b<0\}.$$

4. $$\int_{-\infty}^{+\infty} z\operatorname{erf}(az+\beta)\exp\left(-a_1z^2\right)dz=\frac{a}{a_1\sqrt{a^2+a_1}}\exp\left(-\frac{a_1\beta^2}{a^2+a_1}\right)\qquad\{a\ge 0,\ a_1>0\}.$$

5. $$\int_{-\infty}^{+\infty} z\operatorname{erfi}(az+\beta)\exp\left(-a_1z^2\right)dz=\frac{a}{a_1\sqrt{a_1-a^2}}\exp\left(\frac{a_1\beta^2}{a_1-a^2}\right)\qquad\{a_1>a^2\}.$$

6. $$\int_0^{\infty} z\operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1z^2\right)dz=\frac{\operatorname{erf}(\beta)}{2\alpha_1}-\frac{\alpha}{2\alpha_1\sqrt{\alpha^2+\alpha_1}}\exp\left(-\frac{\alpha_1\beta^2}{\alpha^2+\alpha_1}\right)\times$$
$$\times\left[\operatorname{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1}}\right)\mp 1\right]\qquad\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1z^2+2\alpha\beta z\right)+\ln|z|\right]=$$

$$= \lim_{z\to\infty} \operatorname{Re}\left(\alpha_1 z^2\right) = +\infty\,,\ \lim_{z\to\infty} \operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1}\ z\right) = \pm\infty\ \}.$$

7. $$\int_0^{\infty} z\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(\alpha^2 z^2\right)dz = -\frac{1}{2\alpha^2}\left[\mathrm{erf}(\beta)+\frac{1}{\sqrt{\pi}\,\beta}\exp\left(-\beta^2\right)\right]$$
$$\{\ \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2\right) = -\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha\beta z) = +\infty\ \}.$$

2.6.2.

1. $$\int_0^{+\infty} z^{2m+1}\,\mathrm{erf}\,(az+\beta)\exp\left(-a_1 z^2\right)dz = \frac{m!}{2}\left\{\frac{\mathrm{erf}\,(\beta)}{a_1^{m+1}}+\frac{a}{\sqrt{a^2+a_1}}\times\right.$$
$$\left.\times\left[1-\mathrm{erf}\left(\frac{a\beta}{\sqrt{a^2+a_1}}\right)\right]W_1^{(6)}(a,a_1,\beta)+aW_2^{(6)}(a,a_1,\beta)\right\}\qquad \{a\ge 0,\,a_1>0\}.$$

2. $$\int_0^{+\infty} z^{2m+1}\,\mathrm{erfi}(az+\beta)\exp\left(-a_1 z^2\right)dz = \frac{m!}{2}\left\{\frac{\mathrm{erfi}(\beta)}{a_1^{m+1}}+\frac{a}{\sqrt{a_1-a^2}}\times\right.$$
$$\left.\times\left[1+\mathrm{erf}\left(\frac{a\beta}{\sqrt{a_1-a^2}}\right)\right]W_1^{(6)}(ia,a_1,i\beta)+aW_2^{(6)}(ia,a_1,i\beta)\right\}\qquad \left\{a\ge 0,\,a_1>a^2\right\}.$$

3. $$\int_0^{+\infty} z^{2m+1}\,\mathrm{erfi}\,(az+b)\exp\left(-a^2 z^2\right)dz = \int_{-\infty}^{0} z^{2m+1}\,\mathrm{erfi}\,(az-b)\exp\left(-a^2 z^2\right)dz =$$
$$= \frac{m!}{2a^{2m+2}}\left[\mathrm{erfi}\,(b)-\frac{1}{b}W_3^{(6)}\left(b^2\right)\right]\qquad \{a>0,\,b<0\}.$$

4. $$\int_{-\infty}^{+\infty} z^{2m+1}\,\mathrm{erf}\,(az+\beta)\exp\left(-a_1 z^2\right)dz = \frac{m!a}{\sqrt{a^2+a_1}}W_1^{(6)}(a,a_1,\beta)\quad \{a\ge 0,\,a_1>0\}.$$

5. $$\int_{-\infty}^{+\infty} z^{2m+1}\,\mathrm{erfi}\,(az+\beta)\exp\left(-a_1 z^2\right)dz = \frac{m!a}{\sqrt{a_1-a^2}}W_1^{(6)}(ia,a_1,i\beta)\qquad \{\,a_1>a^2\,\}.$$

6. $$\int_0^{\infty} z^{2m+1}\,\mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha_1 z^2\right)dz = \frac{m!}{2}\left\{\frac{\mathrm{erf}\,(\beta)}{\alpha_1^{m+1}}-\frac{\alpha}{\sqrt{\alpha^2+\alpha_1}}\times\right.$$

$$\times\left[\operatorname{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1}}\right)\mp 1\right]W_1^{(6)}(\alpha,\alpha_1,\beta)+\alpha W_2^{(6)}(\alpha,\alpha_1,\beta)\Bigg\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1z^2+2\alpha\beta z\right)-(2m-1)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1z^2\right)-2m\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1}\,z\right]=\pm\infty\}.$$

7. $$\int_0^\infty z^{2m+1}\operatorname{erf}(\alpha z+\beta)\exp\left(\alpha^2z^2\right)dz=\frac{(-1)^{m+1}m!}{2\alpha^{2m+2}}\left[\operatorname{erf}(\beta)+\frac{1}{\beta}W_3^{(6)}\left(-\beta^2\right)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2\right)+2m\ln|z|\right]=-\infty,\quad \lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta z)-m\ln|z|\right]=+\infty\}.$$

Introduced notations:

1) $$W_1^{(6)}(\alpha,\alpha_1,\beta)=\exp\left(-\frac{\alpha_1\beta^2}{\alpha^2+\alpha_1}\right)\sum_{k=0}^{m}\frac{(2k)!}{k!\alpha_1^{m+1-k}}\sum_{l=0}^{k}\frac{(\alpha\beta)^{2k-2l}}{4^l\,l!(2k-2l)!\left(\alpha^2+\alpha_1\right)^{2k-l}},$$

$$W_1^{(6)}(\alpha,\alpha_1,0)=\sum_{k=0}^{m}\frac{(2k)!}{4^k\,(k!)^2\,\alpha_1^{m+1-k}\left(\alpha^2+\alpha_1\right)^k};$$

2) $$W_2^{(6)}(\alpha,\alpha_1,\beta)=\frac{\exp\left(-\beta^2\right)}{\sqrt{\pi}}\sum_{k=1}^{m}\frac{(2k)!}{k!\alpha_1^{m+1-k}}\sum_{l=1}^{k}\frac{1}{(2k-2l)!}\sum_{r=1}^{l}\frac{(\alpha\beta)^{2k-1-2l+2r}}{\left(\alpha^2+\alpha_1\right)^{2k-l+r}}\times$$

$$\times\left[\frac{r!}{4^{l-r}\,l!(2r)!}-\frac{(l-1)!}{(2k+1-2l)(2l-1)!(r-1)!}\right],\qquad W_2^{(6)}(\alpha,\alpha_1,0)=0;$$

3) $$W_3^{(6)}\left(\beta^2\right)=\frac{1}{\sqrt{\pi}}\exp\left(\beta^2\right)\sum_{k=0}^{m}\frac{(2k)!}{k!\left(4\beta^2\right)^k}.$$

2.7. Integrals of $z^n \exp\left(-\alpha z^2+\beta z\right)\operatorname{erf}(\alpha_1 z+\beta_1)$, $z^n \exp\left(-\alpha z^2\right)\operatorname{erf}(\alpha_1 z+\beta_1)\operatorname{erf}(\alpha_2 z+\beta_2)$

2.7.1.

1. $$\int_0^{+\infty} \exp\left(-az^2\right) \operatorname{erf}\left(a_1 z\right) dz = \frac{1}{\sqrt{\pi a}} \arctan\frac{a_1}{\sqrt{a}} \qquad \{a>0\}.$$

2. $$\int_0^{+\infty} \exp\left(-az^2\right) \operatorname{erfi}(a_1 z) dz = \frac{1}{2\sqrt{\pi a}} \ln\frac{\sqrt{a}+a_1}{\sqrt{a}-a_1} \qquad \{a>a_1^2\}.$$

3. $$\int_0^{+\infty} \exp\left(-az^2\right) \operatorname{erf}\left(a_1 z\right) \operatorname{erf}\left(a_2 z\right) dz = \frac{1}{2}\int_{-\infty}^{+\infty} \exp\left(-az^2\right) \operatorname{erf}\left(a_1 z\right) \operatorname{erf}\left(a_2 z\right) dz =$$
$$= \frac{1}{\sqrt{\pi a}} \arctan\frac{a_1 a_2}{\sqrt{a^2+aa_1^2+aa_2^2}} \qquad \{a>0\}.$$

4. $$\int_0^{+\infty} \exp\left(-az^2\right) \operatorname{erf}(a_1 z) \operatorname{erfi}(a_2 z) dz = \frac{1}{2}\int_{-\infty}^{+\infty} \exp\left(-az^2\right) \operatorname{erf}(a_1 z) \operatorname{erfi}(a_2 z) dz =$$
$$= \frac{1}{2\sqrt{\pi a}} \ln\frac{\sqrt{a^2+aa_1^2-aa_2^2}+a_1 a_2}{\sqrt{a^2+aa_1^2-aa_2^2}-a_1 a_2} \qquad \{a>a_2^2\}.$$

5. $$\int_0^{+\infty} \exp\left(-az^2\right) \operatorname{erfi}(a_1 z) \operatorname{erfi}(a_2 z) dz = \frac{1}{2}\int_{-\infty}^{+\infty} \exp\left(-az^2\right) \operatorname{erfi}(a_1 z) \operatorname{erfi}(a_2 z) dz =$$
$$= \frac{1}{\sqrt{\pi a}} \arctan\frac{a_1 a_2}{\sqrt{a^2-aa_1^2-aa_2^2}} \qquad \{a>a_1^2+a_2^2\}.$$

6. $$\int_{-\infty}^{+\infty} \exp\left(-az^2+\beta z\right) \operatorname{erf}(a_1 z+\beta_1) dz = \sqrt{\frac{\pi}{a}} \exp\left(\frac{\beta^2}{4a}\right) \operatorname{erf}\left(\frac{2a\beta_1+a_1\beta}{2\sqrt{a^2+aa_1^2}}\right) \qquad \{a>0\}.$$

7. $$\int_{-\infty}^{+\infty} \exp\left(-az^2+\beta z\right) \operatorname{erfi}(a_1 z+\beta_1) dz = \sqrt{\frac{\pi}{a}} \exp\left(\frac{\beta^2}{4a}\right) \operatorname{erfi}\left(\frac{2a\beta_1+a_1\beta}{2\sqrt{a^2-aa_1^2}}\right)$$
$$\{a>a_1^2\}.$$

8. $$\int_0^{\infty} \exp\left(-\alpha z^2\right) \operatorname{erf}\left(\alpha_1 z\right) dz = \frac{1}{\sqrt{\pi\alpha}} \arctan\frac{\alpha_1}{\sqrt{\alpha}} \qquad \{\alpha_1^2 \ne -\alpha,\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2\right)+\ln|z|\right]=$$
$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2\right)+2\ln|z|\right]=+\infty\}.$$

9. $\int\limits_{0}^{\infty} \exp\left(-\alpha z^2\right) \operatorname{erf}(\alpha_1 z) \operatorname{erf}(\alpha_2 z)\, dz = \pm\frac{1}{\sqrt{\pi\alpha}} \arctan\frac{\alpha_1\alpha_2}{\sqrt{\alpha}\sqrt{\alpha+\alpha_1^2+\alpha_2^2}}$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2\right)+\ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2\right)+2\ln|z|\right] =$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_2^2 z^2\right)+2\ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2+\alpha_2^2 z^2\right)+3\ln|z|\right] = +\infty,$$

$$\alpha_1^2 \neq -\alpha,\ \alpha_2^2 \neq -\alpha,\quad \lim_{z\to\infty}\left[\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right)\right] = \pm\infty \ \}.$$

10. $\int\limits_{\infty(T_1)}^{\infty(T_2)} \exp\left(-\alpha z^2+\beta z\right) \operatorname{erf}(\alpha_1 z+\beta_1)\, dz = A\frac{\sqrt{\pi}}{\sqrt{\alpha}}\exp\left(\frac{\beta^2}{4\alpha}\right)\operatorname{erf}\left(\frac{2\alpha\beta_1+\alpha_1\beta}{2\sqrt{\alpha}\sqrt{\alpha+\alpha_1^2}}\right)$

$$\{ \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2-\beta z\right)+\ln|z|\right] = \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)+2\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2-\beta z\right)+\ln|z|\right] = \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)+2\ln|z|\right] = +\infty;$$

$$A = \pm 1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = -\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = \pm\infty,$$

$$A = 0 \text{ if } \lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right)\cdot \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = +\infty \ \}.$$

11. $\int\limits_{-\nu}^{\nu} \exp\left(-\alpha z^2\right) \operatorname{erf}(\alpha_1 z)\, dz = 0.$

2.7.2.

1. $\int\limits_{0}^{+\infty} z\exp\left(-a z^2\right) \operatorname{erf}(a_1 z) \operatorname{erf}(a_2 z)\, dz = \frac{a_1}{\pi a\sqrt{a+a_1^2}}\arctan\frac{a_2}{\sqrt{a+a_1^2}} +$

$$+\frac{a_2}{\pi a\sqrt{a+a_2^2}}\arctan\frac{a_1}{\sqrt{a+a_2^2}} \qquad \{a>0\}.$$

2. $\int\limits_{0}^{+\infty} z\exp\left(-a z^2\right) \operatorname{erf}(a_1 z)\operatorname{erfi}(a_2 z)\, dz = \frac{1}{\pi a}\left(\frac{a_1}{2\sqrt{a+a_1^2}}\ln\frac{\sqrt{a+a_1^2}+a_2}{\sqrt{a+a_1^2}-a_2} +\right.$

$$\left.+\frac{a_2}{\sqrt{a-a_2^2}}\arctan\frac{a_1}{\sqrt{a-a_2^2}}\right) \ \{ a > a_2^2 \}.$$

3. $$\int_0^{+\infty} z\exp\left(-az^2\right)\operatorname{erfi}(a_1 z)\operatorname{erfi}(a_2 z)dz = \frac{1}{2\pi a}\left(\frac{a_1}{\sqrt{a-a_1^2}}\ln\frac{\sqrt{a-a_1^2}+a_2}{\sqrt{a-a_1^2}-a_2}+\right.$$

$$\left.+\frac{a_2}{\sqrt{a-a_2^2}}\ln\frac{\sqrt{a-a_2^2}+a_1}{\sqrt{a-a_2^2}-a_1}\right) \qquad \{\ a>a_1^2+a_2^2\}.$$

4. $$\int_{-\infty}^{+\infty} z\exp\left(-az^2+\beta z\right)\operatorname{erf}(a_1 z+\beta_1)dz = \frac{\sqrt{\pi}\,\beta}{2a\sqrt{a}}\exp\left(\frac{\beta^2}{4a}\right)\operatorname{erf}\left(\frac{2a\beta_1+a_1\beta}{2\sqrt{a^2+aa_1^2}}\right)+$$

$$+\frac{a_1}{a\sqrt{a+a_1^2}}\exp\left(\frac{\beta^2-4a\beta_1^2-4a_1\beta\beta_1}{4a+4a_1^2}\right) \qquad \{a>0\}.$$

5. $$\int_{-\infty}^{+\infty} z\exp\left(-az^2+\beta z\right)\operatorname{erfi}(a_1 z+\beta_1)dz = \frac{\sqrt{\pi}\beta}{2a\sqrt{a}}\exp\left(\frac{\beta^2}{4a}\right)\operatorname{erfi}\left(\frac{2a\beta_1+a_1\beta}{2\sqrt{a^2-aa_1^2}}\right)+$$

$$+\frac{a_1}{a\sqrt{a-a_1^2}}\exp\left(\frac{\beta^2+4a\beta_1^2+4a_1\beta\beta_1}{4a-4a_1^2}\right) \qquad \{\ a>a_1^2\}.$$

6. $$\int_{-\infty}^{+\infty} z\exp\left(-az^2\right)\operatorname{erf}(a_1 z+\beta_1)\operatorname{erf}(a_2 z+\beta_2)dz = \frac{a_1}{a\sqrt{a+a_1^2}}\exp\left(-\frac{a\beta_1^2}{a+a_1^2}\right)\times$$

$$\times\operatorname{erf}\left(\frac{a\beta_2+a_1^2\beta_2-a_1a_2\beta_1}{\sqrt{a+a_1^2}\sqrt{a+a_1^2+a_2^2}}\right)+\frac{a_2}{a\sqrt{a+a_2^2}}\exp\left(-\frac{a\beta_2^2}{a+a_2^2}\right)\operatorname{erf}\left(\frac{a\beta_1+a_2^2\beta_1-a_1a_2\beta_2}{\sqrt{a+a_2^2}\sqrt{a+a_1^2+a_2^2}}\right)$$

$$\{a>0\}.$$

7. $$\int_{-\infty}^{+\infty} z\exp\left(-az^2\right)\operatorname{erf}(a_1 z+\beta_1)\operatorname{erfi}(a_2 z+\beta_2)\,dz = \frac{a_1}{a\sqrt{a+a_1^2}}\exp\left(-\frac{a\beta_1^2}{a+a_1^2}\right)\times$$

$$\times\operatorname{erfi}\left(\frac{a\beta_2+a_1^2\beta_2-a_1a_2\beta_1}{\sqrt{a+a_1^2}\sqrt{a+a_1^2-a_2^2}}\right)+\frac{a_2}{a\sqrt{a-a_2^2}}\exp\left(\frac{a\beta_2^2}{a-a_2^2}\right)\operatorname{erf}\left(\frac{a\beta_1-a_2^2\beta_1+a_1a_2\beta_2}{\sqrt{a-a_2^2}\sqrt{a+a_1^2-a_2^2}}\right)$$

$$\{\ a>a_2^2\}.$$

8. $$\int_{-\infty}^{+\infty} z\exp\left(-az^2\right)\operatorname{erfi}(a_1 z+\beta_1)\operatorname{erfi}(a_2 z+\beta_2)dz = \frac{a_1}{a\sqrt{a-a_1^2}}\exp\left(\frac{a\beta_1^2}{a-a_1^2}\right)\times$$

$$\times \operatorname{erfi}\left(\frac{a\beta_2 - a_1^2\beta_2 + a_1a_2\beta_1}{\sqrt{a-a_1^2}\sqrt{a-a_1^2-a_2^2}}\right) + \frac{a_2}{a\sqrt{a-a_2^2}}\exp\left(\frac{a\beta_2^2}{a-a_2^2}\right)\operatorname{erfi}\left(\frac{a\beta_1 - a_2^2\beta_1 + a_1a_2\beta_2}{\sqrt{a-a_2^2}\sqrt{a-a_1^2-a_2^2}}\right)$$

$$\{ a > a_1^2 + a_2^2 \}.$$

9. $$\int_0^\infty z\exp\left(-\alpha z^2\right)\operatorname{erf}\left(\alpha_1 z\right)\operatorname{erf}\left(\alpha_2 z\right)dz = \frac{1}{\pi\alpha}\left(\frac{\alpha_1}{\sqrt{\alpha+\alpha_1^2}}\operatorname{arctg}\frac{\alpha_2}{\sqrt{\alpha+\alpha_1^2}} + \right.$$

$$\left. + \frac{\alpha_2}{\sqrt{\alpha+\alpha_2^2}}\operatorname{arctg}\frac{\alpha_1}{\sqrt{\alpha+\alpha_2^2}}\right) \{ \lim_{z\to\infty}\operatorname{Re}\left(\alpha z^2\right) = \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z^2 + \alpha_1^2 z^2) + \ln|z|\right] =$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_2^2 z^2\right) + \ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + \alpha_2^2 z^2\right) + 2\ln|z|\right] = +\infty,$$

$$\alpha_1^2 \neq -\alpha,\ \alpha_2^2 \neq -\alpha, \alpha_1^2 + \alpha_2^2 \neq -\alpha \}.$$

10. $$\int_{\infty(T_1)}^{\infty(T_2)} z\exp\left(-\alpha z^2 + \beta z\right)\operatorname{erf}\left(\alpha_1 z + \beta_1\right)dz = A\frac{\sqrt{\pi}\beta}{2\alpha\sqrt{\alpha}}\exp\left(\frac{\beta^2}{4\alpha}\right)\times$$

$$\times \operatorname{erf}\left(\frac{2\alpha\beta_1 + \alpha_1\beta}{2\sqrt{\alpha}\sqrt{\alpha+\alpha_1^2}}\right) + A\frac{\alpha_1}{\alpha\sqrt{\alpha+\alpha_1^2}}\exp\left(\frac{\beta^2 - 4\alpha\beta_1^2 - 4\alpha_1\beta\beta_1}{4\alpha + 4\alpha_1^2}\right)$$

$$\{ \lim_{z\to\infty(T_1)}\operatorname{Re}(\alpha z^2 - \beta z) = \lim_{z\to\infty(T_2)}\operatorname{Re}(\alpha z^2 - \beta z) =$$

$$= \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z - \beta z\right) + \ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z - \beta z) + \ln|z|\right] = +\infty;\ A = \pm 1 \text{ if}$$

$$\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = -\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = \pm\infty,$$

$$A = 0 \text{ if } \lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right)\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2}\ z\right) = +\infty \}.$$

11. $$\int_{\infty(T_1)}^{\infty(T_2)} z\exp\left(-\alpha z^2\right)\operatorname{erf}\left(\alpha_1 z + \beta_1\right)\operatorname{erf}\left(\alpha_2 z + \beta_2\right)dz = A\frac{\alpha_1}{\alpha\sqrt{\alpha+\alpha_1^2}}\exp\left(-\frac{\alpha\beta_1^2}{\alpha+\alpha_1^2}\right)\times$$

$$\times \operatorname{erf}\left(\frac{\alpha\beta_2 + \alpha_1^2\beta_2 - \alpha_1\alpha_2\beta_1}{\sqrt{\alpha+\alpha_1^2}\sqrt{\alpha+\alpha_1^2+\alpha_2^2}}\right) + A\frac{\alpha_2}{\alpha\sqrt{\alpha+\alpha_2^2}}\exp\left(-\frac{\alpha\beta_2^2}{\alpha+\alpha_2^2}\right)\times$$

$$\times \operatorname{erf}\left(\frac{\alpha\beta_1+\alpha_2^2\beta_1-\alpha_1\alpha_2\beta_2}{\sqrt{\alpha+\alpha_2^2}\sqrt{\alpha+\alpha_1^2+\alpha_2^2}}\right)\ \{\alpha_1^2\neq-\alpha,\ \alpha_2^2\neq-\alpha,$$

$$\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\alpha z^2\right)=\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\alpha z^2\right)=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2z^2+2\alpha_1\beta_1 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2z^2+2\alpha_1\beta_1 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_2^2z^2+2\alpha_2\beta_2 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_2^2z^2+2\alpha_2\beta_2 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2z^2+\alpha_2^2z^2+2\alpha_1\beta_1 z+2\alpha_2\beta_2 z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2z^2+\alpha_2^2z^2+2\alpha_1\beta_1 z+2\alpha_2\beta_2 z\right)+2\ln|z|\right]=+\infty;$$

$$A=\pm1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right)=\pm\infty,$$

$$A=0 \text{ if } \lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right)\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right)=+\infty\ \}.$$

12. $\int\limits_{-\nu}^{\nu} z\exp\left(-\alpha z^2\right)\operatorname{erf}(\alpha_1 z)\operatorname{erf}(\alpha_2 z)dz=0$.

2.7.3.

1. $\int\limits_0^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erf}(a_1z)dz=\frac{(2m)!}{m!\sqrt{\pi}}W_1^{(7)}(m,a,a_1)$ $\quad\{a>0\}$.

2. $\int\limits_0^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erfi}(a_1z)dz=\frac{(2m)!}{m!\sqrt{\pi}}W_2^{(7)}(m,a,a_1)$ $\quad\{a>a_1^2\}$.

3. $\int\limits_0^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erf}(a_1z)\operatorname{erf}(a_2z)dz=\frac{1}{2}\int\limits_{-\infty}^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erf}(a_1z)\operatorname{erf}(a_2z)dz=$

$$=\frac{(-1)^m}{\sqrt{\pi}}\cdot\frac{\partial^m}{\partial a^m}\left(\frac{1}{\sqrt{a}}\operatorname{arctg}\frac{a_1a_2}{\sqrt{a^2+aa_1^2+aa_2^2}}\right)\quad\{a>0\}.$$

4. $\int_0^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erf}(a_1 z)\operatorname{erfi}(a_2 z)dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erf}(a_1 z)\operatorname{erfi}(a_2 z)dz =$

$$= \frac{(-1)^m}{2\sqrt{\pi}}\cdot\frac{\partial^m}{\partial a^m}\left(\frac{1}{\sqrt{a}}\ln\frac{\sqrt{a^2+aa_1^2-aa_2^2}+a_1a_2}{\sqrt{a^2+aa_1^2-aa_2^2}-a_1a_2}\right) \qquad \{a > a_2^2\}.$$

5. $\int_0^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erfi}(a_1 z)\operatorname{erfi}(a_2 z)dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2m}\exp\left(-az^2\right)\operatorname{erfi}(a_1 z)\operatorname{erfi}(a_2 z)dz =$

$$= \frac{(-1)^m}{\sqrt{\pi}}\cdot\frac{\partial^m}{\partial a^m}\left(\frac{1}{\sqrt{a}}\operatorname{arctg}\frac{a_1a_2}{\sqrt{a^2-aa_1^2-aa_2^2}}\right) \quad \{a > a_1^2+a_2^2\}.$$

6. $\int_0^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erf}(a_1 z)\operatorname{erf}(a_2 z)\,dz = \frac{m!}{\pi}\sum_{k=0}^{m}\frac{(2k)!}{(k!)^2\,a^{m+1-k}}\times$

$$\times\left[a_1W_1^{(7)}\left(k,a+a_1^2,a_2\right)+a_2W_1^{(7)}\left(k,a+a_2^2,a_1\right)\right] \qquad \{a>0\}.$$

7. $\int_0^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erf}(a_1 z)\operatorname{erfi}(a_2 z)\,dz = \frac{m!}{\pi}\sum_{k=0}^{m}\frac{(2k)!}{(k!)^2\,a^{m+1-k}}\times$

$$\times\left[a_1W_2^{(7)}\left(k,a+a_1^2,a_2\right)+a_2W_1^{(7)}\left(k,a-a_2^2,a_1\right)\right] \qquad \{a>a_2^2\}.$$

8. $\int_0^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erfi}(a_1 z)\operatorname{erfi}(a_2 z)\,dz = \frac{m!}{\pi}\sum_{k=0}^{m}\frac{(2k)!}{(k!)^2\,a^{m+1-k}}\times$

$$\times\left[a_1W_2^{(7)}\left(k,a-a_1^2,a_2\right)+a_2W_2^{(7)}\left(k,a-a_2^2,a_1\right)\right] \quad \{a>a_1^2+a_2^2\}.$$

9. $\int_{-\infty}^{+\infty} z^n\exp\left(-az^2+\beta z\right)\operatorname{erf}(a_1 z+\beta_1)dz = \frac{n!}{2^n}W_3^{(7)}(n,a,a_1,\beta,\beta_1) \qquad \{a>0\}.$

10. $\int_{-\infty}^{+\infty} z^n\exp\left(-az^2+\beta z\right)\operatorname{erfi}(a_1 z+\beta_1)dz = \frac{n!}{2^n}W_4^{(7)}(n,a,a_1,\beta,\beta_1) \qquad \{a>a_1^2\}.$

11. $\int_{-\infty}^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erf}(a_1 z+\beta_1)\operatorname{erf}(a_2 z+\beta_2)dz = \frac{m!}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(2k)!}{4^k\,k!\,a^{m+1-k}}\times$

$$\times\Big[a_1\exp\left(-\beta_1^2\right)W_3^{(7)}\left(2k,a+a_1^2,a_2,-2a_1\beta_1,\beta_2\right)+$$

$$+a_2\exp\left(-\beta_2^2\right)W_3^{(7)}\left(2k,a+a_2^2,a_1,-2a_2\beta_2,\beta_1\right)\Big] \{a>0\}.$$

12. $$\int_{-\infty}^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erf}\left(a_1 z+\beta_1\right)\operatorname{erfi}\left(a_2 z+\beta_2\right)dz=\frac{m!}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(2k)!}{4^k\,k!\,a^{m+1-k}}\times$$

$$\times\left[a_1\exp\left(-\beta_1^2\right)W_4^{(7)}\left(2k,\,a+a_1^2,\,a_2,\,-2a_1\beta_1,\,\beta_2\right)+\right.$$

$$\left.+a_2\exp\left(\beta_2^2\right)W_3^{(7)}\left(2k,\,a-a_2^2,\,a_1,\,2a_2\beta_2,\,\beta_1\right)\right]\ \{\,a>a_2^2\,\}.$$

13. $$\int_{-\infty}^{+\infty} z^{2m+1}\exp\left(-az^2\right)\operatorname{erfi}\left(a_1 z+\beta_1\right)\operatorname{erfi}\left(a_2 z+\beta_2\right)dz=\frac{m!}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(2k)!}{4^k\,k!\,a^{m+1-k}}\times$$

$$\times\left[a_1\exp\left(\beta_1^2\right)W_4^{(7)}\left(2k,\,a-a_1^2,\,a_2,\,2a_1\beta_1,\,\beta_2\right)+\right.$$

$$\left.+a_2\exp\left(\beta_2^2\right)W_4^{(7)}\left(2k,\,a-a_2^2,\,a_1,\,2a_2\beta_2,\,\beta_1\right)\right]\{\ a>a_1^2+a_2^2\,\}.$$

14. $$\int_{0}^{\infty} z^{2m}\exp\left(-\alpha z^2\right)\operatorname{erf}\left(\alpha_1 z\right)dz=\frac{(2m)!}{m!\sqrt{\pi}}W_1^{(7)}\left(m,\alpha,\alpha_1\right)\qquad\{\,\alpha\neq 0,\,\alpha_1^2\neq-\alpha,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2\right)-(2m-1)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2\right)-(2m-2)\ln|z|\right]=+\infty\,\}.$$

15. $$\int_{0}^{\infty} z^{2m}\exp\left(-\alpha z^2\right)\operatorname{erf}\left(\alpha_1 z\right)\operatorname{erf}\left(\alpha_2 z\right)dz=\frac{A}{\sqrt{\pi}}\cdot\frac{\partial^m}{\partial\alpha^m}\left(\frac{1}{\sqrt{\alpha}}\arctan\frac{\alpha_1\alpha_2}{\sqrt{\alpha}\sqrt{\alpha+\alpha_1^2+\alpha_2^2}}\right)$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2\right)-(2m-1)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2\right)-(2m-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_2^2 z^2\right)-(2m-2)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2+\alpha_2^2 z^2\right)-(2m-3)\ln|z|\right]=+\infty,$$

$$\alpha_1^2\neq-\alpha,\,\alpha_2^2\neq-\alpha;\ A=(-1)^m\ \text{if}\ \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\,z\right)=+\infty,$$

$$A=(-1)^{m+1}\ \text{if}\ \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\,z\right)=-\infty\,\}.$$

16. $$\int_{0}^{\infty} z^{2m+1}\exp\left(-\alpha z^2\right)\operatorname{erf}\left(\alpha_1 z\right)\operatorname{erf}\left(\alpha_2 z\right)dz=\frac{m!}{\pi}\sum_{k=0}^{m}\frac{(2k)!}{(k!)^2\,\alpha^{m+1-k}}\times$$

$$\times\left[\alpha_1 W_1^{(7)}\left(k,\,\alpha+\alpha_1^2,\,\alpha_2\right)+\alpha_2 W_1^{(7)}\left(k,\,\alpha+\alpha_2^2,\,\alpha_1\right)\right]$$

$$\{\,\alpha_1^2\neq-\alpha,\,\alpha_2^2\neq-\alpha,\,\alpha_1^2+\alpha_2^2\neq-\alpha,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}(\alpha z^2)-2m\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_1^2 z^2\right)-(2m-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2+\alpha_2^2 z^2\right)-(2m-1)\ln|z|\right]=$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + \alpha_2^2 z^2\right) - (2m-2)\ln|z|\right] = +\infty \}.$$

17. $$\int_{\infty(T_1)}^{\infty(T_2)} z^n \exp\left(-\alpha z^2 + \beta z\right) \operatorname{erf}(\alpha_1 z + \beta_1) dz = A\frac{n!}{2^n} W_3^{(7)}(n, \alpha, \alpha_1, \beta, \beta_1)$$

$$\{\alpha \neq 0, \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 - \beta z\right) - (n-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z - \beta z\right) - (n-2)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2 - \beta z\right) - (n-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z - \beta z\right) - (n-2)\ln|z|\right] = +\infty \, ;$$

$$A = \pm 1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha + \alpha_1^2}\ z\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha + \alpha_1^2}\ z\right) = \pm\infty,$$

$$A = 0 \text{ if } \lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha + \alpha_1^2}\ z\right) \cdot \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha + \alpha_1^2}\ z\right) = +\infty \}.$$

18. $$\int_{\infty(T_1)}^{\infty(T_2)} z^{2m+1} \exp\left(-\alpha z^2\right) \operatorname{erf}(\alpha_1 z + \beta_1) \operatorname{erf}(\alpha_2 z + \beta_2) dz = A\frac{m!}{\sqrt{\pi}} \sum_{k=0}^{m} \frac{(2k)!}{4^k\, k!\, \alpha^{m+1-k}} \times$$

$$\times\left[\alpha_1 \exp\left(-\beta_1^2\right) W_3^{(7)}\left(2k, \alpha + \alpha_1^2, \alpha_2, -2\alpha_1\beta_1, \beta_2\right) + \right.$$

$$\left. + \alpha_2 \exp\left(-\beta_2^2\right) W_3^{(7)}\left(2k, \alpha + \alpha_2^2, \alpha_1, -2\alpha_2\beta_2, \beta_1\right)\right]$$

$$\{ \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2\right) - 2m\ln|z|\right] = \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2\right) - 2m\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z\right) - (2m-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + 2\alpha_1\beta_1 z\right) - (2m-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_2^2 z^2 + 2\alpha_2\beta_2 z\right) - (2m-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_2^2 z^2 + 2\alpha_2\beta_2 z\right) - (2m-1)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + \alpha_2^2 z^2 + 2\alpha_1\beta_1 z + 2\alpha_2\beta_2 z\right) - (2m-2)\ln|z|\right] =$$

$$= \lim_{z\to\infty(T_2)} \left[\mathrm{Re}\left(\alpha z^2 + \alpha_1^2 z^2 + \alpha_2^2 z^2 + 2\alpha_1\beta_1 z + 2\alpha_2\beta_2 z\right) - (2m-2)\ln|z| \right] = +\infty,$$

$$\alpha_1^2 \neq -\alpha,\ \alpha_2^2 \neq -\alpha;$$

$$A = \pm 1 \text{ if } \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right) = -\lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right) = \pm\infty,$$

$$A = 0 \text{ if } \lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right) \cdot \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha+\alpha_1^2+\alpha_2^2}\ z\right) = +\infty \}.$$

19. $\int_{-\nu}^{\nu} z^{2m} \exp\left(-\alpha z^2\right) \operatorname{erf}(\alpha_1 z)dz = 0.$

20. $\int_{-\nu}^{\nu} z^{2m+1} \exp\left(-\alpha z^2\right) \operatorname{erf}(\alpha_1 z)\operatorname{erf}(\alpha_2 z)dz = 0.$

Introduced notations:

1) $W_1^{(7)}(m,\alpha,\alpha_1) = \dfrac{1}{4^m \alpha^m \sqrt{\alpha}} \operatorname{arctg}\dfrac{\alpha_1}{\sqrt{\alpha}} + \alpha_1 \sum_{l=0}^{m-1} \dfrac{(l!)^2}{(2l+1)!(4\alpha)^{m-l}\left(\alpha+\alpha_1^2\right)^{l+1}};$

2) $W_2^{(7)}(m,\alpha,\alpha_1) = \dfrac{1}{2^{2m+1} \alpha^m \sqrt{\alpha}} \ln\dfrac{\sqrt{\alpha}+\alpha_1}{\sqrt{\alpha}-\alpha_1} + \alpha_1 \sum_{l=0}^{m-1} \dfrac{(l!)^2}{(2l+1)!(4\alpha)^{m-l}\left(\alpha-\alpha_1^2\right)^{l+1}};$

3) $W_3^{(7)}(n,\alpha,\alpha_1,\beta,\beta_1) = \dfrac{\sqrt{\pi}}{\sqrt{\alpha}} \exp\left(\dfrac{\beta^2}{4\alpha}\right) \operatorname{erf}\left(\dfrac{2\alpha\beta_1+\alpha_1\beta}{2\sqrt{\alpha}\sqrt{\alpha+\alpha_1^2}}\right) \sum_{l=0}^{E(n/2)} \dfrac{\beta^{n-2l}}{l!(n-2l)!\alpha^{n-l}} +$

$$+\frac{1}{\sqrt{\alpha+\alpha_1^2}} \exp\left(\frac{\beta^2 - 4\alpha\beta_1^2 - 4\alpha_1\beta\beta_1}{4\alpha+4\alpha_1^2}\right) \left[\sum_{l=1}^{n-E(n/2)} \frac{l!\beta^{n+1-2l}}{(2l)!(n+1-2l)!} \sum_{r=1}^{l} \frac{4^{l+1-r}(2r-2)!}{(r-1)!} \times \right.$$

$$\times \sum_{q=0}^{r-1} \frac{\alpha_1^{2r-1-2q}\left(2\alpha\beta_1+\alpha_1\beta\right)^{2r-2-2q}}{q!(2r-2-2q)!\alpha^{n-l+r-2q}\left(\alpha+\alpha_1^2\right)^{2r-2-q}} - \sum_{l=1}^{E(n/2)} \frac{\beta^{n-2l}}{l!(n-2l)!} \sum_{r=1}^{l} (r-1)! \times$$

$$\left. \times \sum_{q=0}^{r-1} \frac{\alpha_1^{2r-2q}\left(2\alpha\beta_1+\alpha_1\beta\right)^{2r-1-2q}}{q!(2r-1-2q)!\alpha^{n-l+r-2q}\left(\alpha+\alpha_1^2\right)^{2r-1-q}} \right],$$

$$W_3^{(7)}(2m,\alpha,\alpha_1,0,\beta_1) = \frac{\sqrt{\pi}}{m!\alpha^m\sqrt{\alpha}} \operatorname{erf}\left(\frac{\sqrt{\alpha}\beta_1}{\sqrt{\alpha+\alpha_1^2}}\right) -$$

$$-\frac{1}{m!\sqrt{\alpha+\alpha_1^2}}\exp\left(-\frac{\alpha\beta_1^2}{\alpha+\alpha_1^2}\right)\sum_{l=0}^{m-1}\frac{l!}{\alpha^{m-l}}\sum_{r=0}^{l}\frac{\alpha_1^{2l+2-2r}\left(2\beta_1\right)^{2r+1-2r}}{r!(2l+1-2r)!\left(\alpha+\alpha_1^2\right)^{2l+1-r}},$$

$$W_3^{(7)}(2m,\alpha,\alpha_1,0,0)=0,$$

$$W_3^{(7)}(2m+1,\alpha,\alpha_1,0,\beta_1)=\frac{(m+1)!\alpha_1}{(2m+2)!\sqrt{\alpha+\alpha_1^2}}\exp\left(-\frac{\alpha\beta_1^2}{\alpha+\alpha_1^2}\right)\times$$

$$\times\sum_{l=0}^{m}\frac{(2l)!}{l!\alpha^{m+1-l}}\sum_{r=0}^{l}\frac{4^{m+1-r}\left(\alpha_1\beta_1\right)^{2l-2r}}{r!(2l-2r)!\left(\alpha+\alpha_1^2\right)^{2l-r}},$$

$$W_3^{(7)}(2m+1,\alpha,\alpha_1,0,0)=\frac{(m+1)!\alpha_1}{(2m+2)!\sqrt{\alpha+\alpha_1^2}}\sum_{l=0}^{m}\frac{4^{m+1-l}(2l)!}{(l!)^2\alpha^{m+1-l}\left(\alpha+\alpha_1^2\right)^{l}};$$

4) $$W_4^{(7)}(n,\alpha,\alpha_1,\beta,\beta_1)=\frac{\sqrt{\pi}}{\sqrt{\alpha}}\exp\left(\frac{\beta^2}{4\alpha}\right)\mathrm{erfi}\left(\frac{2\alpha\beta_1+\alpha_1\beta}{2\sqrt{\alpha}\sqrt{\alpha-\alpha_1^2}}\right)\sum_{l=0}^{E(n/2)}\frac{\beta^{n-2l}}{l!(n-2l)!\alpha^{n-l}}+$$

$$+\frac{1}{\sqrt{\alpha-\alpha_1^2}}\exp\left(\frac{\beta^2+4\alpha\beta_1^2+4\alpha_1\beta\beta_1}{4\alpha-4\alpha_1^2}\right)\left[\sum_{l=1}^{n-E(n/2)}\frac{l!\beta^{n+1-2l}}{(2l)!(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l+1-r}(2r-2)!}{(r-1)!}\times\right.$$

$$\times\sum_{q=0}^{r-1}\frac{\alpha_1^{2r-1-2q}\left(2\alpha\beta_1+\alpha_1\beta\right)^{2r-2-2q}}{q!(2r-2-2q)!\alpha^{n-l+r-2q}\left(\alpha-\alpha_1^2\right)^{2r-2-q}}+\sum_{l=1}^{E(n/2)}\frac{\beta^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}(r-1)!\times$$

$$\left.\times\sum_{q=0}^{r-1}\frac{\alpha_1^{2r-2q}\left(2\alpha\beta_1+\alpha_1\beta\right)^{2r-1-2q}}{q!(2r-1-2q)!\alpha^{n-l+r-2q}\left(\alpha-\alpha_1^2\right)^{2r-1-q}}\right],$$

$$W_4^{(7)}(2m,\alpha,\alpha_1,0,\beta_1)=\frac{\sqrt{\pi}}{m!\alpha^m\sqrt{\alpha}}\mathrm{erfi}\left(\frac{\sqrt{\alpha}\beta_1}{\sqrt{\alpha-\alpha_1^2}}\right)+$$

$$+\frac{1}{m!\sqrt{\alpha-\alpha_1^2}}\exp\left(\frac{\alpha\beta_1^2}{\alpha-\alpha_1^2}\right)\sum_{l=0}^{m-1}\frac{l!}{\alpha^{m-l}}\sum_{r=0}^{l}\frac{\alpha_1^{2l+2-2r}\left(2\beta_1\right)^{2l+1-2r}}{r!(2l+1-2r)!\left(\alpha-\alpha_1^2\right)^{2l+1-r}},$$

$$W_4^{(7)}(2m,\alpha,\alpha_1,0,0)=0,$$

$$W_4^{(7)}(2m+1,\alpha,\alpha_1,0,\beta_1)=\frac{(m+1)!\alpha_1}{(2m+2)!\left(\alpha-\alpha_1^2\right)}\exp\left(\frac{\alpha\beta_1^2}{\alpha-\alpha_1^2}\right)\times$$

$$\times\sum_{l=0}^{m}\frac{(2l)!}{l!\alpha^{m+1-l}}\sum_{r=0}^{l}\frac{4^{m+1-r}\left(\alpha_1\beta_1\right)^{2l-2r}}{r!(2l-2r)!\left(\alpha-\alpha_1^2\right)^{2l-r}},$$

$$W_4^{(7)}(2m+1,\alpha,\alpha_1,0,0)=\frac{(m+1)!\alpha_1}{(2m+2)!\sqrt{\alpha-\alpha_1^2}}\sum_{l=0}^{m}\frac{4^{m+1-l}(2l)!}{(l!)^2\alpha^{m+1-l}\left(\alpha-\alpha_1^2\right)^l}.$$

2.8. Integrals of $\sin^{2m+1}\left(\alpha^2 z^2+\beta z+\gamma\right)$, $\sinh^{2m+1}\left(\alpha^2 z^2+\beta z+\gamma\right)$, $\cos^{2m+1}\left(\alpha^2 z^2+\beta z+\gamma\right)$, $\cosh^{2m+1}\left(\alpha^2 z^2+\beta z+\gamma\right)$

2.8.1.

1. $$\int_0^{+\infty}\sin\left(a^2z^2+bz+\gamma\right)dz=\frac{\sqrt{2\pi}}{4a}\left\{\left[\sin\left(\gamma-\frac{b^2}{4a^2}\right)+\cos\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$
$$\left.\times\left[1-\operatorname{Re}\,\operatorname{erf}\left(\frac{i+1}{\sqrt2}\cdot\frac{b}{2a}\right)\right]+\left[\cos\left(\gamma-\frac{b^2}{4a^2}\right)-\sin\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}\,\operatorname{erf}\left(\frac{i+1}{\sqrt2}\cdot\frac{b}{2a}\right)\right\}.$$

2. $$\int_{-\infty}^{+\infty}\sin\left(a^2z^2+bz+\gamma\right)dz=2\int_{-b/\left(2a^2\right)}^{+\infty}\sin\left(a^2z^2+bz+\gamma\right)dz=$$
$$=2\int_{-\infty}^{b/\left(2a^2\right)}\sin\left(a^2z^2-bz+\gamma\right)dz=\frac{\sqrt\pi}{\sqrt2\,a}\left[\cos\left(\frac{b^2}{4a^2}-\gamma\right)-\sin\left(\frac{b^2}{4a^2}-\gamma\right)\right].$$

3. $$\int_0^{\infty}\sin\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(i-1)\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$
$$\left.\times\left[\operatorname{erf}\left(\frac{1-i}{\sqrt2}\cdot\frac{\beta}{2\alpha}\right)-A_1\right]-(i+1)\exp\left[i\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\left[\operatorname{erf}\left(\frac{1+i}{\sqrt2}\cdot\frac{\beta}{2\alpha}\right)-A_2\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|-\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty;\quad A_l=\pm1$$
$$\text{if }\lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(3-2l)\operatorname{Im}(\alpha z)\right]=\pm\infty\}.$$

4. $$\int_0^{\infty}\sinh\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt\pi}{4\alpha}\left\{\exp\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\left[\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)-A_1\right]+\right.$$
$$\left.+\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\left[A_2-\operatorname{erfi}\left(\frac{\beta}{2\alpha}\right)\right]\right\}\quad\{\lim_{z\to\infty}\left[\ln|z|-\left|\operatorname{Re}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty;$$

$$A_1=1\text{ if }\lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty,\quad A_1=-1\text{ if }\lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty,$$

$$A_2=\pm i \text{ if } \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\}.$$

2.8.2.

1. $$\int_0^{+\infty}\cos\left(a^2z^2+bz+\gamma\right)dz=\frac{\sqrt{2\pi}}{4a}\left\{\left[\cos\left(\gamma-\frac{b^2}{4a^2}\right)-\sin\left(\gamma-\frac{b^2}{4a^2}\right)\right]\times\right.$$

$$\left.\times\left[1-\operatorname{Re}\ \operatorname{erf}\left(\frac{i+1}{\sqrt{2}}\cdot\frac{b}{2a}\right)\right]-\left[\cos\left(\gamma-\frac{b^2}{4a^2}\right)+\sin\left(\gamma-\frac{b^2}{4a^2}\right)\right]\operatorname{Im}\ \operatorname{erf}\left(\frac{i+1}{\sqrt{2}}\cdot\frac{b}{2a}\right)\right\}.$$

2. $$\int_{-\infty}^{+\infty}\cos\left(a^2z^2+bz+\gamma\right)dz=2\int_{-b/(2a^2)}^{+\infty}\cos\left(a^2z^2+bz+\gamma\right)dz=$$

$$=2\int_{-\infty}^{b/(2a^2)}\cos\left(a^2z^2-bz+\gamma\right)dz=\frac{\sqrt{\pi}}{\sqrt{2}\,a}\left[\sin\left(\frac{b^2}{4a^2}-\gamma\right)+\cos\left(\frac{b^2}{4a^2}-\gamma\right)\right].$$

3. $$\int_0^{\infty}\cos\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(1+i)\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\left[A_1-\operatorname{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\beta}{2\alpha}\right)\right]+\right.$$

$$\left.+(1-i)\exp\left[i\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\left[A_2-\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\beta}{2\alpha}\right)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|-\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty;$$

$$A_l=\pm1 \text{ if } \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(3-2l)\operatorname{Im}(\alpha z)\right]=\pm\infty\}.$$

4. $$\int_0^{\infty}\cosh\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{\sqrt{\pi}}{4\alpha}\left\{\exp\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\left[A_1-\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)\right]+\right.$$

$$\left.+\exp\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\left[A_2-\operatorname{erfi}\left(\frac{\beta}{2\alpha}\right)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|-\left|\operatorname{Re}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty;\ A_1=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty,$$

$$A_1=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty,\ A_2=\pm i \text{ if } \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\}.$$

2.8.3.

1. $$\int_0^{+\infty}\sin^{2m+1}\left(a^2z^2+bz+\gamma\right)dz=\frac{(2m+1)!\sqrt{2\pi}}{4^{m+1}a}\times$$

$$\times\sum_{k=0}^{m}\frac{(-1)^k}{(m-k)!\,(m+1+k)!\,\sqrt{2k+1}}\left\{\sin\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\times\right.$$

$$\times\left[\operatorname{Re}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)+\operatorname{Im}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)-1\right]+$$

$$+\cos\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\left[\operatorname{Im}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)-\right.$$

$$\left.\left.-\operatorname{Re}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)+1\right]\right\}.$$

2. $$\int_{-\infty}^{+\infty}\sin^{2m+1}\left(a^2z^2+bz+\gamma\right)dz=2\int_{-b/(2a^2)}^{+\infty}\sin^{2m+1}\left(a^2z^2+bz+\gamma\right)dz=$$

$$=2\int_{-\infty}^{b/(2a^2)}\sin^{2m+1}\left(a^2z^2-bz+\gamma\right)dz=\frac{(2m+1)!\sqrt{\pi}}{4^m\,a}\sum_{k=0}^{m}\frac{(-1)^k}{(m-k)!\,(m+1+k)!\,\sqrt{4k+2}}\times$$

$$\times\left\{\cos\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]-\sin\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\right\}.$$

3. $$\int_{0}^{\infty}\sin^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{(2m+1)!\sqrt{\pi}}{4^{m+1}\,\alpha}\times$$

$$\times\sum_{k=0}^{m}\frac{(-1)^k}{(m-k)!\,(m+1+k)!\,\sqrt{4k+2}}\left\{(i-1)\exp\left[i(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\times\left[\operatorname{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)-A_1\right]-(i+1)\exp\left[i\,(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\times$$

$$\left.\times\left[\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)-A_2\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|-(2m+1)\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty;$$

$$A_l=\pm1\ \text{ if }\ \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(3-2l)\operatorname{Im}(\alpha z)\right]=\pm\infty\}.$$

4. $\int\limits_0^{\infty} \sinh^{2m+1}\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{(2m+1)!\sqrt{\pi}}{4^{m+1}\alpha} \sum_{k=0}^{m} \frac{(-1)^{m-k}}{(m-k)!(m+k+1)!\sqrt{2k+1}} \times$

$$\times\left\{\exp\left[(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\left[\operatorname{erf}\left(\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)-A_1\right]+\exp\left[(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\left.\times\left[A_2-\operatorname{erfi}\left(\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)\right]\right\} \quad \{ \lim_{z\to\infty}\left[\ln|z|-(2m+1)\left|\operatorname{Re}\left(\alpha^2 z^2+\beta z\right)\right|\right]=+\infty;$$

$$A_1 = 1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha z) = +\infty,\ A_1 = -1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha z) = -\infty,$$

$$A_2 = \pm i \text{ if } \lim_{z\to\infty} \operatorname{Im}(\alpha z) = \pm\infty \}.$$

2.8.4.

1. $\int\limits_0^{+\infty} \cos^{2m+1}\left(a^2 z^2 + b z + \gamma\right) dz = \frac{(2m+1)!\sqrt{2\pi}}{4^{m+1} a} \sum_{k=0}^{m} \frac{1}{(m-k)!(m+1+k)!\sqrt{2k+1}} \times$

$$\times\left\{\sin\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\left[1+\operatorname{Im}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)-\operatorname{Re}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)\right]+\right.$$

$$\left.+\cos\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\left[1-\operatorname{Re}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)-\operatorname{Im}\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,b}{2a}\right)\right]\right\}.$$

2. $\int\limits_{-\infty}^{+\infty} \cos^{2m+1}\left(a^2 z^2 + b z + \gamma\right) dz = 2 \int\limits_{-b/\left(2a^2\right)}^{+\infty} \cos^{2m+1}\left(a^2 z^2 + b z + \gamma\right) dz =$

$$= 2 \int\limits_{-\infty}^{b/\left(2a^2\right)} \cos^{2m+1}\left(a^2 z^2 - b z + \gamma\right) dz = \frac{(2m+1)!\sqrt{\pi}}{4^m a} \sum_{k=0}^{m} \frac{1}{(m-k)!(m+1+k)!\sqrt{4k+2}} \times$$

$$\times\left\{\sin\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]+\cos\left[(2k+1)\left(\frac{b^2}{4a^2}-\gamma\right)\right]\right\}.$$

3. $\int\limits_0^{\infty} \cos^{2m+1}\left(\alpha^2 z^2 + \beta z + \gamma\right) dz = \frac{(2m+1)!\sqrt{\pi}}{4^{m+1}\alpha} \times$

$$\times\sum_{k=0}^{m} \frac{1}{(m-k)!(m+1+k)!\sqrt{4k+2}} \left\{(1+i)\exp\left[i\,(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\times\left[A_1-\operatorname{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)\right]+(1-i)\exp\left[i(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\times$$

$$\times\left[A_2-\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)\right]\Bigg\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|\;-(2m+1)\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty\,;$$

$$A_l=\pm1 \text{ if } \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(3-2l)\operatorname{Im}(\alpha z)\right]=\pm\infty\,\}.$$

4. $$\int_0^\infty \cosh^{2m+1}\left(\alpha^2z^2+\beta z+\gamma\right)dz=\frac{(2m+1)!\sqrt{\pi}}{4^{m+1}\alpha}\sum_{k=0}^{m}\frac{1}{(m-k)!(m+1+k)!\sqrt{2k+1}}\times$$

$$\times\left\{\exp\left[(2k+1)\left(\frac{\beta^2}{4\alpha^2}-\gamma\right)\right]\left[A_1-\operatorname{erf}\left(\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)\right]+\exp\left[(2k+1)\left(\gamma-\frac{\beta^2}{4\alpha^2}\right)\right]\times\right.$$

$$\left.\times\left[A_2-\operatorname{erfi}\left(\frac{\sqrt{2k+1}\,\beta}{2\alpha}\right)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\ln|z|-(2m+1)\left|\operatorname{Re}\left(\alpha^2z^2+\beta z\right)\right|\right]=+\infty\,;\ A_1=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty\,,$$

$$A_1=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty\,,\ A_2=\pm i \text{ if } \lim_{z\to\infty}\operatorname{Im}(\alpha z)=\pm\infty\,\}.$$

2.9. Integrals of $z^n\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp\left(\beta_1 z\right)$

2.9.1.

1. $$\int_0^{+\infty}\sin\left(a^2z^2+bz+\gamma\right)\exp\left(-b_1z\right)dz=\int_{-\infty}^0\sin\left(a^2z^2-bz+\gamma\right)\exp\left(b_1z\right)dz=$$

$$=\frac{\sqrt{2\pi}}{4a}\exp\left(\frac{bb_1}{2a^2}\right)\left\{\cos\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\left[1+\operatorname{Re}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)-\operatorname{Im}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)\right]-\right.$$

$$\left.-\sin\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\left[1+\operatorname{Re}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)+\operatorname{Im}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)\right]\right\}\ \{b_1\ge0\}.$$

2. $$\int_0^\infty\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp\left(\beta_1z\right)dz=\frac{\sqrt{2\pi}}{8\alpha}\left\{(1+i)\exp\left[\frac{(\beta_1-i\beta)^2}{4i\alpha^2}-i\gamma\right]\times\right.$$

$$\times\left[A_1+\operatorname{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\beta_1-i\beta}{2\alpha}\right)\right]+(1-i)\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\times$$

$$\times\left[A_2+\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\beta_1+i\beta}{2\alpha}\right)\right]\Bigg\}\quad \{\lim_{z\to\infty}\left[\operatorname{Re}(\beta_1 z)+\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|-\ln|z|\right]=-\infty;$$

$$A_k=\pm1 \text{ if } \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(2k-3)\operatorname{Im}(\alpha z)\right]=\pm\infty\}.$$

2.9.2.

1. $$\int_0^{+\infty} z\sin\left(a^2z^2+bz+\gamma\right)\exp(-b_1z)\,dz=-\int_{-\infty}^{0} z\sin\left(a^2z^2-bz+\gamma\right)\exp(b_1z)\,dz=$$

$$=\frac{\cos\gamma}{2a^2}+\frac{\sqrt{2\pi}}{8a^3}\exp\left(\frac{bb_1}{2a^2}\right)\left\{\sin\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\left[(b-b_1)\operatorname{Re}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)+\right.\right.$$

$$\left.+(b+b_1)\operatorname{Im}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)+b-b_1\right]+\cos\left(\frac{b^2-b_1^2}{4a^2}-\gamma\right)\times$$

$$\left.\times\left[(b-b_1)\operatorname{Im}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)-(b+b_1)\operatorname{Re}\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)-b-b_1\right]\right\}\quad\{b_1>0\}.$$

2. $$\int_0^{\infty} z\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp(\beta_1 z)\,dz=\frac{\sqrt{\pi}}{8\sqrt{2}\,\alpha^3}\left\{(1-i)(i\beta_1-\beta)\times\right.$$

$$\times\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\left[A_1+\operatorname{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\beta_1+i\beta}{2\alpha}\right)\right]-(1+i)(\beta+i\beta_1)\times$$

$$\left.\times\exp\left[\frac{(\beta_1-i\beta)^2}{4i\alpha^2}-i\gamma\right]\left[A_2+\operatorname{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\beta_1-i\beta}{2\alpha}\right)\right]\right\}+\frac{\cos\gamma}{2\alpha^2}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}(\beta_1 z)+\left|\operatorname{Im}\left(\alpha^2z^2+\beta z\right)\right|\right]=-\infty;$$

$$A_k=\pm1 \text{ if } \lim_{z\to\infty}\left[\operatorname{Re}(\alpha z)+(3-2k)\operatorname{Im}(\alpha z)\right]=\pm\infty\}.$$

2.9.3.

1. $$\int_0^{+\infty} z^n\sin\left(a^2z^2+bz+\gamma\right)\exp(-b_1z)\,dz=(-1)^n\int_{-\infty}^{0} z^n\sin\left(a^2z^2-bz+\gamma\right)\exp(b_1z)\,dz=$$

$$=\frac{n!}{2^n}\exp\left(\frac{bb_1}{2a^2}\right)\sin\left(\frac{b_1^2-b^2}{4a^2}+\gamma\right)\operatorname{Re}\left\{\left[\operatorname{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)+1\right]W_1^{(9)}(n,a,b,-b_1)\right\}-$$

$$-\frac{n!}{2^n}\exp\left(\frac{bb_1}{2a^2}\right)\cos\left(\frac{b_1^2-b^2}{4a^2}+\gamma\right)\mathrm{Im}\left\{\left[\mathrm{erf}\left(\frac{i-1}{\sqrt{2}}\cdot\frac{b_1+ib}{2a}\right)+1\right]W_1^{(9)}\left(n,a,b,-b_1\right)\right\}+$$

$$+\frac{n!}{2^n}\left[\sin\gamma\cdot\mathrm{Re}W_2^{(9)}\left(n,a^2,b,-b_1\right)-\cos\gamma\cdot\mathrm{Im}W_2^{(9)}\left(n,a^2,b,-b_1\right)\right]$$

$\{b_1>0 \text{ if } n>0\}$.

2. $$\int_0^\infty z^n\sin\left(\alpha^2z^2+\beta z+\gamma\right)\exp\left(\beta_1 z\right)dz=\frac{n!i}{2^{n+1}}\left\{\exp\left[\frac{\left(\beta_1-i\beta\right)^2}{4i\alpha^2}-i\gamma\right]\times\right.$$

$$\times\left[\mathrm{erf}\left(\frac{1-i}{\sqrt{2}}\cdot\frac{\beta_1-i\beta}{2\alpha}\right)+A_1\right]W_1^{(9)}(n,\alpha,\beta,\beta_1)-\exp\left[i\gamma-\frac{(\beta_1+i\beta)^2}{4i\alpha^2}\right]\times$$

$$\times\left[\mathrm{erf}\left(\frac{1+i}{\sqrt{2}}\cdot\frac{\beta_1+i\beta}{2\alpha}\right)+A_2\right]W_3^{(9)}\left(n,\alpha,\beta,\beta_1\right)+$$

$$\left.+\exp(-i\gamma)W_2^{(9)}\left(n,\alpha^2,\beta,\beta_1\right)-\exp(i\gamma)W_2^{(9)}\left(n,-\alpha^2,-\beta,\beta_1\right)\right\}$$

$\{\lim\limits_{z\to\infty}\left[\mathrm{Re}(\beta_1 z)+\left|\mathrm{Im}\left(\alpha^2z^2+\beta z\right)\right|+(n-1)\ln|z|\right]=-\infty;$

$A_k=\pm1$ if $\lim\limits_{z\to\infty}\left[\mathrm{Re}(\alpha z)+(2k-3)\mathrm{Im}(\alpha z)\right]=\pm\infty$ }.

2.9.4.

1. $$\int_0^{+\infty} z^n\sin(bz+\gamma)\exp(-b_1z)dz=(-1)^{n+1}\int_{-\infty}^0 z^n\sin(bz-\gamma)\exp(b_1z)dz=$$

$$=(-1)^n\, n!W_4^{(9)}(b,-b_1)\quad \{b_1>0\}.$$

2. $$\int_0^\infty z^n\sin(\beta z+\gamma)\exp(\beta_1 z)dz=(-1)^n\, n!W_4^{(9)}(\beta,\beta_1)$$

$\{\ \beta_1^2\neq-\beta^2,\ \lim\limits_{z\to\infty}\left[\mathrm{Re}(\beta_1 z)+\left|\mathrm{Im}(\beta z)\right|+n\ln|z|\right]=-\infty$ }.

Introduced notations:

1) $$W_1^{(9)}(n,\alpha,\beta,\beta_1)=\frac{\sqrt{2\pi}(1-i)}{4\alpha}\sum_{l=0}^{E(n/2)}\frac{(\beta_1-i\beta)^{n-2l}}{l!(n-2l)!\left(i\alpha^2\right)^{n-l}},$$

$$W_1^{(9)}(2n_1, \alpha, \beta, i\beta) = \frac{\sqrt{\pi}(1-i)(-i)^{n_1}}{n_1!\sqrt{8}\alpha^{2n_1+1}}, \; W_1^{(9)}(2n_1+1, \alpha, \beta, i\beta) = 0;$$

2) $$W_2^{(9)}\left(n, \alpha^2, \beta, \beta_1\right) = \sum_{l=1}^{n-E(n/2)} \frac{(l-1)!}{(2l-1)!(n+1-2l)!} \sum_{r=1}^{l} \frac{4^{l-r}(\beta_1 - i\beta)^{n-1-2l+2r}}{(r-1)!\left(i\alpha^2\right)^{n-l+r}} -$$

$$-\sum_{l=1}^{E(n/2)} \frac{1}{l!(n-2l)!} \sum_{r=1}^{l} \frac{r!(\beta_1 - i\beta)^{n-1-2l+2r}}{(2r)!\left(i\alpha^2\right)^{n-l+r}},$$

$$W_2^{(9)}\left(2n_1, \alpha^2, \beta, i\beta\right) = 0, \; W_2^{(9)}\left(2n_1+1, \alpha^2, \beta, i\beta\right) = \frac{4^{n_1} n_1!}{(2n_1+1)!\left(i\alpha^2\right)^{n_1+1}};$$

3) $$W_3^{(9)}(n, \alpha, \beta, \beta_1) = \frac{\sqrt{2\pi}(1+i)}{4\alpha} \sum_{l=0}^{E(n/2)} \frac{(\beta_1 + i\beta)^{n-2l}}{l!(n-2l)!\left(-i\alpha^2\right)^{n-l}},$$

$$W_3^{(9)}(2n_1, \alpha, \beta, -i\beta) = \frac{\sqrt{\pi}(1+i)i^{n_1}}{n_1!\sqrt{8}\alpha^{2n_1+1}}, \; W_3^{(9)}(2n_1+1, \alpha, \beta, -i\beta) = 0;$$

4) $$W_4^{(9)}(\beta, \beta_1) = \frac{(n+1)!}{\left(\beta^2 + \beta_1^2\right)^{n+1}} \left[\cos\gamma \cdot \sum_{l=0}^{E(n/2)} \frac{(-1)^l \beta^{2l+1} \beta_1^{n-2l}}{(2l+1)!(n-2l)!} - \right.$$

$$\left. - \sin\gamma \cdot \sum_{l=0}^{n-E(n/2)} \frac{(-1)^l \beta^{2l} \beta_1^{n+1-2l}}{(2l)!(n+1-2l)!} \right] = \frac{i}{2}\left[\frac{\exp(i\gamma)}{(\beta_1 + i\beta)^{n+1}} - \frac{\exp(-i\gamma)}{(\beta_1 - i\beta)^{n+1}}\right].$$

2.10. Integrals of $z^n \exp\left(-\alpha^2 z^2 + \beta z\right) \sin\left(\alpha_1 z^2 + \beta_1 z + \gamma\right)$

2.10.1.

1. $$\int_0^{+\infty} \exp\left(-a^2 z^2 + bz\right) \sin(b_1 z + \gamma) dz = \frac{\sqrt{\pi}}{2a} \exp\left(\frac{b^2 - b_1^2}{4a^2}\right) \times$$

$$\times \left\{ \sin\left(\frac{bb_1}{2a^2} + \gamma\right) \left[1 + \operatorname{Re}\, \operatorname{erf}\left(\frac{b + ib_1}{2a}\right)\right] + \cos\left(\frac{bb_1}{2a^2} + \gamma\right) \operatorname{Im}\, \operatorname{erf}\left(\frac{b + ib_1}{2a}\right) \right\}.$$

2. $$\int_{-\infty}^{+\infty} \exp\left(-a^2 z^2 + bz\right) \sin(b_1 z + \gamma) dz = \frac{\sqrt{\pi}}{a} \exp\left(\frac{b^2 - b_1^2}{4a^2}\right) \sin\left(\frac{bb_1}{2a^2} + \gamma\right).$$

3. $$\int_0^{+\infty} \exp\left(-a^2 z^2 + bz\right) \sin\left(a_1 z^2 + b_1 z + \gamma\right) dz = \frac{\sqrt{\pi}}{2} \exp\left(\frac{a^2 b^2 - a^2 b_1^2 - 2a_1 bb_1}{4a^4 + 4a_1^2}\right) \times$$

$$\times\left\{\sin\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 b b_1}{4a^4 + 4a_1^2} + \gamma\right)\mathrm{Re}\left[\frac{1}{\sqrt{a^2 - i a_1}}\operatorname{erf}\left(\frac{b + i b_1}{2\sqrt{a^2 - i a_1}}\right) + \frac{1}{\sqrt{a^2 - i a_1}}\right] + \right.$$

$$\left. + \cos\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 b b_1}{4a^4 + 4a_1^2} + \gamma\right)\mathrm{Im}\left[\frac{1}{\sqrt{a^2 - i a_1}}\operatorname{erf}\left(\frac{b + i b_1}{2\sqrt{a^2 - i a_1}}\right) + \frac{1}{\sqrt{a^2 - i a_1}}\right]\right\}$$

$$\{a > 0 \text{ or } [a = 0,\, a_1 \neq 0,\, b \leq 0]\}.$$

4. $$\int_{-\infty}^{+\infty} \exp\left(-a^2 z^2 + bz\right)\sin\left(a_1 z^2 + b_1 z + \gamma\right) dz = \frac{\sqrt{2\pi}}{2\sqrt{a^4 + a_1^2}} \times$$

$$\times \exp\left(\frac{a^2 b^2 - a^2 b_1^2 - 2a_1 b b_1}{4a^4 + 4a_1^2}\right)\left[\sqrt{a^2 + \sqrt{a^4 + a_1^2}}\,\sin\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 b b_1}{4a^4 + 4a_1^2} + \gamma\right) + \right.$$

$$\left. + \sqrt{\sqrt{a^4 + a_1^2} - a^2}\,\cos\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 b b_1}{4a^4 + 4a_1^2} + \gamma\right)\right]$$

$$\{a > 0 \text{ or } [a = b = 0,\, a_1 > 0]\}.$$

5. $$\int_0^{\infty} \exp\left(-\alpha^2 z^2 + \beta z\right)\sin\left(\beta_1 z + \gamma\right) dz = \frac{\sqrt{\pi}\, i}{4\alpha}\left\{\exp\left[\frac{(\beta - i\beta_1)^2}{4\alpha^2} - i\gamma\right] \times \right.$$

$$\left. \times\left[\operatorname{erf}\left(\frac{\beta - i\beta_1}{2\alpha}\right) \pm 1\right] - \exp\left[\frac{(\beta + i\beta_1)^2}{4\alpha^2} + i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta + i\beta_1}{2\alpha}\right) \pm 1\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2 - \beta z\right) - \left|\mathrm{Im}\left(\beta_1 z\right)\right| + \ln|z|\right] = +\infty, \quad \lim_{z\to\infty}\mathrm{Re}\left(\alpha z\right) = \pm\infty\}.$$

6. $$\int_0^{\infty} \exp\left(-\alpha^2 z^2 + \beta z\right)\sin\left(\alpha_1 z^2 + \beta_1 z + \gamma\right) dz = \frac{\sqrt{\pi}\, i}{4}\left\{\frac{1}{\sqrt{\alpha^2 + i\alpha_1}} \times \right.$$

$$\times \exp\left[\frac{(\beta - i\beta_1)^2}{4\alpha^2 + 4i\alpha_1} - i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta - i\beta_1}{2\sqrt{\alpha^2 + i\alpha_1}}\right) + A_1\right] - \frac{1}{\sqrt{\alpha^2 - i\alpha_1}} \times$$

$$\left. \times \exp\left[\frac{(\beta + i\beta_1)^2}{4\alpha^2 - 4i\alpha_1} + i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta + i\beta_1}{2\sqrt{\alpha^2 - i\alpha_1}}\right) + A_2\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2 - \beta z\right) - \left|\mathrm{Im}\left(\alpha_1 z^2 + \beta_1 z\right)\right| + \ln|z|\right] = +\infty;$$

$$A_k = \pm 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left[\sqrt{\alpha^2 + (3 - 2k)i\alpha_1}\, z\right] = \pm\infty\}.$$

7. $$\int_0^{\infty} \exp\left(-\alpha^2 z^2 + \beta z\right) \sin\left(\pm i\alpha^2 z^2 + \beta_1 z + \gamma\right) dz = \frac{\exp(\mp i\gamma)}{2(\beta_1 \pm i\beta)} \mp \frac{\sqrt{2\pi}\, i}{8\alpha} \times$$

$$\times \exp\left[\frac{(\beta \pm i\beta_1)^2}{8\alpha^2} \pm i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta \pm i\beta_1}{2\sqrt{2}\alpha}\right) + A\right]$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2 - \beta z \mp i\beta_1 z\right) + \ln|z|\right] = +\infty, \ \lim_{z\to\infty} \operatorname{Re}[(\beta \mp i\beta_1) z] = -\infty;$$

$$A = 1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha z) = +\infty, \ A = -1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha z) = -\infty \}.$$

2.10.2.

1. $$\int_0^{+\infty} z \exp\left(-a^2 z^2 + bz\right) \sin(b_1 z + \gamma) dz = \frac{\sqrt{\pi}}{4a^3} \exp\left(\frac{b^2 - b_1^2}{4a^2}\right) \times$$

$$\times \left\{ \sin\left(\frac{bb_1}{2a^2} + \gamma\right)\left[b \operatorname{Re}\operatorname{erf}\left(\frac{b + ib_1}{2a}\right) - b_1 \operatorname{Im}\operatorname{erf}\left(\frac{b + ib_1}{2a}\right) + b \right] + \cos\left(\frac{bb_1}{2a^2} + \gamma\right) \times \right.$$

$$\left. \times \left[b_1 \operatorname{Re}\operatorname{erf}\left(\frac{b + ib_1}{2a}\right) + b \operatorname{Im}\operatorname{erf}\left(\frac{b + ib_1}{2a}\right) + b_1 \right] \right\} + \frac{\sin\gamma}{2a^2}.$$

2. $$\int_{-\infty}^{+\infty} z \exp\left(-a^2 z^2 + bz\right) \sin(b_1 z + \gamma) dz = \frac{\sqrt{\pi}}{2a^3} \exp\left(\frac{b^2 - b_1^2}{4a^2}\right) \times$$

$$\times \left[b \sin\left(\frac{bb_1}{2a^2} + \gamma\right) + b_1 \cos\left(\frac{bb_1}{2a^2} + \gamma\right) \right].$$

3. $$\int_0^{+\infty} z \exp\left(-a^2 z^2 + bz\right) \sin\left(a_1 z^2 + b_1 z + \gamma\right) dz = \frac{a^2 \sin\gamma + a_1 \cos\gamma}{2a^4 + 2a_1^2} +$$

$$+ \frac{\sqrt{\pi}}{4} \exp\left(\frac{a^2 b^2 - a^2 b_1^2 - 2a_1 bb_1}{4a^4 + 4a_1^2}\right) \left\{ \sin\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 bb_1}{4a^4 + 4a_1^2} + \gamma\right) \times \right.$$

$$\times \operatorname{Re}\left[\frac{b + ib_1}{\left(a^2 - ia_1\right)\sqrt{a^2 - ia_1}} \operatorname{erf}\left(\frac{b + ib_1}{2\sqrt{a^2 - ia_1}}\right) + \frac{b + ib_1}{\left(a^2 - ia_1\right)\sqrt{a^2 - ia_1}} \right] +$$

$$+ \operatorname{Im}\left[\frac{b + ib_1}{\left(a^2 - ia_1\right)\sqrt{a^2 - ia_1}} \operatorname{erf}\left(\frac{b + ib_1}{2\sqrt{a^2 - ia_1}}\right) + \frac{b + ib_1}{\left(a^2 - ia_1\right)\sqrt{a^2 - ia_1}} \right] \times$$

$$\left. \times \cos\left(\frac{a_1 b^2 - a_1 b_1^2 + 2a^2 bb_1}{4a^4 + 4a_1^2} + \gamma\right) \right\} \quad \{a > 0 \text{ or } [a = 0, a_1 \neq 0, b < 0]\}.$$

4. $$\int_{-\infty}^{+\infty} z\exp\left(-a^2z^2+bz\right)\sin\left(a_1z^2+b_1z+\gamma\right)dz=\frac{\sqrt{\pi}}{2}\exp\left(\frac{a^2b^2-a^2b_1^2-2a_1bb_1}{4a^4+4a_1^2}\right)\times$$

$$\times\left\{\sin\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\mathrm{Re}\left[\frac{b+ib_1}{\left(a^2-ia_1\right)\sqrt{a^2-ia_1}}\right]+\right.$$

$$\left.+\cos\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\mathrm{Im}\left[\frac{b+ib_1}{\left(a^2-ia_1\right)\sqrt{a^2-ia_1}}\right]\right\}\quad\{a>0\}.$$

5. $$\int_0^{\infty} z\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\beta_1z+\gamma\right)dz=\frac{\sqrt{\pi}i}{8\alpha^3}\left\{(\beta-i\beta_1)\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\times\right.$$

$$\left.\times\left[\mathrm{erf}\left(\frac{\beta-i\beta_1}{2\alpha}\right)\pm1\right]-(\beta+i\beta_1)\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]\left[\mathrm{erf}\left(\frac{\beta+i\beta_1}{2\alpha}\right)\pm1\right]\right\}+\frac{\sin\gamma}{2\alpha^2}$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2z^2-\beta z\right)-\left|\mathrm{Im}(\beta_1z)\right|\right]=+\infty,\quad \lim_{z\to\infty}\mathrm{Re}(\alpha z)=\pm\infty\}.$$

6. $$\int_0^{\infty} z\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)dz=\frac{\alpha^2\sin\gamma+\alpha_1\cos\gamma}{2\alpha^4+2\alpha_1^2}+\frac{\sqrt{\pi}i}{8}\times$$

$$\times\left\{\frac{\beta-i\beta_1}{\left(\alpha^2+i\alpha_1\right)\sqrt{\alpha^2+i\alpha_1}}\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2+4i\alpha_1}-i\gamma\right]\left[\mathrm{erf}\left(\frac{\beta-i\beta_1}{2\sqrt{\alpha^2+i\alpha_1}}\right)+A_1\right]-\right.$$

$$\left.-\frac{\beta+i\beta_1}{\left(\alpha^2-i\alpha_1\right)\sqrt{\alpha^2-i\alpha_1}}\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2-4i\alpha_1}+i\gamma\right]\left[\mathrm{erf}\left(\frac{\beta+i\beta_1}{2\sqrt{\alpha^2-i\alpha_1}}\right)+A_2\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2z^2-\beta z\right)-\left|\mathrm{Im}\left(\alpha_1z^2+\beta_1z\right)\right|\right]=+\infty;\ A_k=\pm1 \text{ if}$$

$$\lim_{z\to\infty}\mathrm{Re}\left[\sqrt{\alpha^2+(3-2k)i\alpha_1}\ z\right]=\pm\infty\}.$$

7. $$\int_0^{\infty} z\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\pm i\alpha^2z^2+\beta_1z+\gamma\right)dz=\frac{\sqrt{2\pi}(\beta_1\mp i\beta)}{32\alpha^3}\times$$

$$\times\exp\left[\frac{(\beta\pm i\beta_1)^2}{8\alpha^2}\pm i\gamma\right]\left[\mathrm{erf}\left(\frac{\beta\pm i\beta_1}{2\sqrt{2}\alpha}\right)+A\right]\mp\frac{i\exp(\pm i\gamma)}{8\alpha^2}\pm\frac{i\exp(\mp i\gamma)}{2(\beta\mp i\beta_1)^2}$$

$$\{\lim_{z\to\infty}\mathrm{Re}\left(2\alpha^2z^2-\beta z\mp i\beta_1z\right)=+\infty,\ \lim_{z\to\infty}\left[\mathrm{Re}(\beta z\mp i\beta_1z)+\ln|z|\right]=-\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\mathrm{Re}(\alpha z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\mathrm{Re}(\alpha z)=-\infty\}.$$

2.10.3.

1. $$\int_0^{+\infty} z^n \exp\left(-a^2z^2+bz\right)\sin\left(a_1z^2+b_1z+\gamma\right)dz=\frac{n!\sqrt{\pi}}{2^{n+1}}\exp\left(\frac{a^2b^2-a^2b_1^2-2a_1bb_1}{4a^4+4a_1^2}\right)\times$$
$$\times\sin\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\mathrm{Re}\left\{\left[\mathrm{erf}\left(\frac{b+ib_1}{2\sqrt{a^2-ia_1}}\right)+1\right]\frac{W_1^{(10)}(n,a,a_1,b,b_1)}{\sqrt{a^2-ia_1}}\right\}+$$
$$+\frac{n!\sqrt{\pi}}{2^{n+1}}\exp\left(\frac{a^2b^2-a^2b_1^2-2a_1bb_1}{4a^4+4a_1^2}\right)\cos\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\times$$
$$\times\mathrm{Im}\left\{\left[\mathrm{erf}\left(\frac{b+ib_1}{2\sqrt{a^2-ia_1}}\right)+1\right]\frac{W_1^{(10)}(n,a,a_1,b,b_1)}{\sqrt{a^2-ia_1}}\right\}+$$
$$+\frac{n!}{2^n}\left[\mathrm{Re}\,W_2^{(10)}(n,a,a_1,b,b_1)\sin\gamma+\mathrm{Im}\,W_2^{(10)}(n,a,a_1,b,b_1)\cos\gamma\right]$$
$$\{a>0\ \text{ or }\ [a=0,\,a_1\neq 0,\,b<0]\ \text{ for }\ n>0\}.$$

2. $$\int_{-\infty}^{+\infty} z^n \exp\left(-a^2z^2+bz\right)\sin\left(a_1z^2+b_1z+\gamma\right)dz=\frac{n!\sqrt{\pi}}{2^{n}}\exp\left(\frac{a^2b^2-a^2b_1^2-2a_1bb_1}{4a^4+4a_1^2}\right)\times$$
$$\times\left\{\sin\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\mathrm{Re}\left[\frac{W_1^{(10)}(n,a,a_1,b,b_1)}{\sqrt{a^2-ia_1}}\right]+\right.$$
$$\left.+\cos\left(\frac{a_1b^2-a_1b_1^2+2a^2bb_1}{4a^4+4a_1^2}+\gamma\right)\mathrm{Im}\left[\frac{W_1^{(10)}(n,a,a_1,b,b_1)}{\sqrt{a^2-ia_1}}\right]\right\}$$
$$\{a>0\ \text{ for }\ n>0\}.$$

3. $$\int_0^{\infty} z^n \exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\alpha_1z^2+\beta_1z+\gamma\right)dz=\frac{n!\sqrt{\pi}\,i}{2^{n+2}}\left\{\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2+4i\alpha_1}-i\gamma\right]\times\right.$$
$$\times\left[\mathrm{erf}\left(\frac{\beta-i\beta_1}{2\sqrt{\alpha^2+i\alpha_1}}\right)+A_1\right]\frac{W_1^{(10)}(n,\alpha,-\alpha_1,\beta,-\beta_1)}{\sqrt{\alpha^2+i\alpha_1}}-\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2-4i\alpha_1}+i\gamma\right]\times$$
$$\left.\times\left[\mathrm{erf}\left(\frac{\beta+i\beta_1}{2\sqrt{\alpha^2-i\alpha_1}}\right)+A_2\right]\frac{W_1^{(10)}(n,\alpha,\alpha_1,\beta,\beta_1)}{\sqrt{\alpha^2-i\alpha_1}}\right\}+\frac{n!\,i}{2^{n+1}}\times$$

$$\times\left[\exp(-i\gamma)W_2^{(10)}(n,\alpha,-\alpha_1,\beta,-\beta_1)-\exp(i\gamma)W_2^{(10)}(n,\alpha,\alpha_1,\beta,\beta_1)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2-\beta z\right)-\left|\operatorname{Im}\left(\alpha_1z^2+\beta_1z\right)\right|-(n-1)\ln|z|\right]=+\infty\,;$$

$$A_k=\pm1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left[\sqrt{\alpha^2+(3-2k)i\alpha_1z}\right]=\pm\infty\}.$$

4. $$\int_0^\infty z^n\exp\left(-\alpha^2z^2+\beta z\right)\sin\left(\pm i\alpha^2z^2+\beta_1z+\gamma\right)dz=\mp\frac{(-1)^n\,n!i\exp(\mp i\gamma)}{2(\beta\mp i\beta_1)^{n+1}}\mp$$

$$\mp\frac{n!i}{2^{n+1}}\left\{\frac{\sqrt{2\pi}}{4\alpha}\exp\left[\frac{(\beta\pm i\beta_1)^2}{8\alpha^2}\pm i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta\pm i\beta_1}{2\sqrt{2}\alpha}\right)+A\right]\times\right.$$

$$\left.\times W_1^{(10)}\left(n,\alpha,i\alpha^2,\beta,\pm\beta_1\right)+\exp(\pm i\gamma)W_2^{(10)}\left(n,\alpha,i\alpha^2,\beta,\pm\beta_1\right)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\beta z\mp i\beta_1z\right)+n\ln|z|\right]=-\infty\,,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2z^2-\beta z\mp i\beta_1z\right)-(n-1)\ln|z|\right]=+\infty\,;\ A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty\,,$$

$$A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty\}.$$

Introduced notations:

1) $$W_1^{(10)}(n,\alpha,\alpha_1,\beta,\beta_1)=\sum_{l=0}^{E(n/2)}\frac{\left(\beta+i\beta_1\right)^{n-2l}}{l!(n-2l)!\left(\alpha^2-i\alpha_1\right)^{n-l}}\,,$$

$$W_1^{(10)}(2n_1,\alpha,\alpha_1,\beta,i\beta)=\frac{1}{n_1!\left(\alpha^2-i\alpha_1\right)^{n_1}}\,,\ W_1^{(10)}(2n_1+1,\alpha,\alpha_1,\beta,i\beta)=0\,;$$

2) $$W_2^{(10)}(n,\alpha,\alpha_1,\beta,\beta_1)=\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!}{(2l-1)!(n+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}\left(\beta+i\beta_1\right)^{n-1-2l+2r}}{(r-1)!\left(\alpha^2-i\alpha_1\right)^{n-l+r}}-$$

$$-\sum_{l=1}^{E(n/2)}\frac{1}{l!(n-2l)!}\sum_{r=1}^{l}\frac{r!(\beta+i\beta_1)^{n-1-2l+2r}}{(2r)!\left(\alpha^2-i\alpha_1\right)^{n-l+r}}\,,\ W_2^{(10)}(2n_1,\alpha,\alpha_1,\beta,i\beta)=0\,,$$

$$W_2^{(10)}(2n_1+1,\alpha,\alpha_1,\beta,i\beta)=\frac{4^{n_1}\,n_1!}{(2n_1+1)!\left(\alpha^2-i\alpha_1\right)^{n_1+1}}.$$

2.11. Integrals of $z^n \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z)\sin(\beta_2 z+\gamma)$

2.11.1.

1. $$\int_0^{+\infty}\operatorname{erf}(az)\exp(-b_1 z)\sin(b_2 z+\gamma)dz=\int_{-\infty}^{0}\operatorname{erf}(az)\exp(b_1 z)\sin(b_2 z-\gamma)dz=$$

$$=\frac{1}{b_1^2+b_2^2}\exp\left(\frac{b_1^2-b_2^2}{4a^2}\right)\left\{\sin\left(\frac{b_1 b_2}{2a^2}-\gamma\right)\times\right.$$

$$\times\left[b_1\operatorname{Re}\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)-b_2\operatorname{Im}\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)-b_1\right]-\cos\left(\frac{b_1b_2}{2a}-\gamma\right)\times$$

$$\left.\times\left[b_2\operatorname{Re}\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)+b_1\operatorname{Im}\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)-b_2\right]\right\}\quad\{a>0,\,b_1>0\}.$$

2. $$\int_0^{\infty}\operatorname{erf}(\alpha z)\exp(\beta_1 z)\sin(\beta_2 z+\gamma)dz=\frac{1}{2(\beta_2-i\beta_1)}\exp\left[\frac{(\beta_1+i\beta_2)^2}{4\alpha^2}+i\gamma\right]\times$$

$$\times\left[\operatorname{erf}\left(\frac{\beta_1+i\beta_2}{2\alpha}\right)\pm1\right]+\frac{1}{2(\beta_2+i\beta_1)}\exp\left[\frac{(\beta_1-i\beta_2)^2}{4\alpha^2}-i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta_1-i\beta_2}{2\alpha}\right)\pm1\right]$$

$$\{\beta_2^2\neq-\beta_1^2,\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2-\beta_1 z\right)-\left|\operatorname{Im}(\beta_2 z)\right|+2\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}(\beta_1 z)+\left|\operatorname{Im}(\beta_2 z)\right|\right]=-\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.11.2.

1. $$\int_0^{+\infty}\operatorname{erf}(az+b)\exp(-b_1 z)\sin(b_2 z+\gamma)dz=\int_{-\infty}^{0}\operatorname{erf}(az-b)\exp(b_1 z)\sin(b_2 z-\gamma)dz=$$

$$=\frac{\operatorname{erf}(b)}{b_1^2+b_2^2}(b_1\sin\gamma+b_2\cos\gamma)+\frac{1}{b_1^2+b_2^2}\exp\left(\frac{b_1^2-b_2^2+4abb_1}{4a^2}\right)\left\{\sin\left(\gamma-\frac{b_1b_2+2abb_2}{2a^2}\right)\times\right.$$

$$\times\left[b_1+b_2\operatorname{Im}\operatorname{erf}\left(b+\frac{b_1-ib_2}{2a}\right)-b_1\operatorname{Re}\operatorname{erf}\left(b+\frac{b_1-ib_2}{2a}\right)\right]-\cos\left(\gamma-\frac{b_1b_2+2abb_2}{2a^2}\right)\times$$

$$\left.\times\left[b_2\operatorname{Re}\operatorname{erf}\left(b+\frac{b_1-ib_2}{2a}\right)+b_1\operatorname{Im}\operatorname{erf}\left(b+\frac{b_1-ib_2}{2a}\right)-b_2\right]\right\}\quad\{a>0,\,b_1>0\}.$$

2. $$\int_0^{\infty} \operatorname{erf}(\alpha z+\beta)\exp(\beta_1 z)\sin(\beta_2 z+\gamma)dz = \frac{\operatorname{erf}(\beta)}{\beta_1^2+\beta_2^2}(\beta_2\cos\gamma-\beta_1\sin\gamma)+$$

$$+\frac{i}{2}\left\{\frac{1}{\beta_1-i\beta_2}\exp\left[\frac{(\beta_1-i\beta_2)(\beta_1-4\alpha\beta-i\beta_2)}{4\alpha^2}-i\gamma\right]\left[\operatorname{erf}\left(\beta-\frac{\beta_1-i\beta_2}{2\alpha}\right)\mp 1\right]-\right.$$

$$\left.-\frac{1}{\beta_1+i\beta_2}\exp\left[\frac{(\beta_1+i\beta_2)(\beta_1-4\alpha\beta+i\beta_2)}{4\alpha^2}+i\gamma\right]\left[\operatorname{erf}\left(\beta-\frac{\beta_1+i\beta_2}{2\alpha}\right)\mp 1\right]\right\}$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}(\beta_2 z)\right|+2\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}[\operatorname{Re}(\beta_1 z)+|\operatorname{Im}(\beta_2 z)|]=-\infty,\ \beta_2^2\neq-\beta_1^2,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

2.11.3.

1. $$\int_0^{+\infty} z^n \operatorname{erf}(az)\exp(-b_1 z)\sin(b_2 z+\gamma)dz=(-1)^n\int_{-\infty}^{0} z^n\operatorname{erf}(az)\exp(b_1 z)\sin(b_2 z-\gamma)dz=$$

$$=\exp\left(\frac{b_1^2-b_2^2}{4a^2}\right)\sin\left(\frac{b_1 b_2}{2a^2}-\gamma\right)\operatorname{Re}\left\{\left[\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)-1\right]W_1^{(11)}(a,0,-b_1,b_2)\right\}-$$

$$-\exp\left(\frac{b_1^2-b_2^2}{4a^2}\right)\cos\left(\frac{b_1 b_2}{2a^2}-\gamma\right)\operatorname{Im}\left\{\left[\operatorname{erf}\left(\frac{b_1-ib_2}{2a}\right)-1\right]W_1^{(11)}(a,0,-b_1,b_2)\right\}+$$

$$+\sin\gamma\cdot\operatorname{Re}W_2^{(11)}(a,0,-b_1,b_2)+\cos\gamma\cdot\operatorname{Im}W_2^{(11)}(a,0,-b_1,b_2)\ \ \{a>0, b_1>0\}.$$

2. $$\int_0^{\infty} z^n\operatorname{erf}(\alpha z)\exp(\beta_1 z)\sin(\beta_2 z+\gamma)dz=\frac{1}{2i}\exp\left(\frac{\beta_1^2-\beta_2^2}{4\alpha^2}\right)\left\{\exp\left[i\left(\frac{\beta_1\beta_2}{2\alpha^2}+\gamma\right)\right]\times\right.$$

$$\times\left[\operatorname{erf}\left(\frac{\beta_1+i\beta_2}{2\alpha}\right)\pm 1\right]W_1^{(11)}(\alpha,0,\beta_1,\beta_2)-\exp\left[-i\left(\frac{\beta_1\beta_2}{2\alpha^2}+\gamma\right)\times\right.$$

$$\left.\times\left[\operatorname{erf}\left(\frac{\beta_1-i\beta_2}{2\alpha}\right)\pm 1\right]W_1^{(11)}(\alpha,0,\beta_1,-\beta_2)\right\}+\frac{i}{2}\left[\exp(-i\gamma)W_2^{(11)}(\alpha,0,\beta_1,-\beta_2)-\right.$$

$$\left.-\exp(i\gamma)W_2^{(11)}(\alpha,0,\beta_1,\beta_2)\right]\ \{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2-\beta_1 z\right)-\left|\operatorname{Im}(\beta_2 z)\right|-(n-2)\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}[\operatorname{Re}(\beta_1 z)+|\operatorname{Im}(\beta_2 z)|+n\ln|z|]=-\infty,\ \beta_2^2\neq-\beta_1^2,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

2.11.4.

1. $$\int_0^{+\infty} z^n\operatorname{erf}(az+b)\exp(-b_1 z)\sin(b_2 z+\gamma)dz=$$

$$= (-1)^n \int_{-\infty}^{0} z^n \operatorname{erf}(az - b) \exp(b_1 z) \sin(b_2 z - \gamma)\, dz =$$

$$= \exp\left(\frac{b_1^2 - b_2^2 + 4abb_1}{4a^2} \right) \sin\left(\gamma - \frac{b_1 b_2 + 2abb_2}{2a^2} \right) \times$$

$$\times \operatorname{Re}\left\{ \left[1 - \operatorname{erf}\left(b + \frac{b_1 - ib_2}{2a} \right) \right] W_1^{(11)}(a, b, -b_1, b_2) \right\} + \exp\left(\frac{b_1^2 - b_2^2 + 4abb_1}{4a^2} \right) \times$$

$$\times \cos\left(\gamma - \frac{b_1 b_2 + 2abb_2}{2a^2} \right) \operatorname{Im}\left\{ \left[1 - \operatorname{erf}\left(b + \frac{b_1 - ib_2}{2a} \right) \right] W_1^{(11)}(a, b, -b_1, b_2) \right\} +$$

$$+ \sin\gamma \cdot \operatorname{Re} W_2^{(11)}(a, b, -b_1, b_2) + \cos\gamma \cdot \operatorname{Im} W_2^{(11)}(a, b, -b_1, b_2) \quad \{a > 0, b_1 > 0\}.$$

2. $$\int_0^{\infty} z^n \operatorname{erf}(\alpha z + \beta) \exp(\beta_1 z) \sin(\beta_2 z + \gamma)\, dz = \frac{i}{2} \exp\left(\frac{\beta_1^2 - \beta_2^2 - 4\alpha\beta\beta_1}{4\alpha^2} \right) \times$$

$$\times \left\{ \exp\left[i \left(\frac{\beta_1 \beta_2 - 2\alpha\beta\beta_2}{2\alpha^2} + \gamma \right) \right] \left[\operatorname{erf}\left(\beta - \frac{\beta_1 + i\beta_2}{2\alpha} \right) \mp 1 \right] W_1^{(11)}(\alpha, \beta, \beta_1, \beta_2) - \right.$$

$$\left. - \exp\left[i \left(\frac{2\alpha\beta\beta_2 - \beta_1 \beta_2}{2\alpha^2} - \gamma \right) \right] \left[\operatorname{erf}\left(\beta - \frac{\beta_1 - i\beta_2}{2\alpha} \right) \mp 1 \right] W_1^{(11)}(\alpha, \beta, \beta_1, -\beta_2) \right\} +$$

$$+ \frac{1}{2i} \left[\exp(i\gamma) W_2^{(11)}(\alpha, \beta, \beta_1, \beta_2) - \exp(-i\gamma) W_2^{(11)}(\alpha, \beta, \beta_1, -\beta_2) \right]$$

$$\{ \lim_{z\to\infty} \left[\operatorname{Re}\left(\alpha^2 z^2 + 2\alpha\beta z - \beta_1 z \right) - \left| \operatorname{Im}(\beta_2 z) \right| - (n-2) \ln|z| \right] = +\infty,$$

$$\lim_{z\to\infty} \left[\operatorname{Re}(\beta_1 z) + \left| \operatorname{Im}(\beta_2 z) \right| + n \ln|z| \right] = -\infty,\ \beta_2^2 \neq -\beta_1^2,\ \lim_{z\to\infty} \operatorname{Re}(\alpha z) = \pm\infty \}.$$

Introduced notations:

1) $$W_1^{(11)}(\alpha, \beta, \beta_1, \beta_2) = \sum_{k=0}^{n} \frac{n!}{2^k} \left(-\frac{1}{\beta_1 + i\beta_2} \right)^{n+1-k} \sum_{l=0}^{E(k/2)} \frac{(\beta_1 - 2\alpha\beta + i\beta_2)^{k-2l}}{l!(k-2l)!\alpha^{2k-2l}},$$

$$W_1^{(11)}(\alpha, \beta, \beta_1, i\beta_1 - 2i\alpha\beta) = \frac{n!}{(2\alpha)^{n+1}} \sum_{k=0}^{E(n/2)} \frac{1}{k!(-\beta)^{n+1-2k}};$$

2) $$W_2^{(11)}(\alpha, \beta, \beta_1, \beta_2) = n! \operatorname{erf}(\beta) \left(-\frac{1}{\beta_1 + i\beta_2} \right)^{n+1} + \frac{1}{\sqrt{\pi}} \exp\left(-\beta^2 \right) \sum_{k=1}^{n} \frac{n!}{2^{k-1}} \times$$

$$\times\left(-\frac{1}{\beta_1+i\beta_2}\right)^{n+1-k}\left[\sum_{l=1}^{k-E(k/2)}\frac{(l-1)!}{(2l-1)!(k+1-2l)!}\sum_{r=1}^{l}\frac{4^{l-r}(\beta_1-2\alpha\beta+i\beta_2)^{k-1-2l+2r}}{(r-1)!\alpha^{2k-1-2l+2r}}-\right.$$

$$\left.-\sum_{l=1}^{E(k/2)}\frac{1}{l!(k-2l)!}\sum_{r=1}^{l}\frac{r!(\beta_1-2\alpha\beta+i\beta_2)^{k-1-2l+2r}}{(2r)!\alpha^{2k-1-2l+2r}}\right],$$

$$W_2^{(11)}(\alpha,\beta,\beta_1,i\beta_1-2i\alpha\beta)=n!\ \operatorname{erf}(\beta)\left(-\frac{1}{2\alpha\beta}\right)^{n+1}+\frac{(-1)^n\,n!}{\sqrt{\pi}\alpha^{n+1}}\exp\left(-\beta^2\right)\times$$

$$\times\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!}{(2k-1)!(2\beta)^{n+2-2k}}.$$

2.12. Integrals of $z^{2n+1}\operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)$

2.12.1.

1. $$\int_0^{+\infty} z\operatorname{erf}(az+b)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=\frac{a_1\sin\gamma+a_2\cos\gamma}{2a_1^2+2a_2^2}\operatorname{erf}(b)+$$

$$+\frac{a}{2a_1^2+2a_2^2}\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times$$

$$\times\operatorname{Re}\left\{\frac{a_1+ia_2}{\sqrt{a^2+a_1-ia_2}}\left[1-\operatorname{erf}\left(\frac{ab}{\sqrt{a^2+a_1-ia_2}}\right)\right]\right\}+\frac{a}{2a_1^2+2a_2^2}\times$$

$$\times\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times$$

$$\times\operatorname{Im}\left\{\frac{a_1+ia_2}{\sqrt{a^2+a_1-ia_2}}\left[1-\operatorname{erf}\left(\frac{ab}{\sqrt{a^2+a_1-ia_2}}\right)\right]\right\}\ \{a_1>0\}.$$

2. $$\int_0^{+\infty} z\operatorname{erfi}(az+b)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=$$

$$=\int_{-\infty}^{0} z\operatorname{erfi}(az-b)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=$$

$$=\frac{a_1\sin\gamma+a_2\cos\gamma}{2a_1^2+2a_2^2}\operatorname{erfi}(b)+\frac{a}{2a_1^2+2a_2^2}\times$$

$$\times\exp\left(b^2\frac{a_1^2+a_2^2-a^2a_1}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\times$$

$$\times\operatorname{Re}\left\{\frac{a_1+ia_2}{\sqrt{a_1-a^2-ia_2}}\left[1+\operatorname{erf}\left(\frac{ab}{\sqrt{a_1-a^2-ia_2}}\right)\right]\right\}+\frac{a}{2a_1^2+2a_2^2}\times$$

$$\times\exp\left(b^2\frac{a_1^2+a_2^2-a^2a_1}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\times$$

$$\times\operatorname{Im}\left\{\frac{a_1+ia_2}{\sqrt{a_1-a^2-ia_2}}\left[1+\operatorname{erf}\left(\frac{ab}{\sqrt{a_1-a^2-ia_2}}\right)\right]\right\}$$

$$\{a_1>a^2\ \text{ or }\left[a_1=a^2>0,\ a_2\neq 0,\ ab\leq 0\right]\}.$$

3. $$\int_0^{+\infty} z\operatorname{erfi}(az+b)\exp\left(-a^2z^2\right)\sin\left(a_1z^2+\gamma\right)dz=$$

$$=\int_{-\infty}^{0} z\operatorname{erfi}(az-b)\exp\left(-a^2z^2\right)\sin\left(a_1z^2+\gamma\right)dz=$$

$$=\frac{a^2\sin\gamma+a_1\cos\gamma}{2a^4+2a_1^2}\operatorname{erfi}(b)+\frac{a}{\left(a^4+a_1^2\right)\sqrt{8a_1}}\exp\left(b^2\right)\times$$

$$\times\left\{\sin\left(\gamma+\frac{a^2b^2}{a_1}\right)\left[\left(a^2-a_1\right)\operatorname{Re}\ \operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)-\left(a^2+a_1\right)\times\right.\right.$$

$$\left.\times\operatorname{Im}\ \operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)+a^2-a_1\right]+\cos\left(\gamma+\frac{a^2b^2}{a_1}\right)\left[\left(a^2+a_1\right)\operatorname{Re}\ \operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)+\right.$$

$$\left.\left.+\left(a^2-a_1\right)\operatorname{Im}\ \operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)+a^2+a_1\right]\right\}\ \{a>0,a_1>0,b\leq 0\}.$$

4. $$\int_{-\infty}^{+\infty} z\operatorname{erf}(az+b)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=\frac{a}{a_1^2+a_2^2}\times$$

$$\times\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\left[\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times\right.$$

$$\times \mathrm{Re}\left(\frac{a_1+ia_2}{\sqrt{a^2+a_1-ia_2}}\right)+\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\ \mathrm{Im}\left(\frac{a_1+ia_2}{\sqrt{a^2+a_1-ia_2}}\right)\Bigg]$$

$$\{a_1>0\}.$$

5. $$\int_{-\infty}^{+\infty} z\,\mathrm{erfi}(az+b)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=\frac{a}{a_1^2+a_2^2}\times$$

$$\times\exp\left(b^2\frac{a_1^2+a_2^2-a^2a_1}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\left[\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\times\right.$$

$$\left.\times \mathrm{Re}\left(\frac{a_1+ia_2}{\sqrt{a_1-a^2-ia_2}}\right)+\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\mathrm{Im}\left(\frac{a_1+ia_2}{\sqrt{a_1-a^2-ia_2}}\right)\right]$$

$$\{\,a_1>a^2 \text{ or } \left[a_1=a^2>0,\ a_2\neq 0, b=0\right]\}.$$

6. $$\int_{-\infty}^{+\infty} z\,\mathrm{erfi}(az)\exp\left(-a^2z^2\right)\sin\left(a_1z^2+\gamma\right)dz=\frac{a}{\left(a^4+a_1^2\right)\sqrt{2a_1}}\times$$

$$\times\left[\left(a^2-a_1\right)\sin\gamma+\left(a^2+a_1\right)\cos\gamma\right]\ \{a_1>0\}.$$

7. $$\int_{0}^{\infty} z\ \mathrm{erf}\,(\alpha z+\beta)\exp\left(-\alpha_1z^2\right)\sin\left(\alpha_2z^2+\gamma\right)\,dz=\frac{\alpha_1\sin\gamma+\alpha_2\cos\gamma}{2\alpha_1^2+2\alpha_2^2}\times$$

$$\times\mathrm{erf}\,(\beta)+\frac{\alpha}{4}\left\{\frac{1}{(\alpha_2-i\alpha_1)\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\exp\left(-\beta^2\frac{\alpha_1+i\alpha_2}{\alpha^2+\alpha_1+i\alpha_2}-i\gamma\right)\times\right.$$

$$\times\left[A_1-\mathrm{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\right)\right]+\frac{1}{(\alpha_2+i\alpha_1)\sqrt{\alpha^2+\alpha_1-i\alpha_2}}\times$$

$$\left.\times\exp\left(-\beta^2\frac{\alpha_1-i\alpha_2}{\alpha^2+\alpha_1-i\alpha_2}+i\gamma\right)\left[A_2-\mathrm{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1-i\alpha_2}}\right)\right]\right\}$$

$$\{\ \lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2z^2+\alpha_1z^2+2\alpha\beta z\right)-\left|\mathrm{Im}\left(\alpha_2z^2\right)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha_1z^2\right)-\left|\mathrm{Im}\left(\alpha_2z^2\right)\right|\right]=+\infty;$$

$$A_k=\pm1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=\pm\infty\ \}.$$

8. $\int_0^{\infty} z\,\mathrm{erf}(\alpha z+\beta)\exp\left[\left(\alpha^2 \pm i\alpha_1\right)z^2\right]\sin\left(\alpha_1 z^2+\gamma\right)dz = \frac{(1\mp i)\alpha}{8\sqrt{\alpha_1}\left(\alpha^2 \pm 2i\alpha_1\right)}\times$

$$\times\exp\left[\pm i\left(\beta^2\frac{\alpha^2 \pm 2i\alpha_1}{2\alpha_1}+\gamma\right)\right]\left[\mathrm{erf}\left(\frac{1\pm i}{2}\cdot\frac{\alpha\beta}{\sqrt{\alpha_1}}\right)-A\right]+\frac{\mathrm{erf}(\beta)\exp(\pm i\gamma)}{8\alpha_1 \mp 4i\alpha^2}\mp$$

$$\mp\frac{i\exp(\mp i\gamma)}{4\alpha^2}\left[\mathrm{erf}(\beta)+\frac{\exp\left(-\beta^2\right)}{\sqrt{\pi}\beta}\right]\ \{\lim_{z\to\infty}\left[\mathrm{Re}(\alpha\beta z)\pm\mathrm{Im}\left(\alpha_1 z^2\right)+\frac{1}{2}\ln|z|\right]=$$

$$=\lim_{z\to\infty}\mathrm{Re}(\alpha\beta z)=+\infty,\ \lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2\right)\mp 2\,\mathrm{Im}\left(\alpha_1 z^2\right)\right]=\lim_{z\to\infty}\mathrm{Re}\left(\alpha^2 z^2\right)=-\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left[(1\mp i)\sqrt{\alpha_1}\,z\right]=+\infty,\quad A=-1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left[(1\mp i)\sqrt{\alpha_1}\,z\right]=-\infty\ \}.$$

2.12.2.

1. $\int_0^{+\infty} z^{2n+1}\mathrm{erf}(az+b)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz = \frac{n!a}{2}\times$

$$\times\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times$$

$$\times\mathrm{Re}\left\{\left[1-\mathrm{erf}\left(\frac{ab}{\sqrt{a^2+a_1-ia_2}}\right)\right]\frac{W_1^{(12)}(a,a_1,a_2,b)}{\sqrt{a^2+a_1-ia_2}}\right\}+\frac{n!a}{2}\times$$

$$\times\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\mathrm{Im}\left\{\left[1-\mathrm{erf}\left(\frac{ab}{\sqrt{a^2+a_1-ia_2}}\right)\right]\frac{W_1^{(12)}(a,a_1,a_2,b)}{\sqrt{a^2+a_1-ia_2}}\right\}\times$$

$$\times\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)+\frac{n!\,\mathrm{erf}(b)}{2}\left\{\mathrm{Re}\left[\frac{1}{(a_1-ia_2)^{n+1}}\right]\sin\gamma+\right.$$

$$\left.+\mathrm{Im}\left[\frac{1}{(a_1-ia_2)^{n+1}}\right]\cos\gamma\right\}+$$

$$+\frac{n!a}{2\sqrt{\pi}}\left[\mathrm{Re}W_2^{(12)}(a,a_1,a_2,b)\cdot\sin\gamma+\mathrm{Im}W_2^{(12)}(a,a_1,a_2,b)\cdot\cos\gamma\right]\qquad\{a_1>0\}.$$

2. $\int_0^{+\infty} z^{2n+1}\mathrm{erfi}(az+b)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz =$

$$= \int_{-\infty}^{0} z^{2n+1} \operatorname{erfi}(az-b)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz = \frac{n!a}{2}\exp\left(b^2\,\frac{a_1^2+a_2^2-a^2 a_1}{a^4+a_1^2+a_2^2-2a^2 a_1}\right)\times$$

$$\times\sin\left(\gamma+\frac{a^2 a_2 b^2}{a^4+a_1^2+a_2^2-2a^2 a_1}\right)\operatorname{Re}\left\{\left[\operatorname{erf}\left(\frac{ab}{\sqrt{a_1-a^2-ia_2}}\right)+1\right]\frac{W_1^{(12)}(ia,a_1,a_2,ib)}{\sqrt{a_1-a^2-ia_2}}\right\}+$$

$$+\frac{n!a}{2}\exp\left(b^2\,\frac{a_1^2+a_2^2-a^2 a_1}{a^4+a_1^2+a_2^2-2a^2 a_1}\right)\cos\left(\gamma+\frac{a^2 a_2 b^2}{a^4+a_1^2+a_2^2-2a^2 a_1}\right)\times$$

$$\times\operatorname{Im}\left\{\left[1+\operatorname{erf}\left(\frac{ab}{\sqrt{a_1-a^2-ia_2}}\right)\right]\frac{W_1^{(12)}(ia,a_1,a_2,ib)}{\sqrt{a_1-a^2-ia_2}}\right\}+$$

$$+\frac{n!\operatorname{erfi}(b)}{2}\left\{\operatorname{Re}\left[\frac{1}{(a_1-ia_2)^{n+1}}\right]\sin\gamma+\operatorname{Im}\left[\frac{1}{(a_1-ia_2)^{n+1}}\right]\cos\gamma\right\}+\frac{n!a}{2\sqrt{\pi}}\times$$

$$\times\left[\operatorname{Re}W_2^{(12)}(ia,a_1,a_2,ib)\cdot\sin\gamma+\operatorname{Im}W_2^{(12)}(ia,a_1,a_2,ib)\cdot\cos\gamma\right]$$

$\{a_1>a^2$ or $\left[a_1=a^2>0,\ a_2\neq 0,\ ab<0\right]$ for $n>0\}$.

3. $$\int_{0}^{+\infty} z^{2n+1}\operatorname{erfi}(az+b)\exp\left(-a^2 z^2\right)\sin\left(a_1 z^2+\gamma\right)dz=$$

$$= \int_{-\infty}^{0} z^{2n+1}\operatorname{erfi}(az-b)\exp\left(-a^2 z^2\right)\sin\left(a_1 z^2+\gamma\right)dz=\frac{n!a}{\sqrt{2a_1}}\exp\left(b^2\right)\sin\left(\gamma+\frac{a^2b^2}{a_1}\right)\times$$

$$\times\operatorname{Re}\left\{\left[1+\operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)\right]\frac{W_1^{(12)}\left(ia,a^2,a_1,ib\right)}{1-i}\right\}+\frac{n!a}{\sqrt{2a_1}}\exp\left(b^2\right)\times$$

$$\times\cos\left(\gamma+\frac{a^2b^2}{a_1}\right)\operatorname{Im}\left\{\left[1+\operatorname{erf}\left(\frac{1+i}{\sqrt{2a_1}}ab\right)\right]\frac{W_1^{(12)}\left(ia,a^2,a_1,ib\right)}{1-i}\right\}+$$

$$+\frac{n!\operatorname{erfi}(b)}{2}\left\{\operatorname{Re}\left[\frac{1}{\left(a^2-ia_1\right)^{n+1}}\right]\sin\gamma+\operatorname{Im}\left[\frac{1}{\left(a^2-ia_1\right)^{n+1}}\right]\cos\gamma\right\}+$$

$$+\frac{n!a}{2\sqrt{\pi}}\left[\operatorname{Re}W_2^{(12)}\left(ia,a^2,a_1,ib\right)\cdot\sin\gamma+\right.$$

$$\left.+\operatorname{Im}W_2^{(12)}\left(ia,a^2,a_1,ib\right)\cdot\cos\gamma\right]$$ $\{a>0,\ a_1>0,\ b<0$ – for $n>0\}$.

4. $\int\limits_{-\infty}^{+\infty} z^{2n+1}\operatorname{erf}\,(az+b)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz=$

$$=n!a\exp\left(-b^2\frac{a^2a_1+a_1^2+a_2^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\left\{\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times\right.$$

$$\times\operatorname{Re}\left[\frac{W_1^{(12)}(a,a_1,a_2,b)}{\sqrt{a^2+a_1-ia_2}}\right]+\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2+2a^2a_1}\right)\times$$

$$\left.\times\operatorname{Im}\left[\frac{W_1^{(12)}(a,a_1,a_2,b)}{\sqrt{a^2+a_1-ia_2}}\right]\right\}\qquad\{a_1>0\}.$$

5. $\int\limits_{-\infty}^{+\infty} z^{2n+1}\operatorname{erfi}\,(az+b)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz=$

$$=n!a\exp\left(b^2\frac{a_1^2+a_2^2-a^2a_1}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\left\{\sin\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\times\right.$$

$$\times\operatorname{Re}\left[\frac{W_1^{(12)}(ia,a_1,a_2,ib)}{\sqrt{a_1-a^2-ia_2}}\right]+\cos\left(\gamma+\frac{a^2a_2b^2}{a^4+a_1^2+a_2^2-2a^2a_1}\right)\times$$

$$\left.\times\operatorname{Im}\left[\frac{W_1^{(12)}(ia,a_1,a_2,ib)}{\sqrt{a_1-a^2-ia_2}}\right]\right\}\ \{a_1>a^2\ \text{for}\ \ n>0\}.$$

6. $\int\limits_{0}^{\infty} z^{2n+1}\operatorname{erf}\left(\alpha z+\beta\right)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz=\frac{n!i\alpha}{4}\times$

$$\times\left\{\exp\left[-\frac{\beta^2(\alpha_1-i\alpha_2)}{\alpha^2+\alpha_1-i\alpha_2}+i\gamma\right]\left[\operatorname{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1-i\alpha_2}}\right)-A_1\right]\frac{W_1^{(12)}(\alpha,\alpha_1,\alpha_2,\beta)}{\sqrt{\alpha^2+\alpha_1-i\alpha_2}}+\right.$$

$$\left.+\exp\left[-\frac{\beta^2(\alpha_1+i\alpha_2)}{\alpha^2+\alpha_1+i\alpha_2}-i\gamma\right]\left[A_2-\operatorname{erf}\left(\frac{\alpha\beta}{\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\right)\right]\frac{W_1^{(12)}(\alpha,\alpha_1,-\alpha_2,\beta)}{\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\right\}+$$

$$+\frac{n!}{4i}\left\{\exp(i\gamma)\left[\operatorname{erf}(\beta)\left(\frac{1}{\alpha_1-i\alpha_2}\right)^{n+1}+\frac{\alpha}{\sqrt{\pi}}W_2^{(12)}(\alpha,\alpha_1,\alpha_2,\beta)\right]-\right.$$

$$\left.-\exp(-i\gamma)\left[\operatorname{erf}(\beta)\left(\frac{1}{\alpha_1+i\alpha_2}\right)^{n+1}+\frac{\alpha}{\sqrt{\pi}}W_2^{(12)}(\alpha,\alpha_1,-\alpha_2,\beta)\right]\right\}$$

$$\{\ \lim_{z\to\infty} \mathrm{Re}\left(\alpha^2 z^2 + \alpha_1 z^2 + 2\alpha\beta z\right) - \left|\mathrm{Im}\left(\alpha_2 z^2\right)\right| - (2n-1)\ln|z| =$$

$$= \lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha_1 z^2\right) - \left|\mathrm{Im}\left(\alpha_2 z^2\right)\right| - 2n\ln|z|\right] = +\infty\ ;$$

$$A_k = \pm 1 \text{ if } \lim_{z\to\infty} \mathrm{Re}\left[\sqrt{\alpha^2 + \alpha_1 + (2k-3)i\alpha_2}\, z\right] = \pm\infty\ \}.$$

7. $$\int_0^\infty z^{2n+1} \mathrm{erf}\,(\alpha z + \beta)\exp\left[\left(\alpha^2 \pm i\alpha_1\right)z^2\right]\sin\left(\alpha_1 z^2 + \gamma\right) dz = \frac{n!(1\mp i)\alpha}{8\sqrt{\alpha_1}} \times$$

$$\times \exp\left[-\beta^2 \pm i\left(\frac{\alpha^2\beta^2}{2\alpha_1} + \gamma\right)\right]\left[A - \mathrm{erf}\left(\frac{1\pm i}{2}\cdot\frac{\alpha\beta}{\sqrt{\alpha_1}}\right)\right] W_1^{(12)}\left(\alpha, -\alpha^2 \mp i\alpha_1, \pm\alpha_1, \beta\right) \pm$$

$$\pm \frac{n!}{4i}\exp(\pm i\gamma)\left[\frac{\alpha}{\sqrt{\pi}} W_2^{(12)}\left(\alpha, -\alpha^2 \mp i\alpha_1, \pm\alpha_1, \beta\right) + \mathrm{erf}\,(\beta)\left(-\frac{1}{\alpha^2 \pm 2i\alpha_1}\right)^{n+1}\right] \pm$$

$$\pm \frac{n!i}{4}\exp(\mp i\gamma)\left[\frac{\mathrm{erf}\,(\beta)}{\left(-\alpha^2\right)^{n+1}} + \frac{\exp\left(-\beta^2\right)}{\sqrt{\pi}\beta}\sum_{k=0}^{n}\frac{(2k)!}{k!(2\alpha\beta)^{2k}\left(-\alpha^2\right)^{n+1-k}}\right]$$

$$\{\ \lim_{z\to\infty}\left[\mathrm{Re}(2\alpha\beta z) \pm 2\,\mathrm{Im}\left(\alpha_1 z^2\right) - (2n-1)\ln|z|\right] = \lim_{z\to\infty}\left[\mathrm{Re}(\alpha\beta z) - n\ln|z|\right] = +\infty\ ,$$

$$\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2\right) \mp 2\,\mathrm{Im}\left(\alpha_1 z^2\right) + 2n\ln|z|\right] = \lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2\right) + 2n\ln|z|\right] = -\infty\ ;$$

$$A = 1 \text{ if } \lim_{z\to\infty} \mathrm{Re}\left[(1\mp i)\sqrt{\alpha_1}\, z\right] = +\infty\ ,\ A = -1 \text{ if } \lim_{z\to\infty} \mathrm{Re}\left[(1\mp i)\sqrt{\alpha_1}\, z\right] = -\infty\ \}.$$

Introduced notations:

1) $$W_1^{(12)}(\alpha, \alpha_1, \alpha_2, \beta) = \sum_{k=0}^{n}\frac{(2k)!}{k!}\left(\frac{1}{\alpha_1 - i\alpha_2}\right)^{n+1-k} \times$$

$$\times \sum_{l=0}^{k}\frac{(\alpha\beta)^{2k-2l}}{4^l\, l!(2k-2l)!\left(\alpha^2 + \alpha_1 - i\alpha_2\right)^{2k-l}},$$

$$W_1^{(12)}(\alpha, \alpha_1, \alpha_2, 0) = \sum_{k=0}^{n}\frac{(2k)!}{4^k\,(k!)^2\left(\alpha^2 + \alpha_1 - i\alpha_2\right)^k}\left(\frac{1}{\alpha_1 - i\alpha_2}\right)^{n+1-k};$$

2) $$W_2^{(12)}(\alpha, \alpha_1, \alpha_2, \beta) = \exp\left(-\beta^2\right)\sum_{k=1}^{n}\frac{(2k)!}{k!}\left(\frac{1}{\alpha_1 - i\alpha_2}\right)^{n+1-k} \times$$

$$\times\left[\sum_{l=1}^{k}\frac{1}{l!(2k-2l)!}\sum_{r=1}^{l}\frac{r!(\alpha\beta)^{2k-1-2l+2r}}{4^{l-r}(2r)!\left(\alpha^2+\alpha_1-i\alpha_2\right)^{2k-l+r}}-\sum_{l=1}^{k}\frac{(l-1)!}{(2l-1)!(2k+1-2l)!}\times\right.$$

$$\left.\times\sum_{r=1}^{l}\frac{(\alpha\beta)^{2k-1-2l+2r}}{(r-1)!\left(\alpha^2+\alpha_1-i\alpha_2\right)^{2k-l+r}}\right],\quad W_2^{(12)}(\alpha,\alpha_1,\alpha_2,0)=0.$$

2.13. Integrals of $z^n \operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1 z^2+\beta_1 z\right)\sin\left(\alpha_2 z^2+\beta_2 z+\gamma\right)$, $z^n \operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin(\beta z)$, $z^n \operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\cos(\beta z)$

2.13.1.

1. $$\int_0^{+\infty}\operatorname{erf}(az)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=-\int_{-\infty}^{0}\operatorname{erf}(az)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=$$

$$=\frac{1}{2\sqrt{\pi}}\left[\operatorname{Re}\left(\frac{1}{\sqrt{a_1+ia_2}}\ln\frac{\sqrt{a_1+ia_2}+ia}{\sqrt{a_1+ia_2}-ia}\right)\cos\gamma+\operatorname{Im}\left(\frac{1}{\sqrt{a_1+ia_2}}\ln\frac{\sqrt{a_1+ia_2}+ia}{\sqrt{a_1+ia_2}-ia}\right)\sin\gamma\right]$$

$\{a_1>0$ or $[a_1=0,\,a_2\neq 0]\}$.

2. $$\int_0^{+\infty}\operatorname{erf}(az)\exp\left(-a^2z^2+bz\right)\sin(b_1z+\gamma)\,dz=\frac{\sqrt{\pi}}{4a}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\times$$

$$\times\operatorname{Re}\left\{\left[\operatorname{erf}\left(\frac{b+ib_1}{2\sqrt{2}a}\right)+1\right]^2\right\}+\frac{\sqrt{\pi}}{4a}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\times$$

$$\times\operatorname{Im}\left\{\left[\operatorname{erf}\left(\frac{b+ib_1}{2\sqrt{2}a}\right)+1\right]^2\right\}\qquad\{a>0\}.$$

3. $$\int_0^{+\infty}\operatorname{erfi}(az)\exp\left(-a_1z^2\right)\sin\left(a_2z^2+\gamma\right)dz=\frac{1}{2\sqrt{\pi}}\times$$

$$\times\left[\operatorname{Re}\left(\frac{1}{\sqrt{a_1+ia_2}}\ln\frac{\sqrt{a_1+ia_2}+a}{\sqrt{a_1+ia_2}-a}\right)\sin\gamma-\operatorname{Im}\left(\frac{1}{\sqrt{a_1+ia_2}}\ln\frac{\sqrt{a_1+ia_2}+a}{\sqrt{a_1+ia_2}-a}\right)\cos\gamma\right]$$

$\{a_1>a^2\}$.

4. $$\int_0^{+\infty}\operatorname{erf}(az)\exp\left(-a_1z^2\right)\sin(bz)dz=\frac{1}{2}\int_{-\infty}^{+\infty}\operatorname{erf}(az)\exp\left(-a_1z^2\right)\sin(bz)dz=$$

$$= \sqrt{\frac{\pi}{4a_1}} \exp\left(-\frac{b^2}{4a_1}\right) \operatorname{erfi}\left(\frac{ab}{2\sqrt{a_1^2 + a^2 a_1}}\right) \quad \{ a_1 > 0 \}.$$

5. $$\int_0^{+\infty} \operatorname{erfi}(az) \exp\left(-a_1 z^2\right) \sin(bz)\, dz = \frac{1}{2} \int_{-\infty}^{+\infty} \operatorname{erfi}(az) \exp\left(-a_1 z^2\right) \sin(bz)\, dz =$$

$$= \sqrt{\frac{\pi}{4a_1}} \exp\left(-\frac{b^2}{4a_1}\right) \operatorname{erf}\left(\frac{ab}{2\sqrt{a_1^2 - a^2 a_1}}\right) \quad \{a_1 > a^2\}.$$

6. $$\int_{-\infty}^{+\infty} \operatorname{erf}(az + b) \exp\left(-a_1 z^2 + b_1 z\right) \sin\left(a_2 z^2 + b_2 z + \gamma\right) dz =$$

$$= \sqrt{\pi} \exp\left(\frac{a_1 b_1^2 - a_1 b_2^2 - 2a_2 b_1 b_2}{4a_1^2 + 4a_2^2}\right) \left\{ \sin\left(\frac{2a_1 b_1 b_2 + a_2 b_1^2 - a_2 b_2^2}{4a_1^2 + 4a_2^2} + \gamma\right) \times\right.$$

$$\times \operatorname{Re}\left[\frac{1}{\sqrt{a_1 + ia_2}} \operatorname{erf}\left(\frac{2a_1 b + ab_1 + 2ia_2 b - iab_2}{2\sqrt{a_1 + ia_2}\sqrt{a^2 + a_1 + ia_2}}\right)\right] - \cos\left(\frac{2a_1 b_1 b_2 + a_2 b_1^2 - a_2 b_2^2}{4a_1^2 + 4a_2^2} + \gamma\right) \times$$

$$\left. \times \operatorname{Im}\left[\frac{1}{\sqrt{a_1 + ia_2}} \operatorname{erf}\left(\frac{2a_1 b + ab_1 + 2ia_2 b - iab_2}{2\sqrt{a_1 + ia_2}\sqrt{a^2 + a_1 + ia_2}}\right)\right]\right\}$$

$$\{a_1 > 0 \text{ or } [a_1 = b_1 = 0, a_2 \neq 0]\}.$$

7. $$\int_{-\infty}^{+\infty} \operatorname{erf}(az) \exp\left(-a^2 z^2 + bz\right) \sin(b_1 z + \gamma)\, dz = \frac{\sqrt{\pi}}{a} \exp\left(\frac{b^2 - b_1^2}{2\sqrt{2}a}\right) \times$$

$$\times \left[\sin\left(\frac{bb_1}{2a^2} + \gamma\right) \operatorname{Re} \operatorname{erf}\left(\frac{b + ib_1}{2\sqrt{2}a}\right) + \cos\left(\frac{bb_1}{2a^2} + \gamma\right) \operatorname{Im} \operatorname{erf}\left(\frac{b + ib_1}{2\sqrt{2}a}\right)\right] \quad \{a > 0\}.$$

8. $$\int_{-\infty}^{+\infty} \operatorname{erfi}(az + b) \exp\left(-a_1 z^2 + b_1 z\right) \sin\left(a_2 z^2 + b_2 z + \gamma\right) dz =$$

$$= \sqrt{\pi} \exp\left(\frac{a_1 b_1^2 - a_1 b_2^2 - 2a_2 b_1 b_2}{4a_1^2 + 4a_2^2}\right) \left\{ \sin\left(\frac{2a_1 b_1 b_2 + a_2 b_1^2 - a_2 b_2^2}{4a_1^2 + 4a_2^2} + \gamma\right) \times\right.$$

$$\times \operatorname{Re}\left[\frac{1}{\sqrt{a_1 + ia_2}} \operatorname{erfi}\left(\frac{2a_1 b + ab_1 + 2ia_2 b - iab_2}{2\sqrt{a_1 + ia_2}\sqrt{a_1 - a^2 + ia_2}}\right)\right] -$$

$$\left. - \cos\left(\frac{2a_1 b_1 b_2 + a_2 b_1^2 - a_2 b_2^2}{4a_1^2 + 4a_2^2} + \gamma\right) \operatorname{Im}\left[\frac{1}{\sqrt{a_1 + ia_2}} \operatorname{erfi}\left(\frac{2a_1 b + ab_1 + 2ia_2 b - iab_2}{2\sqrt{a_1 + ia_2}\sqrt{a_1 - a^2 + ia_2}}\right)\right]\right\}$$

$$\{ a_1 > a^2 \}.$$

9. $$\int_0^\infty \operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz=\frac{1}{4\sqrt{\pi}}\left[\frac{\exp(i\gamma)}{\sqrt{\alpha_1-i\alpha_2}}\times\right.$$

$$\left.\times\ln\frac{\sqrt{\alpha_1-i\alpha_2}-i\alpha}{\sqrt{\alpha_1-i\alpha_2}+i\alpha}+\frac{\exp(-i\gamma)}{\sqrt{\alpha_1+i\alpha_2}}\ln\frac{\sqrt{\alpha_1+i\alpha_2}+i\alpha}{\sqrt{\alpha_1+i\alpha_2}-i\alpha}\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right)-\left|\operatorname{Im}\left(\alpha_2 z^2\right)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2\right)-\left|\operatorname{Im}\left(\alpha_2 z^2\right)\right|+2\ln|z|\right]=+\infty,$$

$$\alpha_2^2\neq-\alpha_1^2,\ \alpha_2^2\neq-\left(\alpha^2+\alpha_1\right)^2\ \}.$$

10. $$\int_0^\infty \operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2+\beta z\right)\sin(\beta_1 z+\gamma)\,dz=\frac{\sqrt{\pi}}{8i\alpha}\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]\times$$

$$\times\left[\operatorname{erf}\left(\frac{\beta+i\beta_1}{2\sqrt{2}\alpha}\right)\pm1\right]^2-\frac{\sqrt{\pi}}{8i\alpha}\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\left[\operatorname{erf}\left(\frac{\beta-i\beta_1}{2\sqrt{2}\alpha}\right)\pm1\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2-\beta z\right)-\left|\operatorname{Im}(\beta_1 z)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2-\beta z\right)-\left|\operatorname{Im}(\beta_1 z)\right|+2\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

11. $$\int_0^\infty \operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin(\beta z)\,dz=\pm\frac{\sqrt{\pi}}{2\sqrt{\alpha_1}}\exp\left(-\frac{\beta^2}{4\alpha_1}\right)\times$$

$$\times\operatorname{erfi}\left(\frac{\alpha\beta}{2\sqrt{\alpha_1}\sqrt{\alpha^2+\alpha_1}}\right)\ \{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right)-\left|\operatorname{Im}(\beta z)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2\right)-\left|\operatorname{Im}(\beta z)\right|+2\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1}\,z\right)=\pm\infty\ \}.$$

12. $$\int_{\infty(T_1)}^{\infty(T_2)} \operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1 z^2+\beta_1 z\right)\sin\left(\alpha_2 z^2+\beta_2 z+\gamma\right)dz=\frac{\sqrt{\pi}i}{2}\times$$

$$\times\left\{\frac{A_1}{\sqrt{\alpha_1+i\alpha_2}}\exp\left[\frac{(\beta_1-i\beta_2)^2}{4\alpha_1+4i\alpha_2}-i\gamma\right]\operatorname{erf}\left[\frac{\alpha(\beta_1-i\beta_2)+2\beta(\alpha_1+i\alpha_2)}{2\sqrt{\alpha_1+i\alpha_2}\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\right]-\right.$$

$$\left.-\frac{A_2}{\sqrt{\alpha_1-i\alpha_2}}\exp\left[\frac{(\beta_1+i\beta_2)^2}{4\alpha_1-4i\alpha_2}+i\gamma\right]\operatorname{erf}\left[\frac{\alpha(\beta_1+i\beta_2)+2\beta(\alpha_1-i\alpha_2)}{2\sqrt{\alpha_1-i\alpha_2}\sqrt{\alpha^2+\alpha_1-i\alpha_2}}\right]\right\}$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+2\ln|z|\right]=+\infty,$$

$$\alpha_2^2\neq-\alpha_1^2;\ A_k=\pm1\ \text{ if }\lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2 z}\right]=$$

$$=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2 z}\right]=\pm\infty,\ A_k=0\ \text{ if}$$

$$\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2 z}\right]\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2 z}\right]=+\infty\}.$$

13. $\int_{-\nu}^{\nu}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz=0.$

14. $\int_{-\nu}^{\nu}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\cos(\beta z)\,dz=0.$

2.13.2.

1. $$\int_0^{+\infty} z\operatorname{erf}(az)\exp\left(-a^2 z^2+bz\right)\sin\left(b_1 z+\gamma\right)dz=\frac{1}{4a^2}\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\times$$

$$\times\left[\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\operatorname{Re}\theta+\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\operatorname{Im}\theta\right]\qquad\{\theta=\frac{\sqrt{\pi}\left(b+ib_1\right)}{2a}\times$$

$$\times\left[1+\operatorname{erf}\left(\frac{b+ib_1}{2\sqrt{2}a}\right)\right]^2+\sqrt{2}\exp\left[-\frac{\left(b+ib_1\right)^2}{8a^2}\right]\left[1+\operatorname{erf}\left(\frac{b+ib_1}{2\sqrt{2}a}\right)\right],\quad a>0\}.$$

2. $$\int_0^{+\infty} z\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz=\frac{1}{2}\int_{-\infty}^{+\infty} z\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz=$$

$$=\frac{1}{4a_1}\left[\frac{2a}{\sqrt{a^2+a_1}}\exp\left(-\frac{b^2}{4a^2+4a_1}\right)-\frac{\sqrt{\pi}b}{\sqrt{a_1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{erfi}\left(\frac{ab}{2\sqrt{a^2a_1+a_1^2}}\right)\right]$$

$$\{a_1>0\}.$$

3. $$\int_0^{+\infty} z\operatorname{erfi}(az)\exp\left(-a_1z^2\right)\cos(bz)\,dz=\frac{1}{2}\int_{-\infty}^{+\infty} z\operatorname{erfi}(az)\exp\left(-a_1z^2\right)\cos(bz)\,dz=$$

$$=\frac{1}{4a_1}\left[\frac{2a}{\sqrt{a_1-a^2}}\exp\left(\frac{b^2}{4a^2-4a_1}\right)-\frac{\sqrt{\pi}\,b}{\sqrt{a_1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{erf}\left(\frac{ab}{2\sqrt{a_1^2-a^2a_1}}\right)\right]$$

$$\{\ a_1>a^2\ \}.$$

4. $$\int_{-\infty}^{+\infty} z\ \operatorname{erf}\,(az+b)\exp\left(-a_1z^2+b_1z\right)\sin\left(a_2z^2+b_2z+\gamma\right)dz=$$

$$=\frac{1}{2}\exp\left[\frac{a_1\left(b_1^2-b_2^2\right)-2a_2b_1b_2}{4a_1^2+4a_2^2}\right]\left\{\sin\left[\frac{a_2\left(b_1^2-b_2^2\right)+2a_1b_1b_2}{4a_1^2+4a_2^2}+\gamma\right]\operatorname{Re}\theta-\right.$$

$$\left.-\cos\left[\frac{a_2\left(b_1^2-b_2^2\right)+2a_1b_1b_2}{4a_1^2+4a_2^2}+\gamma\right]\operatorname{Im}\theta\right\}\qquad\{\,\theta=\frac{\sqrt{\pi}\left(b_1-ib_2\right)}{\left(a_1+ia_2\right)\sqrt{a_1+ia_2}}\times$$

$$\times\operatorname{erf}\left(\frac{2a_1b+ab_1+2ia_2b-iab_2}{2\sqrt{a_1+ia_2}\,\sqrt{a^2+a_1+ia_2}}\right)+\frac{2a}{\left(a_1+ia_2\right)\sqrt{a^2+a_1+ia_2}}\times$$

$$\times\exp\left[-\frac{\left(2a_1b+ab_1+2ia_2b-iab_2\right)^2}{4\left(a_1+ia_2\right)\left(a^2+a_1+ia_2\right)}\right],\ a_1>0\ \}.$$

5. $$\int_{-\infty}^{+\infty} z\ \operatorname{erfi}(az+b)\exp\left(-a_1z^2+b_1z\right)\sin\left(a_2z^2+b_2z+\gamma\right)dz=$$

$$=\frac{1}{2}\exp\left[\frac{a_1\left(b_1^2-b_2^2\right)-2a_2b_1b_2}{4a_1^2+4a_2^2}\right]\left\{\sin\left[\frac{a_2\left(b_1^2-b_2^2\right)+2a_1b_1b_2}{4a_1^2+4a_2^2}+\gamma\right]\operatorname{Im}\theta+\right.$$

$$\left.+\cos\left[\frac{a_2\left(b_1^2-b_2^2\right)+2a_1b_1b_2}{4a_1^2+4a_2^2}+\gamma\right]\operatorname{Re}\theta\right\}\qquad\{\,\theta=\frac{\sqrt{\pi}\left(b_1-ib_2\right)}{\left(a_1+ia_2\right)\sqrt{a_1+ia_2}}\times$$

$$\times \operatorname{erf}\left(\frac{ab_2 - 2a_2 b + 2ia_1 b + iab_1}{2\sqrt{a_1 + ia_2}\sqrt{a_1 - a^2 + ia_2}}\right) + \frac{2ia}{(a_1 + ia_2)\sqrt{a_1 - a^2 + ia_2}} \times$$

$$\times \exp\left[\frac{\left(2a_1 b + ab_1 + 2ia_2 b - iab_2\right)^2}{4\left(a_1 + ia_2\right)\left(a_1 - a^2 + ia_2\right)}\right], \; a_1 > a^2 \}.$$

6. $$\int_0^\infty z \operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2 + \beta z\right)\sin(\beta_1 z + \gamma)\, dz = \frac{1}{8i\alpha^2}\exp\left[\frac{(\beta + i\beta_1)^2}{4\alpha^2} + i\gamma\right] \times$$

$$\times \left\{\frac{\sqrt{\pi}}{2\alpha}(\beta + i\beta_1)\left[\operatorname{erf}\left(\frac{\beta + i\beta_1}{2\sqrt{2}\alpha}\right) \pm 1\right]^2 + \sqrt{2}\exp\left[-\frac{(\beta + i\beta_1)^2}{8\alpha^2}\right]\left[\operatorname{erf}\left(\frac{\beta + i\beta_1}{2\sqrt{2}\alpha}\right) \pm 1\right]\right\} -$$

$$- \frac{1}{8i\alpha^2}\exp\left[\frac{(\beta - i\beta_1)^2}{4\alpha^2} - i\gamma\right] \left\{\frac{\sqrt{\pi}}{2\alpha}(\beta - i\beta_1)\left[\operatorname{erf}\left(\frac{\beta - i\beta_1}{2\sqrt{2}\alpha}\right) \pm 1\right]^2 + \right.$$

$$\left. + \sqrt{2}\exp\left[-\frac{(\beta - i\beta_1)^2}{8\alpha^2}\right]\left[\operatorname{erf}\left(\frac{\beta - i\beta_1}{2\sqrt{2}\alpha}\right) \pm 1\right] \right\}$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 - \beta z\right) - \left|\operatorname{Im}(\beta_1 z)\right|\right] =$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2 - \beta z\right) - \left|\operatorname{Im}(\beta_1 z)\right| + \ln|z|\right] = +\infty, \; \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty \}.$$

7. $$\int_0^\infty z \operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\cos(\beta z)\, dz = \pm\frac{1}{4\alpha_1}\left[\frac{2\alpha}{\sqrt{\alpha^2 + \alpha_1}}\exp\left(-\frac{\beta^2}{4\alpha^2 + 4\alpha_1}\right) - \right.$$

$$\left. - \frac{\sqrt{\pi}\beta}{\sqrt{\alpha_1}}\exp\left(-\frac{\beta^2}{4\alpha_1}\right)\operatorname{erfi}\left(\frac{\alpha\beta}{2\sqrt{\alpha_1}\sqrt{\alpha^2 + \alpha_1}}\right)\right] \qquad \{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right) - \left|\operatorname{Im}(\beta z)\right|\right] =$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + \alpha_1 z^2\right) - \left|\operatorname{Im}(\beta z)\right| + \ln|z|\right] = +\infty, \; \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2 + \alpha_1}\, z\right) = \pm\infty \}.$$

8. $$\int_{\infty(T_1)}^{\infty(T_2)} z \operatorname{erf}(\alpha z + \beta)\exp\left(-\alpha_1 z^2 + \beta_1 z\right)\sin\left(\alpha_2 z^2 + \beta_2 z + \gamma\right) dz =$$

$$= A_1 \frac{i}{4}\exp\left[\frac{(\beta_1 - i\beta_2)^2}{4\alpha_1 + 4i\alpha_2} - i\gamma\right] W_1^{(13)}(1, \alpha, \alpha_1, \alpha_2, \beta, \beta_1, \beta_2) - A_2 \frac{i}{4} \times$$

$$\times \exp\left[\frac{(\beta_1 + i\beta_2)^2}{4\alpha_1 - 4i\alpha_2} + i\gamma\right] W_1^{(13)}(1, \alpha, \alpha_1, -\alpha_2, \beta, \beta_1, -\beta_2)$$

$$\{ \lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+\ln|z|\right]\ =$$

$$=\lim_{z\to\infty(T_2)}[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|\Big]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|+\ln|z|\right]\ =+\infty,$$

$$\alpha_2^2\neq-\alpha_1^2;\ A_k=\pm1\ \text{ if }\lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=$$

$$=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=\pm\infty,\ A_k=0\ \text{ if}$$

$$\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=+\infty\ \}.$$

9. $\int\limits_{-\nu}^{\nu} z\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin(\beta z)\,dz=0.$

2.13.3.

1. $$\int\limits_0^{+\infty} z^{2n}\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2+\gamma\right)dz=\frac{(2n)!}{n!\sqrt{\pi}}\left[\sin\gamma\cdot\operatorname{Re}W_2^{(13)}(a,a_1,a_2)-\right.$$
$$\left.-\cos\gamma\cdot\operatorname{Im}W_2^{(13)}(a,a_1,a_2)\right]\qquad\{a_1>0\ \text{ for }n>0\}.$$

2. $$\int\limits_0^{+\infty} z^n\operatorname{erf}(az)\exp\left(-a^2 z^2+bz\right)\sin\left(b_1 z+\gamma\right)dz=\exp\left(\frac{b^2-b_1^2}{4a^2}\right)\times$$
$$\times\left[\sin\left(\frac{bb_1}{2a^2}+\gamma\right)\operatorname{Re}W_3^{(13)}(a,b,b_1,1)+\cos\left(\frac{bb_1}{2a^2}+\gamma\right)\operatorname{Im}W_3^{(13)}(a,b,b_1,1)\right]$$
$$\{a>0\}.$$

3. $$\int\limits_0^{+\infty} z^{2n}\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\sin(bz)\,dz=\frac{1}{2}\int\limits_{-\infty}^{+\infty} z^{2n}\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\sin(bz)\,dz=$$
$$=-\frac{(2n)!}{2^{2n+1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{Im}W_4^{(13)}(2n,a,a_1,b)\qquad\{a_1>0\}.$$

4. $$\int_0^{+\infty} z^{2n+1}\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2n+1}\operatorname{erf}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz =$$

$$= \frac{(2n+1)!}{4^{n+1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{Re}W_4^{(13)}(2n+1, a, a_1, b) \qquad \{a_1 > 0\}.$$

5. $$\int_0^{+\infty} z^{2n}\operatorname{erfi}(az)\exp\left(-a_1 z^2\right)\sin\left(a_2 z^2 + \gamma\right)dz = \frac{(2n)!}{n!\sqrt{\pi}}\left[\sin\gamma\cdot\operatorname{Im}W_2^{(13)}(ia, a_1, a_2) + \right.$$

$$\left. + \cos\gamma\cdot\operatorname{Re}W_2^{(13)}(ia, a_1, a_2)\right] \qquad \{a_1 > a^2\}.$$

6. $$\int_0^{+\infty} z^{2n}\operatorname{erfi}(az)\exp\left(-a_1 z^2\right)\sin(bz)\,dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2n}\operatorname{erfi}(az)\exp\left(-a_1 z^2\right)\sin(bz)\,dz =$$

$$= \frac{(2n)!}{2^{2n+1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{Re}W_4^{(13)}(2n, ia, a_1, b) \qquad \{a_1 > a^2\}.$$

7. $$\int_0^{+\infty} z^{2n+1}\operatorname{erfi}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2n+1}\operatorname{erfi}(az)\exp\left(-a_1 z^2\right)\cos(bz)\,dz =$$

$$= \frac{(2n+1)!}{4^{n+1}}\exp\left(-\frac{b^2}{4a_1}\right)\operatorname{Im}W_4^{(13)}(2n+1, ia, a_1, b) \qquad \{a_1 > a^2\}.$$

8. $$\int_{-\infty}^{+\infty} z^n\operatorname{erf}(az+b)\exp\left(-a_1 z^2 + b_1 z\right)\sin\left(a_2 z^2 + b_2 z + \gamma\right)dz =$$

$$= \frac{n!}{2^n}\sin\gamma\cdot\operatorname{Re}\left\{\exp\left[\frac{(b_1 - ib_2)^2}{4a_1 + 4ia_2}\right]W_1^{(13)}(n, a, a_1, a_2, b, b_1, b_2)\right\} -$$

$$- \frac{n!}{2^n}\cos\gamma\cdot\operatorname{Im}\left\{\exp\left[\frac{(b_1 - ib_2)^2}{4a_1 + 4ia_2}\right]W_1^{(13)}(n, a, a_1, a_2, b, b_1, b_2)\right\}$$

$\{a_1 > 0$ for $n > 0\}$.

9. $$\int_{-\infty}^{+\infty} z^n\operatorname{erfi}(az+b)\exp\left(-a_1 z^2 + b_1 z\right)\sin\left(a_2 z^2 + b_2 z + \gamma\right)dz =$$

$$= \frac{n!}{2^n}\cos\gamma\cdot\operatorname{Re}\left\{\exp\left[\frac{(b_1 - ib_2)^2}{4a_1 + 4ia_2}\right]W_1^{(13)}(n, ia, a_1, a_2, ib, b_1, b_2)\right\} +$$

$$+ \frac{n!}{2^n}\sin\gamma\cdot\operatorname{Im}\left\{\exp\left[\frac{(b_1 - ib_2)^2}{4a_1 + 4ia_2}\right]W_1^{(13)}(n, ia, a_1, a_2, ib, b_1, b_2)\right\} \quad \{a_1 > a^2\}.$$

10. $\int_0^\infty z^{2n}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz = \frac{(2n)!i}{2\sqrt{\pi}n!}\times$

$$\times\left[\exp(-i\gamma)W_2^{(13)}(\alpha,\alpha_1,\alpha_2)-\exp(i\gamma)W_2^{(13)}(\alpha,\alpha_1,-\alpha_2)\right]$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right)-\left|\operatorname{Im}\left(\alpha_2 z^2\right)\right|-(2n-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2\right)-\left|\operatorname{Im}\left(\alpha_2 z^2\right)\right|-(2n-2)\ln|z|\right]=+\infty,$$

$$\alpha_2^2\neq-\alpha_1^2,\ \alpha_2^2\neq-(\alpha^2+\alpha_1)^2\}.$$

11. $\int_0^\infty z^n\operatorname{erf}(\alpha z)\exp\left(-\alpha^2 z^2+\beta z\right)\sin(\beta_1 z+\gamma)dz=$

$$=\frac{1}{2i}\exp\left[\frac{(\beta+i\beta_1)^2}{4\alpha^2}+i\gamma\right]W_3^{(13)}(\alpha,\beta,\beta_1,\pm1)-\frac{1}{2i}\exp\left[\frac{(\beta-i\beta_1)^2}{4\alpha^2}-i\gamma\right]\times$$

$$\times W_3^{(13)}(\alpha,\beta,-\beta_1,\pm1)\qquad\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2-\beta z\right)-\left|\operatorname{Im}(\beta_1 z)\right|-(n-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2-\beta z\right)-\left|\operatorname{Im}(\beta_1 z)\right|-(n-2)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

12. $\int_0^\infty z^{2n}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin(\beta z)dz=\pm\frac{(2n)!i}{2^{2n+1}}\exp\left(-\frac{\beta^2}{4\alpha_1}\right)W_4^{(13)}(2n,\alpha,\alpha_1,\beta)$

$$\left\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right)-|\operatorname{Im}(\beta z)|-(2n-1)\ln|z|\right]=\right.$$

$$\left.=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2\right)-\left|\operatorname{Im}(\beta z)\right|-(2n-2)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1}\ z\right)=\pm\infty\right\}.$$

13. $\int_0^\infty z^{2n+1}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\cos(\beta z)dz=\pm\frac{(2n+1)!}{4^{n+1}}\exp\left(-\frac{\beta^2}{4\alpha_1}\right)\times$

$$\times W_4^{(13)}(2n+1,\alpha,\alpha_1,\beta)\qquad\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1 z^2\right)-\left|\operatorname{Im}(\beta z)\right|-2n\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2\right)-\left|\operatorname{Im}(\beta z)\right|-(2n-1)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1}\ z\right)=\pm\infty\ \}.$$

14. $\int_{\infty(T_1)}^{\infty(T_2)} z^n\operatorname{erf}(\alpha z+\beta)\exp\left(-\alpha_1 z^2+\beta_1 z\right)\sin\left(\alpha_2 z^2+\beta_2 z+\gamma\right)dz=$

$$=\frac{n!i}{2^{n+1}}\left\{A_1\exp\left[\frac{(\beta_1-i\beta_2)^2}{4\alpha_1+4i\alpha_2}-i\gamma\right]W_1^{(13)}(n,\alpha,\alpha_1,\alpha_2,\beta,\beta_1,\beta_2)-\right.$$

$$\left.-A_2\exp\left[\frac{(\beta_1+i\beta_2)^2}{4\alpha_1-4i\alpha_2}+i\gamma\right]W_1^{(13)}(n,\alpha,\alpha_1,-\alpha_2,\beta,\beta_1,-\beta_2)\right\}$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|-(n-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_1 z^2-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|-(n-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1 z^2+2\alpha\beta z-\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha_2 z^2+\beta_2 z\right)\right|-(n-2)\ln|z|\right]=+\infty,$$

$$\alpha_2^2\neq-\alpha_1^2;\ A_k=\pm1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=$$

$$=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=\pm\infty,\ A_k=0 \text{ if}$$

$$\lim_{z\to\infty(T_1)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left[\sqrt{\alpha^2+\alpha_1+(3-2k)i\alpha_2}\ z\right]=+\infty\}.$$

15. $\int_{-\nu}^{\nu} z^{2n}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin\left(\alpha_2 z^2+\gamma\right)dz=0.$

16. $\int_{-\nu}^{\nu} z^{2n}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\cos(\beta z)dz=0.$

17. $\int_{-\nu}^{\nu} z^{2n+1}\operatorname{erf}(\alpha z)\exp\left(-\alpha_1 z^2\right)\sin(\beta z)dz=0.$

Introduced notations:

1) $W_1^{(13)}(n,\alpha,\alpha_1,\alpha_2,\beta,\beta_1,\beta_2)=\frac{\sqrt{\pi}}{\sqrt{\alpha_1+i\alpha_2}}\times$

$$\times\operatorname{erf}\left[\frac{2\beta(\alpha_1+i\alpha_2)+\alpha(\beta_1-i\beta_2)}{2\sqrt{\alpha_1+i\alpha_2}\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\right]\sum_{l=0}^{E(n/2)}\frac{(\beta_1-i\beta_2)^{n-2l}}{l!(n-2l)!(\alpha_1+i\alpha_2)^{n-l}}+$$

$$+\frac{1}{\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\exp\left\{-\frac{\left[2\beta\ (\alpha_1+i\alpha_2)+\alpha(\beta_1-i\beta_2)\right]^2}{4(\alpha_1+i\alpha_2)(\alpha^2+\alpha_1+i\,\alpha_2)}\right\}\times$$

$$\times\left\{\sum_{l=1}^{n-E(n/2)}\frac{(l-1)!\,(\beta_1-i\,\beta_2)^{n+1-2l}}{(2l-1)!(n+1-2l)!}\sum_{r=1}^{l}\frac{2^{2l+1-2r}\,(2r-2)!}{(r-1)!}\times\right.$$

$$\times\sum_{q=0}^{r-1}\frac{\alpha^{2r-1-2q}\left[2\beta(\alpha_1+i\,\alpha_2)+\alpha(\beta_1-i\,\beta_2)\right]^{2r-2-2q}}{q!(2r-2-2q)!\,(\alpha_1+i\,\alpha_2)^{n-l+r-2q}\left(\alpha^2+\alpha_1+i\,\alpha_2\right)^{2r-2-q}}-$$

$$-\sum_{l=1}^{E(n/2)}\frac{(\beta_1-i\,\beta_2)^{n-2l}}{l!(n-2l)!}\sum_{r=1}^{l}(r-1)!\times$$

$$\left.\times\sum_{q=0}^{r-1}\frac{\left(\alpha^2\right)^{r-q}\left[2\beta(\alpha_1+i\,\alpha_2)+\alpha(\beta_1-i\,\beta_2)\right]^{2r-1-2q}}{q!(2r-1-2q)!\,(\alpha_1+i\alpha_2)^{n-l+r-2q}\left(\alpha^2+\alpha_1+i\,\alpha_2\right)^{2r-1-q}}\right\},$$

$$W_1^{(13)}(2n_1,\alpha,\alpha_1,\alpha_2,0,0,0)=W_1^{(13)}(2n_1,\alpha,\alpha_1,\alpha_2,0,i\,\beta_2,\beta_2)=0\,,$$

$$W_1^{(13)}(2n_1+1,\alpha,\alpha_1,\alpha_2,0,0,0)=W_1^{(13)}(2n_1+1,\alpha,\alpha_1,\alpha_2,0,i\,\beta_2,\beta_2)=$$

$$=\frac{n_1!\,\alpha}{(2n_1+1)!\sqrt{\alpha^2+\alpha_1+i\,\alpha_2}}\sum_{l=0}^{n_1}\frac{2^{2n_1+1-2l}\,(2l)!}{(l!)^2\,(\alpha_1+i\,\alpha_2)^{n_1+1-l}\left(\alpha^2+\alpha_1+i\,\alpha_2\right)^{l}},$$

$$W_1^{(13)}(2n_1,\alpha,\alpha_1,\alpha_2,\beta,0,0)=W_1^{(13)}(2n_1,\alpha,\alpha_1,\alpha_2,\beta,i\beta_2,\beta_2)=$$

$$=\frac{\sqrt{\pi}}{n_1!\sqrt{\alpha_1+i\alpha_2}\,(\alpha_1+i\alpha_2)^{n_1}}\operatorname{erf}\left(\frac{\beta\sqrt{\alpha_1+i\,\alpha_2}}{\sqrt{\alpha^2+\alpha_1+i\,\alpha_2}}\right)-\frac{1}{n_1!\sqrt{\alpha^2+\alpha_1+i\alpha_2}}\times$$

$$\times\exp\left[-\frac{\beta^2(\alpha_1+i\,\alpha_2)}{\alpha^2+\alpha_1+i\alpha_2}\right]\sum_{l=0}^{n_1-1}\frac{l!}{(\alpha_1+i\,\alpha_2)^{n_1-l}}\sum_{r=0}^{l}\frac{\left(\alpha^2\right)^{l+1-r}(2\beta)^{2l+1-2r}}{r!\,(2l+1-2r)!\left(\alpha^2+\alpha_1+i\alpha_2\right)^{2l+1-r}},$$

$$W_1^{(13)}(2n_1+1,\alpha,\alpha_1,\alpha_2,\beta,0,0)=W_1^{(13)}(2n_1+1,\alpha,\alpha_1,\alpha_2,\beta,i\beta_2,\beta_2)=$$

$$=\frac{n_1!}{(2n_1+1)!\sqrt{\alpha^2+\alpha_1+i\,\alpha_2}}\exp\left[-\frac{\beta^2(\alpha_1+i\,\alpha_2)}{\alpha^2+\alpha_1+i\,\alpha_2}\right]\times$$

$$\times\sum_{l=0}^{n_1}\frac{(2l)!}{l!\,(\alpha_1+i\,\alpha_2)^{n_1+1-l}}\sum_{r=0}^{l}\frac{2^{2n_1+1-2r}\,\alpha^{2l+1-2r}\left(\beta^2\right)^{l-r}}{r!\,(2l-2r)!\left(\alpha^2+\alpha_1+i\alpha_2\right)^{2l-r}};$$

2) $$W_2^{(13)}(\alpha,\alpha_1,\alpha_2)=\frac{i}{\left(2\sqrt{\alpha_1+i\alpha_2}\right)^{2n+1}}\ln\frac{\sqrt{\alpha_1+i\,\alpha_2}-i\alpha}{\sqrt{\alpha_1+i\,\alpha_2}+i\alpha}+$$

$$+\alpha\sum_{l=0}^{n-1}\frac{(l!)^2}{(2l+1)!\left[4(\alpha_1+i\alpha_2)\right]^{n-l}\left(\alpha^2+\alpha_1+i\alpha_2\right)^{l+1}};$$

3) $$W_3^{(13)}(\alpha,\beta,\beta_1,A)=\frac{\sqrt{\pi}n!}{4\alpha^{n+1}}\left[\operatorname{erf}\left(\frac{\beta+i\beta_1}{2\sqrt{2}\alpha}\right)+A\right]^2\sum_{k=0}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2k}}{4^k\,k!(n-2k)!(2\alpha)^{n-2k}}+$$

$$+2\frac{n!}{\alpha^{n+1}}\left[\sum_{k=1}^{E(n/2)}\frac{(\beta+i\beta_1)^{n-2k}}{k!(n-2k)!(2\alpha)^{n-2k}}\sum_{l=0}^{k-1}\frac{l!}{4^{k-l}\,(2l+1)!}W_5^{(13)}(2l+1,\alpha,\beta,\beta_1,A)+\right.$$

$$\left.+\sum_{k=1}^{n-E(n/2)}\frac{k!(\beta+i\beta_1)^{n+1-2k}}{(2k)!(n+1-2k)!(2\alpha)^{n+1-2k}}\sum_{l=0}^{k-1}\frac{1}{l!}W_5^{(13)}(2l,\alpha,\beta,\beta_1,A)\right];$$

4) $$W_4^{(13)}(n,\alpha,\alpha_1,\beta)=\frac{\sqrt{\pi}}{i^{n+2}\sqrt{\alpha_1}}\operatorname{erf}\left(\frac{i\alpha\beta}{2\sqrt{\alpha_1}\sqrt{\alpha_1+\alpha^2}}\right)\sum_{k=0}^{E(n/2)}\frac{(-1)^k\,\beta^{n-2k}}{k!(n-2k)!\alpha_1^{n-k}}+$$

$$+\frac{\alpha}{i^{n+1}\sqrt{\alpha_1+\alpha^2}}\exp\left[\frac{\alpha^2\beta^2}{4\alpha_1\left(\alpha_1+\alpha^2\right)}\right]\left[\sum_{k=1}^{E(n/2)}\frac{\beta^{n-2k}}{k!(n-2k)!}\sum_{l=1}^{k}(l-1)!\times\right.$$

$$\times\sum_{r=0}^{l-1}\frac{(-1)^{k+l-r}\left(\alpha^2\beta\right)^{2l-1-2r}}{r!(2l-1-2r)!\alpha_1^{n-k+l-2r}\left(\alpha_1+\alpha^2\right)^{2l-1-r}}-\sum_{k=1}^{n-E(n/2)}\frac{(k-1)!\beta^{n+1-2k}}{(2k-1)!(n+1-2k)!}\times$$

$$\left.\times\sum_{l=1}^{k}\frac{2^{2k+1-2l}(2l-2)!}{(l-1)!}\sum_{r=0}^{l-1}\frac{(-1)^{k+l-r}\left(\alpha^2\beta\right)^{2l-2-2r}}{r!(2l-2-2r)!\alpha_1^{n-k+l-2r}\left(\alpha_1+\alpha^2\right)^{2l-2-r}}\right];$$

5) $$W_5^{(13)}(n_1,\alpha,\beta,\beta_1,A)=\frac{n_1!\sqrt{2}}{(-2)^{n_1+2}}\exp\left[-\frac{(\beta+i\beta_1)^2}{8\alpha^2}\right]\left[\operatorname{erf}\left(\frac{\beta+i\beta_1}{2\sqrt{2}\alpha}\right)+A\right]\times$$

$$\times \sum_{r=0}^{E(n_1/2)} \frac{(\beta + i\beta_1)^{n_1-2r}}{2^r\, r!(n_1-2r)!(2\alpha)^{n_1-2r}} + \frac{n_1!}{(-2)^{n_1+1}\sqrt{\pi}} \exp\left[-\frac{(\beta+i\beta_1)^2}{4\alpha^2}\right] \times$$

$$\times \left[\sum_{r=1}^{E(n_1/2)} \frac{(\beta + i\beta_1)^{n_1-2r}}{r!(n_1-2r)!(2\alpha)^{n_1-2r}} \sum_{q=0}^{r-1} \frac{q!(\beta+i\beta_1)^{2q+1}}{2^{r-q}(2q+1)!(2\alpha)^{2q+1}} + \right.$$

$$\left. + \sum_{r=1}^{n_1-E(n_1/2)} \frac{r!(\beta+i\beta_1)^{n_1+1-2r}}{(2r)!(n_1+1-2r)!(2\alpha)^{n_1+1-2r}} \sum_{q=0}^{r-1} \frac{2^{r-q}(\beta+i\beta_1)^{2q}}{q!(2\alpha)^{2q}} \right].$$

2.14. Integrals of $z^n[\pm 1-\operatorname{erf}(\alpha z+\beta)]$, $z^n[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)]$

2.14.1.

1. $$\int_0^{+\infty}[1-\operatorname{erf}(az+\beta)]\,dz=-\int_{-\infty}^{0}[-1-\operatorname{erf}(az-\beta)]\,dz=\frac{1}{\sqrt{\pi}a}\exp\left(-\beta^2\right)+\frac{\beta}{a}[\operatorname{erf}(\beta)-1]$$
$$\{a>0\}.$$

2. $$\int_0^{+\infty}[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)]dz=\frac{\beta_1}{a}[1-\operatorname{erf}(\beta_1)]+\frac{\beta_2}{a}[\operatorname{erf}(\beta_2)-1]+$$
$$+\frac{1}{\sqrt{\pi}\,a}\left[\exp\left(-\beta_2^2\right)-\exp\left(-\beta_1^2\right)\right]\quad\{a>0\}.$$

3. $$\int_0^{+\infty}\left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}(a_2 z+\beta_2)\right]dz=\frac{\beta_1}{a_1}\left[1-\operatorname{erf}(\beta_1)\right]+\frac{\beta_2}{a_2}\left[\operatorname{erf}(\beta_2)-1\right]+$$
$$+\frac{1}{\sqrt{\pi}a_2}\exp\left(-\beta_2^2\right)-\frac{1}{\sqrt{\pi}a_1}\exp\left(-\beta_1^2\right)\quad\{a_1>0,\,a_2>0\}.$$

4. $$\int_{-\infty}^{+\infty}[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)]\,dz=\frac{2}{a}(\beta_1-\beta_2)\qquad\{a>0\}.$$

5. $$\int_{-\infty}^{+\infty}\left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}(a_2 z+\beta_2)\right]dz=2\left(\frac{\beta_1}{a_1}-\frac{\beta_2}{a_2}\right)\qquad\{a_1>0,\,a_2>0\}.$$

6. $$\int_0^{\infty}\left[\pm 1-\operatorname{erf}(\alpha z+\beta)\right]dz=\frac{1}{\sqrt{\pi}\,\alpha}\exp\left(-\beta^2\right)+\frac{\beta}{\alpha}\left[\operatorname{erf}(\beta)\mp 1\right]$$
$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+2\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

7. $$\int_0^{\infty}[\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)]\,dz=\frac{1}{\sqrt{\pi}\alpha}\left[\exp\left(-\beta_2^2\right)-\exp\left(-\beta_1^2\right)\right]+$$
$$+\frac{\beta_2}{\alpha}\left[\operatorname{erf}(\beta_2)\mp 1\right]-\frac{\beta_1}{\alpha}\left[\operatorname{erf}(\beta_1)\mp 1\right]\qquad\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_1 z\right)+2\ln|z|\right]=$$
$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_2 z\right)+2\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $$\int_0^{\infty}[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)]\,dz=\frac{\beta_1}{\alpha_1}[A_1-\operatorname{erf}(\beta_1)]\mp\frac{\beta_2}{\alpha_2}[A_2-\operatorname{erf}(\beta_2)]-$$

$$-\frac{1}{\sqrt{\pi}\alpha_1}\exp\left(-\beta_1^2\right)\pm\frac{1}{\sqrt{\pi}\alpha_2}\exp\left(-\beta_2^2\right)$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)+2\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)+2\ln|z|\right]=+\infty\,;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\alpha_k z\right)=+\infty\,,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\alpha_k z\right)=-\infty\,;\ \lim_{z\to\infty}\operatorname{Re}\left(\alpha_1 z\right)\cdot\lim_{z\to\infty}\operatorname{Re}\left(\alpha_2 z\right)=\pm\infty\,\}.$$

2.14.2.

1. $$\int_0^{+\infty} z\left[1-\operatorname{erf}\left(az+\beta\right)\right]dz=\int_{-\infty}^{0} z\left[-1-\operatorname{erf}\left(az-\beta\right)\right]dz=\frac{2\beta^2+1}{4a^2}\times$$

$$\times\left[1-\operatorname{erf}\left(\beta\right)\right]-\frac{\beta}{2\sqrt{\pi}a^2}\exp\left(-\beta^2\right)\quad\{a>0\}.$$

2. $$\int_0^{+\infty} z\left[\operatorname{erf}\left(az+\beta_1\right)-\operatorname{erf}\left(az+\beta_2\right)\right]dz=\frac{1}{2\sqrt{\pi}a^2}\left[\beta_1\exp\left(-\beta_1^2\right)-\beta_2\exp\left(-\beta_2^2\right)\right]+$$

$$+\frac{2\beta_1^2+1}{4a^2}\left[\operatorname{erf}\left(\beta_1\right)-1\right]+\frac{2\beta_2^2+1}{4a^2}\left[1-\operatorname{erf}\left(\beta_2\right)\right]\qquad\{a>0\}.$$

3. $$\int_0^{+\infty} z\left[\operatorname{erf}\left(a_1 z+\beta_1\right)-\operatorname{erf}\left(a_2 z+\beta_2\right)\right]dz=\frac{1}{2\sqrt{\pi}}\left[\frac{\beta_1}{a_1^2}\exp\left(-\beta_1^2\right)-\frac{\beta_2}{a_2^2}\exp\left(-\beta_2^2\right)\right]+$$

$$+\frac{2\beta_1^2+1}{4a_1^2}\left[\operatorname{erf}\left(\beta_1\right)-1\right]+\frac{2\beta_2^2+1}{4a_2^2}\left[1-\operatorname{erf}\left(\beta_2\right)\right]\qquad\{a_1>0,\,a_2>0\}.$$

4. $$\int_{-\infty}^{+\infty} z\left[\operatorname{erf}\left(az+\beta_1\right)-\operatorname{erf}\left(az+\beta_2\right)\right]dz=\frac{\beta_2^2-\beta_1^2}{a^2}\qquad\{a>0\}.$$

5. $$\int_{-\infty}^{+\infty} z\left[\operatorname{erf}\left(a_1 z+\beta_1\right)-\operatorname{erf}\left(a_2 z+\beta_2\right)\right]dz=\frac{2\beta_2^2+1}{2a_2^2}-\frac{2\beta_1^2+1}{2a_1^2}\qquad\{a_1>0,\,a_2>0\}.$$

6. $$\int_0^{\infty} z\left[\pm1-\operatorname{erf}\left(\alpha z+\beta\right)\right]dz=\frac{2\beta^2+1}{4\alpha^2}\left[\pm1-\operatorname{erf}\left(\beta\right)\right]-\frac{\beta}{2\sqrt{\pi}\alpha^2}\exp\left(-\beta^2\right)$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty\ \}.$$

7. $\int\limits_0^\infty z[\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)]\,dz=\frac{1}{2\sqrt{\pi}\alpha^2}\left[\beta_1\exp\left(-\beta_1^2\right)-\beta_2\exp\left(-\beta_2^2\right)\right]+$

$$+\frac{2\beta_1^2+1}{4\alpha^2}[\operatorname{erf}(\beta_1)\mp 1]-\frac{2\beta_2^2+1}{4\alpha^2}[\operatorname{erf}(\beta_2)\mp 1]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_2 z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

8. $\int\limits_0^\infty z\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=\frac{1}{2\sqrt{\pi}}\left[\frac{\beta_1}{\alpha_1^2}\exp\left(-\beta_1^2\right)\mp\frac{\beta_2}{\alpha_2^2}\exp\left(-\beta_2^2\right)\right]+$

$$+\frac{2\beta_1^2+1}{4\alpha_1^2}\left[\operatorname{erf}(\beta_1)-A_1\right]\pm\frac{2\beta_2^2+1}{4\alpha_2^2}\left[A_2-\operatorname{erf}(\beta_2)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1 z\right)+\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2 z\right)+\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\ \}.$$

2.14.3.

1. $\int\limits_0^{+\infty} z^2[1-\operatorname{erf}(az+\beta)]\,dz=-\int\limits_{-\infty}^{0} z^2[-1-\operatorname{erf}(az-\beta)]\,dz=$

$$=\frac{\beta^2+1}{3\sqrt{\pi}a^3}\exp\left(-\beta^2\right)+\frac{2\beta^3+3\beta}{6a^3}[\operatorname{erf}(\beta)-1]\qquad\{a>0\}.$$

2. $\int\limits_0^{+\infty} z^2\left[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)\right]dz=\frac{1}{3\sqrt{\pi}a^3}\left[\left(\beta_2^2+1\right)\exp\left(-\beta_2^2\right)-\left(\beta_1^2+1\right)\times\right.$

$$\left.\times\exp\left(-\beta_1^2\right)\right]+\frac{2\beta_1^3+3\beta_1}{6a^3}[1-\operatorname{erf}(\beta_1)]+\frac{2\beta_2^3+3\beta_2}{6a^3}[\operatorname{erf}(\beta_2)-1]\ \ \{a>0\}.$$

3. $\int\limits_0^{+\infty} z^2[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]\,dz=\frac{2\beta_1^3+3\beta_1}{6a_1^3}[1-\operatorname{erf}(\beta_1)]-$

$$-\frac{2\beta_2^3+3\beta_2}{6a_2^3}[1-\operatorname{erf}(\beta_2)]-\frac{\beta_1^2+1}{3\sqrt{\pi}a_1^3}\exp\left(-\beta_1^2\right)+\frac{\beta_2^2+1}{3\sqrt{\pi}a_2^3}\exp\left(-\beta_2^2\right)\qquad\{a_1>0,\,a_2>0\}.$$

4. $\int\limits_{-\infty}^{+\infty} z^2 \left[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)\right] dz = \frac{2\beta_1^3+3\beta_1-2\beta_2^3-3\beta_2}{3a^3}$ $\{a>0\}$.

5. $\int\limits_{-\infty}^{+\infty} z^2 \left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}(a_2 z+\beta_2)\right] dz = \frac{2\beta_1^3+3\beta_1}{3a_1^3}-\frac{2\beta_2^3+3\beta_2}{3a_2^3}$

$\{a_1>0,\, a_2>0\}$.

6. $\int\limits_{0}^{\infty} z^2 \left[\pm 1-\operatorname{erf}(\alpha z+\beta)\right] dz = \frac{\beta^2+1}{3\sqrt{\pi}\alpha^3}\exp\left(-\beta^2\right)+\frac{2\beta^3+3\beta}{6\alpha^3}\left[\operatorname{erf}(\beta)\mp 1\right]$

$\{\lim\limits_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)=+\infty,\quad \lim\limits_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}$.

7. $\int\limits_{0}^{\infty} z^2 [\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)]\, dz = \frac{2\beta_2^3+3\beta_2}{6\alpha^3}[\operatorname{erf}(\beta_2)\mp 1]-\frac{2\beta_1^3+3\beta_1}{6\alpha^3}\times$

$\times[\operatorname{erf}(\beta_1)\mp 1]+\frac{1}{3\sqrt{\pi}\alpha^3}\left[\left(\beta_2^2+1\right)\exp\left(-\beta_2^2\right)-\left(\beta_1^2+1\right)\exp\left(-\beta_1^2\right)\right]$

$\{\lim\limits_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_1 z\right)=\lim\limits_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_2 z\right)=+\infty,\quad \lim\limits_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}$.

8. $\int\limits_{0}^{\infty} z^2 [\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)]dz = \frac{2\beta_1^3+3\beta_1}{6\alpha_1^3}[A_1-\operatorname{erf}(\beta_1)]\mp$

$\mp\frac{2\beta_2^3+3\beta_2}{6\alpha_2^3}\left[A_2-\operatorname{erf}(\beta_2)\right]-\frac{1}{3\sqrt{\pi}}\left[\frac{\beta_1^2+1}{\alpha_1^3}\exp\left(-\beta_1^2\right)\mp\frac{\beta_2^2+1}{\alpha_2^3}\exp\left(-\beta_2^2\right)\right]$

$\{\lim\limits_{z\to\infty}\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)=$

$=\lim\limits_{z\to\infty}\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)=+\infty;\ A_k=1$ if $\lim\limits_{z\to\infty}\operatorname{Re}(\alpha_k z)=+\infty,$

$A_k=-1$ if $\lim\limits_{z\to\infty}\operatorname{Re}(\alpha_k z)=-\infty;\ \lim\limits_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim\limits_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\}$.

2.14.4.

1. $\int\limits_{0}^{+\infty} z^n [1-\operatorname{erf}(az+\beta)]dz = (-1)^{n+1}\int\limits_{-\infty}^{0} z^n [-1-\operatorname{erf}(az-\beta)]dz = W_1^{(14)}(n, a, \beta, 1)$

$\{a>0\}$.

2. $\int\limits_{0}^{+\infty} z^n [\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)]\, dz = W_1^{(14)}(n, a, \beta_2, 1)-W_1^{(14)}(n, a, \beta_1, 1)$

$\{a>0\}$.

3. $\int_0^{+\infty} z^n \left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}(a_2 z+\beta_2)\right] dz = W_1^{(14)}(n, a_2, \beta_2, 1)-$

$$-W_1^{(14)}(n, a_1, \beta_1, 1) \qquad \{a_1>0,\, a_2>0\}.$$

4. $\int_{-\infty}^{+\infty} z^n \left[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)\right] dz = n!\left[W_2^{(14)}(n, a, \beta_1)-W_2^{(14)}(n, a, \beta_2)\right]$

$$\{a>0\}.$$

5. $\int_{-\infty}^{+\infty} z^n \left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}(a_2 z+\beta_2)\right] dz =$

$$= n!\left[W_2^{(14)}(n, a_1, \beta_1)-W_2^{(14)}(n, a_2, \beta_2)\right] \qquad \{a_1>0,\, a_2>0\}.$$

6. $\int_0^{\infty} z^n \left[\pm 1-\operatorname{erf}(\alpha z+\beta)\right] dz = W_1^{(14)}(n, \alpha, \beta, \pm 1)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-(n-2)\ln|z|\right]=+\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

7. $\int_0^{\infty} z^n \left[\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)\right] dz = W_1^{(14)}(n, \alpha, \beta_2, \pm 1)-W_1^{(14)}(n, \alpha, \beta_1, \pm 1)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_1 z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_2 z\right)-(n-2)\ln|z|\right]=+\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $\int_0^{\infty} z^n \left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right] dz =$

$$=\pm W_1^{(14)}(n, \alpha_2, \beta_2, A_2)-W_1^{(14)}(n, \alpha_1, \beta_1, A_1)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)-(n-2)\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\}.$$

Introduced notations:

1) $$W_1^{(14)}(n,\alpha,\beta,A)=\frac{n!}{(-\alpha)^{n+1}}\left\{\left[A-\operatorname{erf}(\beta)\right]\sum_{k=0}^{n-E(n/2)}\frac{\beta^{n+1-2k}}{4^k\,k!(n+1-2k)!}+\frac{\exp\left(-\beta^2\right)}{\sqrt{\pi}}\times\right.$$
$$\left.\times\left[\sum_{k=1}^{n-E(n/2)}\frac{1}{k!(n+1-2k)!}\sum_{l=1}^{k}\frac{l!\beta^{n-2k+2l}}{4^{k-l}(2l)!}-\sum_{k=0}^{E(n/2)}\frac{k!}{(2k+1)!(n-2k)!}\sum_{l=0}^{k}\frac{\beta^{n-2k+2l}}{l!}\right]\right\},$$

$$W_1^{(14)}(2n_1,\alpha,0,A)=\frac{n_1!}{(2n_1+1)\sqrt{\pi}\alpha^{2n_1+1}},$$

$$W_1^{(14)}(2n_1+1,\alpha,0,A)=A\frac{(2n_1+1)!}{(n_1+1)!(2\alpha)^{2n_1+2}};$$

2) $$W_2^{(14)}(n,\alpha,\beta)=\frac{(-1)^n}{\alpha^{n+1}}\sum_{k=0}^{n-E(n/2)}\frac{\beta^{n+1-2k}}{2^{2k-1}k!(n+1-2k)!},$$

$$W_2^{(14)}(2n_1,\alpha,0)=0,\quad W_2^{(14)}(2n_1+1,\alpha,0)=-\frac{1}{2^{2n_1+1}(n_1+1)!\alpha^{2n_1+2}}.$$

2.15. Integrals of $z^n[\pm 1-\operatorname{erf}(\alpha z+\beta)]^2$, $z^n\left[1-\operatorname{erf}^2(\alpha z+\beta)\right]$, $z^n\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]$, $z^n\left[\operatorname{erf}^2(\alpha_1 z+\beta_1)-\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]$

2.15.1.

1. $$\int_0^{+\infty}\left[1-\operatorname{erf}(az+\beta)\right]^2dz=\int_{-\infty}^{0}\left[-1-\operatorname{erf}(az-\beta)\right]^2dz=\frac{2}{\sqrt{2\pi}a}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]-$$
$$-\frac{2}{\sqrt{\pi}a}\exp\left(-\beta^2\right)\left[\operatorname{erf}(\beta)-1\right]-\frac{\beta}{a}\left[\operatorname{erf}(\beta)-1\right]^2\qquad\{a>0\}.$$

2. $$\int_0^{+\infty}\left[1-\operatorname{erf}^2(az+\beta)\right]dz=\frac{2}{\sqrt{\pi}\,a}\operatorname{erf}(\beta)\exp\left(-\beta^2\right)+\frac{\beta}{a}\left[\operatorname{erf}^2(\beta)-1\right]-$$
$$-\frac{2}{\sqrt{2\pi}\,a}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]\qquad\{a>0\}.$$

3. $$\int_0^{+\infty}\left[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}^2(a_2z+\beta_2)\right]dz=-\int_{-\infty}^{0}\left[\operatorname{erf}(a_1z-\beta_1)+\operatorname{erf}^2(a_2z-\beta_2)\right]dz=$$

$$=\frac{\beta_1}{a_1}[1-\operatorname{erf}(\beta_1)]-\frac{1}{\sqrt{\pi}a_1}\exp\left(-\beta_1^2\right)+\frac{1}{a_2}\left\{\frac{2}{\sqrt{\pi}}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)+\right.$$

$$\left.+\frac{2}{\sqrt{2\pi}}\left[1-\operatorname{erf}\left(\sqrt{2}\beta_2\right)\right]-\beta_2\left[1-\operatorname{erf}^2(\beta_2)\right]\right\} \qquad \{a_1>0,\, a_2>0\}.$$

4. $$\int_0^{+\infty}\left[\operatorname{erf}^2(a_1z+\beta_1)-\operatorname{erf}^2(a_2z+\beta_2)\right]dz=\frac{\beta_1}{a_1}\left[1-\operatorname{erf}^2(\beta_1)\right]-\frac{\beta_2}{a_2}\left[1-\operatorname{erf}^2(\beta_2)\right]+$$

$$+\frac{2}{\sqrt{\pi}}\left[\frac{\operatorname{erf}(\beta_2)}{a_2}\exp\left(-\beta_2^2\right)-\frac{\operatorname{erf}(\beta_1)}{a_1}\exp\left(-\beta_1^2\right)\right]+$$

$$+\frac{2}{\sqrt{2\pi}}\left[\frac{1-\operatorname{erf}\left(\sqrt{2}\beta_2\right)}{a_2}+\frac{\operatorname{erf}\left(\sqrt{2}\beta_1\right)-1}{a_1}\right] \qquad \{a_1>0,\, a_2>0\}.$$

5. $$\int_{-\infty}^{+\infty}\left[1-\operatorname{erf}^2(az+\beta)\right]dz=\frac{4}{\sqrt{2\pi}a} \qquad \{a>0\}.$$

6. $$\int_{-\infty}^{+\infty}\left[\operatorname{erf}^2(a_1z+\beta_1)-\operatorname{erf}^2(a_2z+\beta_2)\right]dz=\frac{4}{\sqrt{2\pi}}\left(\frac{1}{a_2}-\frac{1}{a_1}\right) \qquad \{a_1>0,\, a_2>0\}.$$

7. $$\int_0^{\infty}[\pm 1-\operatorname{erf}(\alpha z+\beta)]^2\,dz=\frac{2}{\sqrt{2\pi}\alpha}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]-\frac{2}{\sqrt{\pi}\alpha}\exp\left(-\beta^2\right)\times$$

$$\times[\operatorname{erf}(\beta)\mp 1]-\frac{\beta}{\alpha}[\operatorname{erf}(\beta)\mp 1]^2$$

$$\left\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+3\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\right\}.$$

8. $$\int_0^{\infty}\left[1-\operatorname{erf}^2(\alpha z+\beta)\right]dz=\frac{2}{\sqrt{\pi}\alpha}\operatorname{erf}(\beta)\exp\left(-\beta^2\right)+\frac{\beta}{\alpha}\left[\operatorname{erf}^2(\beta)-1\right]-$$

$$-\frac{2}{\sqrt{2\pi}\alpha}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]$$

$$\left\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+3\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\right\}.$$

9. $$\int_0^{\infty}\left[\operatorname{erf}(\alpha_1z+\beta_1)\mp\operatorname{erf}^2(\alpha_2z+\beta_2)\right]dz=\frac{\beta_1}{\alpha_1}[\pm 1-\operatorname{erf}(\beta_1)]-\frac{1}{\sqrt{\pi}\alpha_1}\exp\left(-\beta_1^2\right)\pm$$

$$\pm\frac{1}{\alpha_2}\left\{\frac{2}{\sqrt{\pi}}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)+\frac{2}{\sqrt{2\pi}}\left[A-\operatorname{erf}\left(\sqrt{2}\beta_2\right)\right]+\beta_2\left[\operatorname{erf}^2(\beta_2)-1\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)+3\ln|z|\right]=+\infty;\ A=1 \text{ if}$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=\pm\infty\}.$$

10. $$\int_0^\infty\left[\operatorname{erf}^2(\alpha_1 z+\beta_1)-\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]dz=\frac{\beta_1}{\alpha_1}\left[1-\operatorname{erf}^2(\beta_1)\right]-\frac{\beta_2}{\alpha_2}\left[1-\operatorname{erf}^2(\beta_2)\right]+$$

$$+\frac{2}{\sqrt{\pi}}\left[\frac{\operatorname{erf}(\beta_2)}{\alpha_2}\exp\left(-\beta_2^2\right)-\frac{\operatorname{erf}(\beta_1)}{\alpha_1}\exp\left(-\beta_1^2\right)\right]+$$

$$+\frac{2}{\sqrt{2\pi}}\left[\frac{A-\operatorname{erf}\left(\sqrt{2}\beta_2\right)}{\alpha_2}+\frac{\operatorname{erf}\left(\sqrt{2}\beta_1\right)\mp 1}{\alpha_1}\right]$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)+3\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)+3\ln|z|\right]=+\infty;\ A=1 \text{ if}$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=\pm\infty\}.$$

2.15.2.

1. $$\int_0^{+\infty} z[1-\operatorname{erf}(az+\beta)]^2\,dz=-\int_{-\infty}^0 z[-1-\operatorname{erf}(az-\beta)]^2\,dz=\frac{2\beta^2+1}{4a^2}[\operatorname{erf}(\beta)-1]^2+$$

$$+\frac{\beta}{\sqrt{\pi}a^2}\exp\left(-\beta^2\right)\left[\operatorname{erf}(\beta)-1\right]-\frac{2\beta}{\sqrt{2\pi}a^2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]-\frac{1}{2\pi a^2}\exp\left(-2\beta^2\right)\ \{a>0\}.$$

2. $$\int_0^{+\infty} z\left[1-\operatorname{erf}^2(az+\beta)\right]dz=\frac{2\beta^2+1}{4a^2}\left[1-\operatorname{erf}^2(\beta)\right]+\frac{2\beta}{\sqrt{2\pi}a^2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]+$$

$$+\frac{1}{2\pi a^2}\exp\left(-2\beta^2\right)-\frac{\beta}{\sqrt{\pi}a^2}\operatorname{erf}(\beta)\exp\left(-\beta^2\right)\ \{a>0\}.$$

3. $$\int_0^{+\infty} z\left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz=\int_{-\infty}^0 z\left[\operatorname{erf}(a_1 z-\beta_1)+\operatorname{erf}^2(a_2 z-\beta_2)\right]dz=$$

$$=\frac{2\beta_1^2+1}{4a_1^2}[\operatorname{erf}(\beta_1)-1]+\frac{\beta_1}{2\sqrt{\pi}a_1^2}\exp\left(-\beta_1^2\right)+\frac{2\beta_2^2+1}{4a_2^2}\left[1-\operatorname{erf}^2(\beta_2)\right]+$$

$$+\frac{1}{2\pi a_2^2}\exp\left(-2\beta_2^2\right)+\frac{2\beta_2}{\sqrt{2\pi}a_2^2}\left[\operatorname{erf}\left(\sqrt{2}\beta_2\right)-1\right]-\frac{\beta_2}{\sqrt{\pi}a_2^2}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)$$

$$\{a_1 > 0,\, a_2 > 0\}.$$

4. $$\int_0^{+\infty} z\left[\operatorname{erf}^2(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz = \frac{2\beta_2+1}{4a_2^2}\left[1-\operatorname{erf}^2(\beta_2)\right]+$$

$$+\frac{2\beta_2}{\sqrt{2\pi}a_2^2}\left[\operatorname{erf}\left(\sqrt{2}\beta_2\right)-1\right]+\frac{1}{2\pi a_2^2}\exp\left(-2\beta_2^2\right)-\frac{\beta_2}{\sqrt{\pi}a_2^2}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)-$$

$$-\frac{2\beta_1+1}{4a_1^2}\left[1-\operatorname{erf}^2(\beta_1)\right]-\frac{2\beta_1}{\sqrt{2\pi}a_1^2}\left[\operatorname{erf}\left(\sqrt{2}\beta_1\right)-1\right]-\frac{1}{2\pi a_1^2}\exp\left(-2\beta_1^2\right)+$$

$$+\frac{\beta_1}{\sqrt{\pi}a_1^2}\operatorname{erf}(\beta_1)\exp\left(-\beta_1^2\right) \qquad \{a_1 > 0,\, a_2 > 0\}.$$

5. $$\int_{-\infty}^{+\infty} z\left[1-\operatorname{erf}^2(az+\beta)\right]dz = -\frac{4\beta}{\sqrt{2\pi}a^2} \qquad \{a > 0\}.$$

6. $$\int_{-\infty}^{+\infty} z\left[\operatorname{erf}^2(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz = \frac{4}{\sqrt{2\pi}}\left(\frac{\beta_1}{a_1^2}-\frac{\beta_2}{a_2^2}\right) \quad \{a_1 > 0,\, a_2 > 0\}.$$

7. $$\int_0^{\infty} z[\pm 1-\operatorname{erf}(\alpha z+\beta)]^2\,dz = \frac{2\beta^2+1}{4\alpha^2}[\operatorname{erf}(\beta)\mp 1]^2+\frac{\beta}{\sqrt{\pi}\alpha^2}\exp\left(-\beta^2\right)\times$$

$$\times[\operatorname{erf}(\beta)\mp 1]-\frac{2\beta}{\sqrt{2\pi}\alpha^2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]-\frac{1}{2\pi\alpha^2}\exp\left(-2\beta^2\right)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $$\int_0^{\infty} z\left[1-\operatorname{erf}^2(\alpha z+\beta)\right]dz = \frac{2\beta^2+1}{4\alpha^2}\left[1-\operatorname{erf}^2(\beta)\right]+\frac{2\beta}{\sqrt{2\pi}\alpha^2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]+$$

$$+\frac{1}{2\pi\alpha^2}\exp\left(-2\beta^2\right)-\frac{\beta}{\sqrt{\pi}\alpha^2}\operatorname{erf}(\beta)\exp\left(-\beta^2\right)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

9. $$\int_0^{\infty} z\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]dz = \frac{2\beta_1^2+1}{4\alpha_1^2}[\operatorname{erf}(\beta_1)\mp 1]+$$

$$+\frac{\beta_1}{2\sqrt{\pi}\alpha_1^2}\exp\left(-\beta_1^2\right)\pm\frac{2\beta_2^2+1}{4\alpha_2^2}\left[1-\operatorname{erf}^2(\beta_2)\right]\pm\frac{1}{2\pi\alpha_2^2}\exp\left(-2\beta_2^2\right)\mp\frac{2\beta_2}{\sqrt{2\pi}\alpha_2^2}\times$$

$$\times\left[A-\operatorname{erf}\left(\sqrt{2}\beta_2\right)\right]\mp\frac{\beta_2}{\sqrt{\pi}\alpha_2^2}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)$$

$$\left\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)+\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)+\ln|z|\right]=+\infty;\right.$$

$$\left. A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=\pm\infty \right\}.$$

10. $$\int_0^\infty z\left[\operatorname{erf}^2(\alpha_1 z+\beta_1)-\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]dz=\frac{2\beta_2^2+1}{4\alpha_2^2}\left[1-\operatorname{erf}^2(\beta_2)\right]+$$

$$+\frac{2\beta_2}{\sqrt{2\pi}\alpha_2^2}\left[\operatorname{erf}\left(\sqrt{2}\beta_2\right)-A\right]+\frac{1}{2\pi\alpha_2^2}\exp\left(-2\beta_2^2\right)-\frac{\beta_2}{\sqrt{\pi}\alpha_2^2}\operatorname{erf}(\beta_2)\exp\left(-\beta_2^2\right)-$$

$$-\frac{2\beta_1^2+1}{4\alpha_1^2}\left[1-\operatorname{erf}^2(\beta_1)\right]-\frac{2\beta_1}{\sqrt{2\pi}\alpha_1^2}\left[\operatorname{erf}\left(\sqrt{2}\beta_1\right)\mp 1\right]-$$

$$-\frac{1}{2\pi\alpha_1^2}\exp\left(-2\beta_1^2\right)+\frac{\beta_1}{\sqrt{\pi}\alpha_1^2}\operatorname{erf}(\beta_1)\exp\left(-\beta_1^2\right)$$

$$\{\lim_{z\to\infty}[\operatorname{Re}(\alpha_1^2 z^2+2\alpha_1\beta_1 z)+\ln|z|]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)+\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=-\infty;\quad \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=\pm\infty\}.$$

2.15.3.

1. $$\int_0^{+\infty} z^n[1-\operatorname{erf}(az+\beta)]^2dz=(-1)^n\int_{-\infty}^0 z^n[-1-\operatorname{erf}(az-\beta)]^2dz=$$

$$=\frac{n!}{(-a)^{n+1}}\left\{[\operatorname{erf}(\beta)-1]^2W_1^{(15)}(n,\beta)-\frac{2}{\sqrt{\pi}}W_2^{(15)}(n,\beta)-\frac{2}{\sqrt{2\pi}}\times\right.$$

$$\left.\times\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1]W_3^{(15)}(n,\beta)-\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta)\right]\right\}\qquad\{a>0\}.$$

2. $$\int_0^{+\infty} z^n\left[1-\operatorname{erf}^2(az+\beta)\right]dz=\frac{n!}{(-a)^{n+1}}\left\{\left[1-\operatorname{erf}^2(\beta)\right]W_1^{(15)}(n,\beta)+\right.$$

$$\left.+\frac{2}{\sqrt{2\pi}}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)-1\right]W_3^{(15)}(n,\beta)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta)\right\}\ \{a>0\}.$$

3. $\int\limits_{0}^{+\infty} z^n \left[\operatorname{erf}(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz=(-1)^{n+1}\times$

$$\times\int\limits_{-\infty}^{0} z^n \left[\operatorname{erf}(a_1 z-\beta_1)+\operatorname{erf}^2(a_2 z-\beta_2)\right]dz=\frac{n!}{(-a_1)^{n+1}}\times$$

$$\times\left\{\left[\operatorname{erf}(\beta_1)-1\right]W_1^{(15)}(n,\beta_1)+\frac{1}{\sqrt{\pi}}W_2^{(15)}(n,\beta_1)\right\}+\frac{n!}{(-a_2)^{n+1}}\left\{\left[1-\operatorname{erf}^2(\beta_2)\right]\times\right.$$

$$\left.\times W_1^{(15)}(n,\beta_2)+\frac{2}{\sqrt{2\pi}}\left[\operatorname{erf}\left(\sqrt{2}\beta_2\right)-1\right]W_3^{(15)}(n,\beta_2)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_2)\right\}$$

$$\{a_1>0,\ a_2>0\}.$$

4. $\int\limits_{0}^{+\infty} z^n \left[\operatorname{erf}^2(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz=\frac{n!}{(-a_2)^{n+1}}\left\{\left[1-\operatorname{erf}^2(\beta_2)\right]\times\right.$

$$\left.\times W_1^{(15)}(n,\beta_2)+\frac{2}{\sqrt{2\pi}}\left[\operatorname{erf}\left(\sqrt{2}\,\beta_2\right)-1\right]W_3^{(15)}(n,\beta_2)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_2)\right\}-$$

$$-\frac{n!}{(-a_1)^{n+1}}\left\{\left[1-\operatorname{erf}^2(\beta_1)\right]W_1^{(15)}(n,\beta_1)+\frac{2}{\sqrt{2\pi}}\times\right.$$

$$\left.\times\left[\operatorname{erf}\left(\sqrt{2}\beta_1\right)-1\right]W_3^{(15)}(n,\beta_1)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_1)\right\}\quad\{a_1>0,\ a_2>0\}.$$

5. $\int\limits_{-\infty}^{+\infty} z^n \left[1-\operatorname{erf}^2(az+\beta)\right]dz=(-1)^n\,n!\frac{4}{\sqrt{2\pi}a^{n+1}}W_3^{(15)}(n,\beta)\qquad\{a>0\}.$

6. $\int\limits_{-\infty}^{+\infty} z^n \left[\operatorname{erf}^2(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz=(-1)^n\,n!\frac{4}{\sqrt{2\pi}}\left[\frac{1}{a_2^{n+1}}W_3^{(15)}(n,\beta_2)-\right.$

$$\left.-\frac{1}{a_1^{n+1}}W_3^{(15)}(n,\beta_1)\right]\qquad\{a_1>0,\ a_2>0\}.$$

7. $\int\limits_{0}^{\infty} z^n \left[\pm1-\operatorname{erf}(\alpha z+\beta)\right]^2dz=\frac{n!}{(-\alpha)^{n+1}}\left\{\left[\operatorname{erf}(\beta)\mp1\right]^2W_1^{(15)}(n,\beta)\mp\frac{2}{\sqrt{\pi}}W_2^{(15)}(n,\beta)-\right.$

$$\left.-\frac{2}{\sqrt{2\pi}}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp1\right]W_3^{(15)}(n,\beta)-\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta)\right\}$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+(3-n)\ln|z|\right]=+\infty;\quad\lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $$\int_0^\infty z^n\left[1-\text{erf}^2(\alpha z+\beta)\right]dz=\frac{n!}{(-\alpha)^{n+1}}\times$$

$$\times\left\{\left[1-\text{erf}^2(\beta)\right]W_1^{(15)}(n,\beta)+\frac{2}{\sqrt{2\pi}}\left[\text{erf}\left(\sqrt{2}\beta\right)\mp 1\right]W_3^{(15)}(n,\beta)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta)\right\}$$

$$\{\lim_{z\to\infty}\left[\text{Re}\left(\alpha^2z^2+2\alpha\beta z\right)-(n-2)\ln|z|\right]=+\infty\text{ for } n>0;\ \lim_{z\to\infty}\text{Re}(\alpha z)=\pm\infty\}.$$

9. $$\int_0^\infty z^n\left[\text{erf}(\alpha_1 z+\beta_1)\mp\text{erf}^2(\alpha_2 z+\beta_2)\right]dz=\frac{n!}{(-\alpha_1)^{n+1}}\times$$

$$\times\left\{\left[\text{erf}(\beta_1)\mp 1\right]W_1^{(15)}(n,\beta_1)+\frac{1}{\sqrt{\pi}}W_2^{(15)}(n,\beta_1)\right\}\pm\frac{n!}{(-\alpha_2)^{n+1}}\times$$

$$\times\left\{[1-\text{erf}^2(\beta_2)]W_1^{(15)}(n,\beta_2)+\frac{2}{\sqrt{2\pi}}[\text{erf}(\sqrt{2}\beta)-A]\times\right.$$

$$\left.\times W_3^{(15)}(n,\beta_2)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_2)\right\}$$

$$\{\lim_{z\to\infty}\left[\text{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1 z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\text{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2 z\right)-(n-2)\ln|z|\right]=+\infty\text{ for } n>0;\ A=1\text{ if}$$

$$\lim_{z\to\infty}\text{Re}(\alpha_2 z)=+\infty,\ A=-1\text{ if }\lim_{z\to\infty}\text{Re}(\alpha_2 z)=-\infty;\ \lim_{z\to\infty}\text{Re}(\alpha_1 z)=\pm\infty\}.$$

10. $$\int_0^\infty z^n\left[\text{erf}^2(\alpha_1 z+\beta_1)-\text{erf}^2(\alpha_2 z+\beta_2)\right]dz=\frac{(-1)^n n!}{\alpha_1^{n+1}}\times$$

$$\times\left\{\left[1-\text{erf}^2(\beta_1)\right]W_1^{(15)}(n,\beta_1)+\frac{2}{\sqrt{2\pi}}\left[\text{erf}\left(\sqrt{2}\beta_1\right)\mp 1\right]W_3^{(15)}(n,\beta_1)+\right.$$

$$\left.+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_1)\right\}+\frac{n!}{(-\alpha_2)^{n+1}}\left\{\left[1-\text{erf}^2(\beta_2)\right]W_1^{(15)}(n,\beta_2)+\frac{2}{\sqrt{2\pi}}\times\right.$$

$$\left.\times\left[\text{erf}\left(\sqrt{2}\beta_2\right)-A\right]W_3^{(15)}(n,\beta_2)+\frac{1}{\sqrt{\pi}}W_4^{(15)}(n,\beta_2)\right\}$$

$$\{\lim_{z\to\infty}\left[\text{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1 z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\text{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2 z\right)-(n-2)\ln|z|\right]=+\infty\text{ for } n>0;\ A=1\text{ if}$$

$$\lim_{z\to\infty}\text{Re}(\alpha_2 z)=+\infty,\ A=-1\text{ if }\lim_{z\to\infty}\text{Re}(\alpha_2 z)=-\infty;\ \lim_{z\to\infty}\text{Re}(\alpha_1 z)=\pm\infty\}.$$

Introduced notations:

1) $W_1^{(15)}(n,\beta)=\sum_{k=0}^{n-E(n/2)}\frac{\beta^{n+1-2k}}{4^k k!(n+1-2k)!}$, $W_1^{(15)}(2n_1,0)=0$,

$$W_1^{(15)}(2n_1+1,0)=\frac{1}{4^{n_1+1}(n_1+1)!};$$

2) $$W_2^{(15)}(n,\beta)=\exp\left(-\beta^2\right)\left[\sum_{k=0}^{E(n/2)}\frac{k!}{(2k+1)!(n-2k)!}\sum_{l=0}^{k}\frac{\beta^{n-2k+2l}}{l!}-\right.$$

$$\left.-\sum_{k=1}^{n-E(n/2)}\frac{1}{k!(n+1-2k)!}\sum_{l=1}^{k}\frac{l!\beta^{n-2k+2l}}{4^{k-l}(2l)!}\right],$$

$W_2^{(15)}(2n_1,0)=\frac{n_1!}{(2n_1+1)!}$, $W_2^{(15)}(2n_1+1,0)=0$;

3) $$W_3^{(15)}(n,\beta)=\sum_{k=0}^{E(n/2)}\frac{k!\beta^{n-2k}}{(2k+1)!(n-2k)!}\sum_{l=0}^{k}\frac{(2l)!}{8^l(l!)^2},$$

$W_3^{(15)}(2n_1,0)=\frac{n_1!}{(2n_1+1)!}\sum_{k=0}^{n_1}\frac{(2k)!}{8^k(k!)^2}$, $W_3^{(15)}(2n_1+1,0)=0$;

4) $$W_4^{(15)}(n,\beta)=\sum_{k=1}^{n-E(n/2)}\frac{\beta^{n+1-2k}}{k!(n+1-2k)!}\left[\operatorname{erf}(\beta)\exp\left(-\beta^2\right)\sum_{l=1}^{k}\frac{(l-1)!\,\beta^{2l-1}}{4^{k-l}(2l-1)!}+\right.$$

$$\left.+\frac{\exp\left(-2\beta^2\right)}{\sqrt{\pi}}\sum_{l=0}^{k-1}\frac{(l!)^2}{(2l+1)!}\sum_{r=1}^{l+1}\frac{\beta^{2r-2}}{2^{2k-l-r}(r-1)!}\right]-\sum_{k=0}^{E(n/2)}\frac{(k+1)!\beta^{n-2k}}{(2k+2)!(n-2k)!}\times$$

$$\times\left[4\operatorname{erf}(\beta)\exp\left(-\beta^2\right)\sum_{l=0}^{k}\frac{\beta^{2l}}{l!}+\frac{\exp\left(-2\beta^2\right)}{\sqrt{\pi}}\sum_{l=1}^{k}\frac{(2l)!}{(l!)^2}\sum_{r=1}^{l}\frac{(r-1)!\beta^{2r-1}}{8^{l-r}(2r-1)!}\right],$$

$W_4^{(15)}(2n_1,0)=0$,

$$W_4^{(15)}(2n_1+1,0)=\frac{1}{(n_1+1)!\sqrt{\pi}}\sum_{k=0}^{n_1}\frac{(k!)^2}{2^{2n_1+1-k}(2k+1)!}.$$

2.16. Integrals of $z^n \exp(\beta z)[\pm 1-\operatorname{erf}(\alpha z+\beta_1)]$, $z^n \exp(\beta z)[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)]$

2.16.1.

1. $$\int_0^{+\infty}\exp(\beta z)[1-\operatorname{erf}(az+\beta_1)]\,dz=-\int_{-\infty}^{0}\exp(-\beta z)[-1-\operatorname{erf}(az-\beta_1)]\,dz=$$
$$=\frac{1}{\beta}[\operatorname{erf}(\beta_1)-1]+\frac{1}{\beta}\exp\left(\frac{\beta^2-4a\beta\beta_1}{4a^2}\right)\left[\operatorname{erf}\left(\frac{\beta-2a\beta_1}{2a}\right)+1\right]\quad\{a>0\}.$$

2. $$\int_0^{+\infty}\exp(\beta z)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]dz=\frac{1}{\beta}\exp\left(\frac{\beta^2-4a_1\beta\beta_1}{4a_1^2}\right)\times$$
$$\times\left[\operatorname{erf}\left(\frac{2a_1\beta_1-\beta}{2a_1}\right)-1\right]-\frac{1}{\beta}\exp\left(\frac{\beta^2-4a_2\beta\beta_2}{4a_2^2}\right)\left[\operatorname{erf}\left(\frac{2a_2\beta_2-\beta}{2a_2}\right)-1\right]+$$
$$+\frac{1}{\beta}\left[\operatorname{erf}(\beta_2)-\operatorname{erf}(\beta_1)\right]\qquad\{a_1>0,\,a_2>0\}.$$

3. $$\int_{-\infty}^{+\infty}\exp(\beta z)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]\,dz=$$
$$=\frac{2}{\beta}\left[\exp\left(\frac{\beta^2-4a_2\beta\beta_2}{4a_2^2}\right)-\exp\left(\frac{\beta^2-4a_1\beta\beta_1}{4a_1^2}\right)\right]\qquad\{a_1>0,\,a_2>0\}.$$

4. $$\int_0^{\infty}\exp(\beta z)\left[\pm1-\operatorname{erf}(\alpha z+\beta_1)\right]dz=\frac{1}{\beta}\left[\operatorname{erf}(\beta_1)\mp1\right]-\frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$
$$\times\left[\operatorname{erf}\left(\frac{2\alpha\beta_1-\beta}{2\alpha}\right)\mp1\right]$$
$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1z-\beta z\right)+2\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

5. $$\int_0^{\infty}\exp(\beta z)[\operatorname{erf}(\alpha_1z+\beta_1)\mp\operatorname{erf}(\alpha_2z+\beta_2)]\,dz=\frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha_1\beta\beta_1}{4\alpha_1^2}\right)\times$$
$$\times\left[\operatorname{erf}\left(\frac{2\alpha_1\beta_1-\beta}{2\alpha_1}\right)-A_1\right]\mp\frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha_2\beta\beta_2}{4\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{2\alpha_2\beta_2-\beta}{2\alpha_2}\right)-A_2\right]-$$
$$-\frac{1}{\beta}[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)]\qquad\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1z-\beta z\right)+2\ln|z|\right]=$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2 + 2\alpha_2\beta_2 z - \beta z\right) + 2\ln|z|\right] = +\infty;\ A_k = 1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z) = +\infty,$$

$$A_k = -1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z) = -\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z) = \pm\infty\}.$$

2.16.2.

1. $$\int_0^{+\infty} z\exp(\beta z)[1-\operatorname{erf}(az+\beta_1)]\,dz = \int_{-\infty}^{0} z\exp(-\beta z)[-1-\operatorname{erf}(az-\beta_1)]\,dz =$$

$$= \frac{\beta^2 - 2a^2 - 2a\beta\beta_1}{2a^2\beta^2}\exp\left(\frac{\beta^2 - 4a\beta\beta_1}{4a^2}\right)\left[\operatorname{erf}\left(\frac{\beta - 2a\beta_1}{2a}\right)+1\right]+$$

$$+\frac{1}{\beta^2}[1-\operatorname{erf}(\beta_1)] + \frac{1}{\sqrt{\pi}a\beta}\exp\left(-\beta_1^2\right) \qquad \{a>0\}.$$

2. $$\int_0^{+\infty} z\exp(\beta z)[\operatorname{erf}(a_1 z+\beta_1) - \operatorname{erf}(a_2 z+\beta_2)]dz = \frac{1}{\beta^2}[\operatorname{erf}(\beta_1) - \operatorname{erf}(\beta_2)] + \frac{1}{\sqrt{\pi}\beta}\times$$

$$\times\left[\frac{1}{a_2}\exp\left(-\beta_2^2\right) - \frac{1}{a_1}\exp\left(-\beta_1^2\right)\right] + \frac{\beta^2 - 2a_1^2 - 2a_1\beta\beta_1}{2a_1^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2 - 4a_1\beta\beta_1}{4a_1^2}\right)\left[\operatorname{erf}\left(\frac{2a_1\beta_1 - \beta}{2a_1}\right)-1\right] + \frac{\beta^2 - 2a_2^2 - 2a_2\beta\beta_2}{2a_2^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2 - 4a_2\beta\beta_2}{4a_2^2}\right)\left[1-\operatorname{erf}\left(\frac{2a_2\beta_2 - \beta}{2a_2}\right)\right] \qquad \{a_1>0,\ a_2>0\}.$$

3. $$\int_{-\infty}^{+\infty} z\exp(\beta z)[\operatorname{erf}(a_1 z+\beta_1) - \operatorname{erf}(a_2 z+\beta_2)]\,dz = \frac{2a_1^2 + 2a_1\beta\beta_1 - \beta^2}{a_1^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2 - 4a_1\beta\beta_1}{4a_1^2}\right) - \frac{2a_2^2 + 2a_2\beta\beta_2 - \beta^2}{a_2^2\beta^2}\exp\left(\frac{\beta^2 - 4a_2\beta\beta_2}{4a_2^2}\right) \qquad \{a_1>0,\ a_2>0\}.$$

4. $$\int_0^{\infty} z\exp(\beta z)[\pm 1-\operatorname{erf}(\alpha z+\beta_1)]\,dz = \frac{1}{\sqrt{\pi}\alpha\beta}\exp\left(-\beta_1^2\right) + \frac{\beta^2 - 2\alpha^2 - 2\alpha\beta\beta_1}{2\alpha^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2 - 4\alpha\beta\beta_1}{4\alpha^2}\right)\left[\operatorname{erf}\left(\frac{\beta - 2\alpha\beta_1}{2\alpha}\right)\pm 1\right] - \frac{1}{\beta^2}[\operatorname{erf}(\beta_1)\mp 1]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + 2\alpha\beta_1 z - \beta z\right) + \ln|z|\right] = +\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

5. $$\int_0^{\infty} z\exp(\beta z)[\operatorname{erf}(\alpha_1 z+\beta_1) \mp \operatorname{erf}(\alpha_2 z+\beta_2)]\,dz = \frac{\beta^2 - 2\alpha_1^2 - 2\alpha_1\beta\beta_1}{2\alpha_1^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2-4\alpha_1\beta\beta_1}{4\alpha_1^2}\right)\left[\operatorname{erf}\left(\frac{2\alpha_1\beta_1-\beta}{2\alpha_1}\right)-A_1\right]\mp\frac{\beta^2-2\alpha_2^2-2\alpha_2\beta\beta_2}{2\alpha_2^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2-4\alpha_2\beta\beta_2}{4\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{2\alpha_2\beta_2-\beta}{2\alpha_2}\right)-A_2\right]+\frac{1}{\beta^2}\left[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)\right]-$$

$$-\frac{1}{\sqrt{\pi}\beta}\left[\frac{1}{\alpha_1}\exp\left(-\beta_1^2\right)\mp\frac{1}{\alpha_2}\exp\left(-\beta_2^2\right)\right]\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1z-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2z-\beta z\right)+\ln|z|\right]=+\infty;\ A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_kz)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_kz)=-\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha_1z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2z)=\pm\infty\}.$$

2.16.3.

1 $$\int_0^{+\infty}z^n\exp(\beta z)\left[1-\operatorname{erf}(az+\beta_1)\right]dz=(-1)^{n+1}\int_{-\infty}^{0}z^n\exp(-\beta z)\left[-1-\operatorname{erf}(az-\beta)\right]dz=$$

$$=(-1)^{n+1}n!\left\{\frac{1}{\beta^{n+1}}\left[1-\operatorname{erf}(\beta_1)\right]+W_1^{(16)}(a,\beta,\beta_1,1)\right\}\quad\{a>0\}.$$

2. $$\int_0^{+\infty}z^n\exp(\beta z)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]dz=$$

$$=(-1)^{n+1}n!\left\{\frac{1}{\beta^{n+1}}\left[\operatorname{erf}(\beta_1)-\operatorname{erf}(\beta_2)\right]-\right.$$

$$\left.-W_1^{(16)}(a_1,\beta,\beta_1,1)+W_1^{(16)}(a_2,\beta,\beta_2,1)\right\}\quad\{a_1>0,\ a_2>0\}.$$

3. $$\int_{-\infty}^{+\infty}z^n\exp(\beta z)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]dz=$$

$$=(-1)^{n+1}n!\left[W_2^{(16)}(a_1,\beta,\beta_1)-W_2^{(16)}(a_2,\beta,\beta_2)\right]\quad\{\beta\neq0,\ a_1>0,\ a_2>0\}.$$

4. $$\int_0^{\infty}z^n\exp(\beta z)[\pm1-\operatorname{erf}(\alpha z+\beta_1)]dz=$$

$$=(-1)^{n+1}n!\left\{\frac{1}{\beta^{n+1}}[\pm1-\operatorname{erf}(\beta_1)]+W_1^{(16)}(\alpha,\beta,\beta_1,\pm1)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1z-\beta z\right)-(n-2)\ln|z|\right]=+\infty,\quad\lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

5. $\int\limits_0^\infty z^n \exp(\beta z)[\operatorname{erf}(\alpha_1 z + \beta_1) \mp \operatorname{erf}(\alpha_2 z + \beta_2)]dz =$

$$= (-1)^{n+1} n! \left\{ \frac{1}{\beta^{n+1}} \left[\operatorname{erf}(\beta_1) \mp \operatorname{erf}(\beta_2) \right] - \right.$$

$$\left. - W_1^{(16)}(\alpha_1, \beta, \beta_1, A_1) \pm W_1^{(16)}(\alpha_2, \beta, \beta_2, A_2) \right\}$$

$$\{ \lim_{z\to\infty} \left[\operatorname{Re}\left(\alpha_1^2 z^2 + 2\alpha_1\beta_1 z - \beta z\right) - (n-2)\ln|z| \right] =$$

$$= \lim_{z\to\infty} \left[\operatorname{Re}\left(\alpha_2^2 z^2 + 2\alpha_2\beta_2 z - \beta z\right) - (n-2)\ln|z| \right] = +\infty ;$$

$$A_k = 1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha_k z) = +\infty , \quad A_k = -1 \text{ if } \lim_{z\to\infty} \operatorname{Re}(\alpha_k z) = -\infty ;$$

$$\lim_{z\to\infty} \operatorname{Re}(\alpha_1 z) \cdot \lim_{z\to\infty} \operatorname{Re}(\alpha_2 z) = \pm\infty \}.$$

Introduced notations:

1) $$W_1^{(16)}(\alpha, \beta, \beta_1, A) = \exp\left(\frac{\beta^2 - 4\alpha\beta\beta_1}{4\alpha^2} \right) \left[\operatorname{erf}\left(\frac{2\alpha\beta_1 - \beta}{2\alpha} \right) - A \right] \sum_{k=0}^{n} \frac{1}{2^k \beta^{n+1-k}} \times$$

$$\times \sum_{l=0}^{E(k/2)} \frac{(2\alpha\beta_1 - \beta)^{k-2l}}{l!(k-2l)!\alpha^{2k-2l}} + \frac{1}{\sqrt{\pi}} \exp\left(-\beta_1^2\right) \sum_{k=1}^{n} \frac{1}{2^k \beta^{n+1-k}} \times$$

$$\times \left[\sum_{l=1}^{k-E(k/2)} \frac{l!}{(2l)!(k+1-2l)!} \sum_{r=1}^{l} \frac{4^{l+1-r} (2\alpha\beta_1 - \beta)^{k-1-2l+2r}}{(r-1)!\alpha^{2k-1-2l+2r}} - \right.$$

$$\left. - \sum_{l=1}^{E(k/2)} \frac{1}{l!\,(k-2l)!} \sum_{r=1}^{l} \frac{(r-1)!\,(2\alpha\beta_1 - \beta)^{k-1-2l+2r}}{(2r-1)!\alpha^{2k-1-2l+2r}} \right],$$

$$W_1^{(16)}\left(\alpha, \beta, \frac{\beta}{2\alpha}, A\right) = \exp\left(-\frac{\beta^2}{4\alpha^2} \right) \times$$

$$\times \left[\frac{2}{\sqrt{\pi}} \sum_{k=1}^{n-E(n/2)} \frac{k!}{(2k)!\,\beta^{n+2-2k}\alpha^{2k-1}} - A \sum_{k=0}^{E(n/2)} \frac{1}{k!\beta^{n+1-2k}(2\alpha)^{2k}} \right];$$

2) $$W_2^{(16)}(\alpha, \beta, \beta_1) = 2\exp\left(\frac{\beta^2 - 4\alpha\beta\beta_1}{4\alpha^2} \right) \sum_{k=0}^{n} \frac{1}{2^k \beta^{n+1-k}} \sum_{l=0}^{E(k/2)} \frac{(2\alpha\beta_1 - \beta)^{k-2l}}{l!(k-2l)!\alpha^{2k-2l}},$$

$$W_2^{(16)}(\alpha,\beta,\frac{\beta}{2\alpha}) = 2\exp\left(-\frac{\beta^2}{4\alpha^2}\right)\sum_{k=0}^{E(n/2)}\frac{1}{k!\,\beta^{n+1-2k}\,(2\alpha)^{2k}}.$$

2.17. Integrals of $z^n \exp\left(-\alpha^2 z^2+\beta z\right)[\pm 1-\operatorname{erf}(\alpha_1 z+\beta_1)]$, $z^n \exp\left(-\alpha^2 z^2+\beta z\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]$

2.17.1.

1. $$\int_0^{+\infty}\exp\left(-a^2z^2\right)\left[1-\operatorname{erf}(az+\beta)\right]dz = -\int_{-\infty}^{0}\exp\left(-a^2z^2\right)\left[-1-\operatorname{erf}(az-\beta)\right]dz =$$
$$= \frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

2. $$\int_0^{+\infty}\exp\left(-a^2z^2\right)\left[1-\operatorname{erf}(a_1z)\right]dz = -\int_{-\infty}^{0}\exp\left(-a^2z^2\right)\left[-1-\operatorname{erf}(a_1z)\right]dz =$$
$$= \frac{1}{\sqrt{\pi}a}\arctan\frac{a}{a_1} \qquad \{a_1>0\}.$$

3. $$\int_0^{+\infty}\exp\left(a^2z^2\right)[1-\operatorname{erf}(a_1z)]dz = -\int_{-\infty}^{0}\exp\left(a^2z^2\right)[-1-\operatorname{erf}(a_1z)]dz =$$
$$= \frac{1}{2\sqrt{\pi}a}\ln\frac{a_1+a}{a_1-a} \qquad \{a_1>a\}.$$

4. $$\int_0^{+\infty}\exp\left(-a^2z^2\right)[\operatorname{erf}(az+\beta)-\operatorname{erf}(a_1z)]dz =$$
$$= \frac{1}{\sqrt{\pi}a}\arctan\frac{a}{a_1}-\frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \qquad \{a>0,\, a_1>0\}.$$

5. $$\int_0^{+\infty}\exp\left(-a^2z^2\right)\left[\operatorname{erf}(a_1z)-\operatorname{erf}(a_2z)\right]dz = \frac{1}{\sqrt{\pi}a}\left(\arctan\frac{a}{a_2}-\arctan\frac{a}{a_1}\right)$$
$$\{a_1>0,\, a_2>0\}.$$

6. $$\int_0^{+\infty}\exp\left(a^2z^2\right)[\operatorname{erf}(a_1z)-\operatorname{erf}(a_2z)]dz = \frac{1}{2\sqrt{\pi}\,a}\left(\ln\frac{a_1-a}{a_1+a}-\ln\frac{a_2-a}{a_2+a}\right)$$
$$\{a_1>a,\, a_2>a\}.$$

7. $\int_0^{+\infty}\exp\left(-a^2z^2\right)[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)]dz=$

$$=\frac{\sqrt{\pi}}{4a}\left\{\left[1-\operatorname{erf}\left(\frac{\beta_2}{\sqrt{2}}\right)\right]^2-\left[1-\operatorname{erf}\left(\frac{\beta_1}{\sqrt{2}}\right)\right]^2\right\}\qquad\{a>0\}$$

8. $\int_{-\infty}^{+\infty}\exp\left(-a^2z^2+\beta z\right)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]dz=\frac{\sqrt{\pi}}{a}\exp\left(\frac{\beta^2}{4a^2}\right)\times$

$$\times\left[\operatorname{erf}\left(\frac{a_1\beta+2a^2\beta_1}{2a\sqrt{a^2+a_1^2}}\right)-\operatorname{erf}\left(\frac{a_2\beta+2a^2\beta_2}{2a\sqrt{a^2+a_2^2}}\right)\right]\qquad\{a_1>0,\,a_2>0\}.$$

9. $\int_{-\infty}^{+\infty}\exp\left(a^2z^2+\beta z\right)\left[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)\right]dz=\frac{\sqrt{\pi}}{a}\exp\left(-\frac{\beta^2}{4a^2}\right)\times$

$$\times\left[\operatorname{erfi}\left(\frac{2a^2\beta_1-a_1\beta}{2a\sqrt{a_1^2-a^2}}\right)-\operatorname{erfi}\left(\frac{2a^2\beta_2-a_2\beta}{2a\sqrt{a_2^2-a^2}}\right)\right]\qquad\{a_1>a,\,a_2>a\}.$$

10. $\int_0^{\infty}\exp\left(-\alpha^2z^2\right)[\pm1-\operatorname{erf}(\alpha z+\beta)]dz=\frac{\sqrt{\pi}}{4\alpha}\left[1\mp\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad\lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

11. $\int_0^{\infty}\exp\left(-\alpha^2z^2\right)\left[\pm1-\operatorname{erf}(\alpha_1z)\right]dz=\frac{1}{\sqrt{\pi}\alpha}\arctan\frac{\alpha}{\alpha_1}\qquad\{\alpha_1^2\neq-\alpha^2,$

$$\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2\right)+2\ln|z|\right]=+\infty;\quad\pm\operatorname{Re}(\alpha_1z)>0\text{ if }z\to\infty\}.$$

12. $\int_0^{\infty}\exp\left(-\alpha^2z^2\right)\left[\operatorname{erf}(\alpha z+\beta)\mp\operatorname{erf}(\alpha_1z)\right]dz=-\frac{\sqrt{\pi}}{4\alpha}\left[A-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2\pm$

$$\pm\frac{1}{\sqrt{\pi}\alpha}\arctan\frac{\alpha}{\alpha_1}\qquad\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2\right)+2\ln|z|\right]=+\infty,\ \alpha_1^2\neq-\alpha^2;$$

$$A=1\text{ if }\lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty,\ A=-1\text{ if }\lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty;$$

$$\pm\operatorname{Re}(\alpha z+\beta)\cdot\operatorname{Re}(\alpha_1z)>0\text{ if }z\to\infty\}.$$

13. $\int_0^{\infty}\exp\left(-\alpha^2z^2\right)\left[\operatorname{erf}(\alpha_1z)\mp\operatorname{erf}(\alpha_2z)\right]dz=-\frac{1}{\sqrt{\pi}\alpha}\left(\arctan\frac{\alpha}{\alpha_1}\mp\arctan\frac{\alpha}{\alpha_2}\right)$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2\right)+2\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2\right)+2\ln|z|\right]=+\infty,$$

$$\alpha_1^2\neq-\alpha^2,\ \alpha_2^2\neq-\alpha^2;\ \pm\operatorname{Re}(\alpha_1 z)\cdot\operatorname{Re}(\alpha_2 z)>0 \text{ if } z\to\infty\}.$$

14. $$\int_0^\infty \exp\left(-\alpha^2 z^2\right)\left[\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)\right]dz=$$

$$=\frac{\sqrt{\pi}}{4\alpha}\left[1\mp\operatorname{erf}\left(\frac{\beta_2}{\sqrt{2}}\right)\right]^2-\frac{\sqrt{\pi}}{4\alpha}\left[1\mp\operatorname{erf}\left(\frac{\beta_1}{\sqrt{2}}\right)\right]^2\ \{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta_1 z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta_2 z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

15. $$\int_{\infty(T_1)}^{\infty(T_2)} \exp\left(-\alpha^2 z^2+\beta z\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=$$

$$=\frac{\sqrt{\pi}}{\alpha}\exp\left(\frac{\beta^2}{4\alpha^2}\right)\left[A_1\operatorname{erf}\left(\frac{\alpha_1\beta+2\alpha^2\beta_1}{2\alpha\sqrt{\alpha^2+\alpha_1^2}}\right)\mp A_2\operatorname{erf}\left(\frac{\alpha_2\beta+2\alpha^2\beta_2}{2\alpha\sqrt{\alpha^2+\alpha_2^2}}\right)\right]$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2+2\alpha_2\beta_2 z-\beta z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2+2\alpha_2\beta_2 z-\beta z\right)+2\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\infty,$$

$$A_k=0 \text{ if } \lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=+\infty;$$

$$\pm\operatorname{Re}(\alpha_1 z+\beta_1)\cdot\operatorname{Re}(\alpha_2 z+\beta_2)>0 \text{ if } z\to\infty(T_1) \text{ and } z\to\infty(T_2)\}.$$

2.17.2.

1. $$\int_0^{+\infty} z\exp\left(-a^2 z^2\right)\left[1-\operatorname{erf}(a_1 z+\beta)\right]dz=\int_{-\infty}^0 z\exp\left(-a^2 z^2\right)\left[-1-\operatorname{erf}(a_1 z-\beta)\right]dz=$$

$$= \frac{1}{2a^2}\left[1 - \operatorname{erf}(\beta)\right] + \frac{a_1}{2a^2\sqrt{a^2 + a_1^2}} \exp\left(-\frac{a^2\beta^2}{a^2 + a_1^2}\right)\left[\operatorname{erf}\left(\frac{a_1\beta}{\sqrt{a^2 + a_1^2}}\right) - 1\right]$$

$\{a_1 > 0\}$.

2. $$\int_0^{+\infty} z \exp\left(a^2 z^2\right)[1 - \operatorname{erf}(a_1 z + \beta)]dz = \int_{-\infty}^{0} z \exp\left(a^2 z^2\right)[-1 - \operatorname{erf}(a_1 z - \beta)]dz =$$

$$= \frac{1}{2a^2}[\operatorname{erf}(\beta) - 1] + \frac{a_1}{2a^2\sqrt{a_1^2 - a^2}} \exp\left(\frac{\alpha^2\beta^2}{a_1^2 - a^2}\right)\left[1 - \operatorname{erf}\left(\frac{a_1\beta}{\sqrt{a_1^2 - a^2}}\right)\right]$$

$\{a_1 > a\}$.

3. $$\int_0^{+\infty} z \exp\left(a^2 z^2\right)\left[1 - \operatorname{erf}(az + b)\right]dz = \int_{-\infty}^{0} z \exp\left(a^2 z^2\right)\left[-1 - \operatorname{erf}(az - b)\right]dz =$$

$$= \frac{1}{2a^2}[\operatorname{erf}(b) - 1] + \frac{1}{2\sqrt{\pi}a^2 b} \exp\left(-b^2\right) \qquad \{a > 0, b > 0\}.$$

4. $$\int_0^{+\infty} z \exp\left(-a^2 z^2\right)[\operatorname{erf}(a_1 z + \beta_1) - \operatorname{erf}(a_2 z + \beta_2)]dz = \frac{1}{2a^2}[\operatorname{erf}(\beta_1) - \operatorname{erf}(\beta_2)] +$$

$$+ \frac{a_1}{2a^2\sqrt{a^2 + a_1^2}} \exp\left(-\frac{a^2\beta_1^2}{a^2 + a_1^2}\right)\left[1 - \operatorname{erf}\left(\frac{a_1\beta_1}{\sqrt{a^2 + a_1^2}}\right)\right] - \frac{a_2}{2a^2\sqrt{a^2 + a_2^2}} \times$$

$$\times \exp\left(-\frac{a^2\beta_2^2}{a^2 + a_2^2}\right)\left[1 - \operatorname{erf}\left(\frac{a_2\beta_2}{\sqrt{a^2 + a_2^2}}\right)\right] \qquad \{a_1 > 0, a_2 > 0\}.$$

5. $$\int_0^{+\infty} z \exp\left(a^2 z^2\right)[\operatorname{erf}(a_1 z + \beta_1) - \operatorname{erf}(a_2 z + \beta_2)]dz = \frac{a_1}{2a^2\sqrt{a_1^2 - a^2}} \times$$

$$\times \exp\left(\frac{a^2\beta_1^2}{a_1^2 - a^2}\right)\left[\operatorname{erf}\left(\frac{a_1\beta_1}{\sqrt{a_1^2 - a^2}}\right) - 1\right] - \frac{a_2}{2a^2\sqrt{a_2^2 - a^2}} \exp\left(\frac{a^2\beta_2^2}{a_2^2 - a^2}\right) \times$$

$$\times \left[\operatorname{erf}\left(\frac{a_2\beta_2}{\sqrt{a_2^2 - a^2}}\right) - 1\right] + \frac{1}{2a^2}\left[\operatorname{erf}(\beta_2) - \operatorname{erf}(\beta_1)\right] \qquad \{a_1 > a, a_2 > a\}.$$

6. $$\int_0^{+\infty} z \exp\left(a^2 z^2\right)\left[\operatorname{erf}(a_1 z + \beta) - \operatorname{erf}(az + b)\right]dz =$$

$$= \int_{-\infty}^{0} z\exp\left(a^2z^2\right)\left[\operatorname{erf}\left(a_1 z-\beta\right)-\operatorname{erf}\left(a\,z-b\right)\right]dz = \frac{a_1}{2a^2\sqrt{a_1^2-a^2}}\exp\left(\frac{a^2\beta^2}{a_1^2-a^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{a_1\beta}{\sqrt{a_1^2-a^2}}\right)-1\right]+\frac{\exp\left(-b^2\right)}{2\sqrt{\pi}a^2b}+\frac{\operatorname{erf}\left(b\right)-\operatorname{erf}\left(\beta\right)}{2a^2} \qquad \left\{a>0,\, a_1>a,\, b>0\right\}.$$

7. $$\int_{0}^{+\infty} z\exp\left(a^2z^2\right)\left[\operatorname{erf}\left(az+b_1\right)-\operatorname{erf}\left(az+b_2\right)\right]dz =$$

$$= \int_{-\infty}^{0} z\exp\left(a^2z^2\right)\left[\operatorname{erf}\left(a\,z-b_1\right)-\operatorname{erf}\left(az-b_2\right)\right]dz =$$

$$= \frac{1}{2a^2}\left[\operatorname{erf}\left(b_2\right)-\operatorname{erf}\left(b_1\right)\right]+\frac{1}{2\sqrt{\pi}a^2}\left[\frac{1}{b_2}\exp\left(-b_2^2\right)-\frac{1}{b_1}\exp\left(-b_1^2\right)\right]$$

$$\left\{a>0,\, b_1>0,\, b_2>0\right\}.$$

8. $$\int_{-\infty}^{+\infty} z\exp\left(-a^2z^2+\beta z\right)\left[\operatorname{erf}\left(a_1 z+\beta_1\right)-\operatorname{erf}\left(a_2 z+\beta_2\right)\right]dz = \frac{\sqrt{\pi}\beta}{2a^3}\exp\left(\frac{\beta^2}{4a^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{2a^2\beta_1+a_1\beta}{2a\sqrt{a^2+a_1^2}}\right)-\operatorname{erf}\left(\frac{2a^2\beta_2+a_2\beta}{2a\sqrt{a^2+a_2^2}}\right)\right]+\frac{a_1}{a^2\sqrt{a^2+a_1^2}}\times$$

$$\times\exp\left(\frac{\beta^2-4a^2\beta_1^2-4a_1\beta\beta_1}{4a^2+4a_1^2}\right)-\frac{a_2}{a^2\sqrt{a^2+a_2^2}}\exp\left(\frac{\beta^2-4a^2\beta_2^2-4a_2\beta\beta_2}{4a^2+4a_2^2}\right)$$

$$\left\{a_1>0,\, a_2>0\right\}.$$

9. $$\int_{-\infty}^{+\infty} z\exp\left(a^2z^2+\beta z\right)\left[\operatorname{erf}\left(a_1 z+\beta_1\right)-\operatorname{erf}\left(a_2 z+\beta_2\right)\right]dz = \frac{\sqrt{\pi}\,\beta}{2a^3}\exp\left(-\frac{\beta^2}{4a^2}\right)\times$$

$$\times\left[\operatorname{erfi}\left(\frac{a_1\beta-2a^2\beta_1}{2a\sqrt{a_1^2-a^2}}\right)-\operatorname{erfi}\left(\frac{a_2\beta-2a^2\beta_2}{2a\sqrt{a_2^2-a^2}}\right)\right]-\frac{a_1}{a^2\sqrt{a_1^2-a^2}}\times$$

$$\times\exp\left(\frac{\beta^2+4a^2\beta_1^2-4a_1\beta\beta_1}{4a_1^2-4a^2}\right)+\frac{a_2}{a^2\sqrt{a_2^2-a^2}}\exp\left(\frac{\beta^2+4a^2\beta_2^2-4a_2\beta\beta_2}{4a_2^2-4a^2}\right)\Bigg]$$

$$\left\{a_1>a,\, a_2>a\right\}.$$

10. $$\int_{0}^{\infty} z\exp\left(-\alpha^2z^2\right)\left[\pm1-\operatorname{erf}\left(\alpha_1 z+\beta\right)\right]dz = \frac{1}{2\alpha^2}\left[\pm1-\operatorname{erf}\left(\beta\right)\right]+\frac{\alpha_1}{2\alpha^2\sqrt{\alpha^2+\alpha_1^2}}\times$$

$$\times\exp\left(-\frac{\alpha^2\beta^2}{\alpha^2+\alpha_1^2}\right)\left[\operatorname{erf}\left(\frac{\alpha_1\beta}{\sqrt{\alpha^2+\alpha_1^2}}\right)-A\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2+2\alpha_1\beta z\right)+\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1^2}\ z\right)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_1^2}\ z\right)=-\infty;$$

$$\pm\operatorname{Re}\left(\alpha_1z+\beta\right)>0 \text{ if } z\to\infty\}.$$

11. $$\int_0^\infty z\exp\left(\alpha^2z^2\right)\left[\pm1-\operatorname{erf}\left(\alpha z+\beta\right)\right]dz=\frac{1}{2\alpha^2}\left[\operatorname{erf}\left(\beta\right)\mp1\right]+\frac{1}{2\sqrt{\pi}\alpha^2\beta}\exp\left(-\beta^2\right)$$

$$\{\lim_{z\to\infty}\operatorname{Re}\left(\alpha\beta z\right)=+\infty,\ \pm\operatorname{Re}\left(\alpha z+\beta\right)>0 \text{ if } z\to\infty\}.$$

12. $$\int_0^\infty z\exp\left(-\alpha^2z^2\right)\left[\operatorname{erf}\left(\alpha_1z+\beta_1\right)\mp\operatorname{erf}\left(\alpha_2z+\beta_2\right)\right]dz=\frac{1}{2\alpha^2}\left[\operatorname{erf}\left(\beta_1\right)\mp\operatorname{erf}\left(\beta_2\right)\right]-$$

$$-\frac{\alpha_1}{2\alpha^2\sqrt{\alpha^2+\alpha_1^2}}\exp\left(-\frac{\alpha^2\beta_1^2}{\alpha^2+\alpha_1^2}\right)\left[\operatorname{erf}\left(\frac{\alpha_1\beta_1}{\sqrt{\alpha^2+\alpha_1^2}}\right)-A_1\right]\pm\frac{\alpha_2}{2\alpha^2\sqrt{\alpha^2+\alpha_2^2}}\times$$

$$\times\exp\left(-\frac{\alpha^2\beta_2^2}{\alpha^2+\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{\alpha_2\beta_2}{\sqrt{\alpha^2+\alpha_2^2}}\right)-A_2\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2+2\alpha_1\beta_1z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_2^2z^2+2\alpha_2\beta_2z\right)+\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=+\infty,\ A_k=-1 \text{ if}$$

$$\lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\infty;\ \pm\operatorname{Re}\left(\alpha_1z+\beta_1\right)\cdot\operatorname{Re}\left(\alpha_2z+\beta_2\right)>0 \text{ if } z\to\infty\}.$$

13. $$\int_0^\infty z\exp\left(\alpha^2z^2\right)\left[\operatorname{erf}\left(\alpha_1z+\beta_1\right)\mp\operatorname{erf}\left(\alpha z+\beta_2\right)\right]dz=\frac{\alpha_1}{2\alpha^2\sqrt{\alpha_1^2-\alpha^2}}\times$$

$$\times\exp\left(\frac{\alpha^2\beta_1^2}{\alpha_1^2-\alpha^2}\right)\left[\operatorname{erf}\frac{\alpha_1\beta_1}{\sqrt{\alpha_1^2-\alpha^2}}-A\right]-\frac{\operatorname{erf}\left(\beta_1\right)\mp\operatorname{erf}\left(\beta_2\right)}{2\alpha^2}\pm\frac{\exp\left(-\beta_2^2\right)}{2\sqrt{\pi}\alpha^2\beta_2}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2-\alpha^2z^2+2\alpha_1\beta_1z\right)+\ln|z|\right]=\lim_{z\to\infty}\operatorname{Re}\left(\alpha\beta_2z\right)=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty} \operatorname{Re}\left(\sqrt{\alpha_1^2-\alpha^2}\,z\right)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty} \operatorname{Re}\left(\sqrt{\alpha_1^2-\alpha^2}\,z\right)=-\infty;$$

$$\pm\operatorname{Re}(\alpha z+\beta_2)\cdot\operatorname{Re}(\alpha_1 z+\beta_1)>0 \text{ if } z\to\infty\}.$$

14. $$\int_0^{\infty} z\exp\left(\alpha^2 z^2\right)\left[\operatorname{erf}(\alpha z+\beta_1)-\operatorname{erf}(\alpha z+\beta_2)\right]dz=\frac{1}{2\alpha^2}\left[\operatorname{erf}(\beta_2)-\operatorname{erf}(\beta_1)\right]+$$

$$+\frac{1}{2\sqrt{\pi}\,\alpha^2}\left[\frac{1}{\beta_2}\exp\left(-\beta_2^2\right)-\frac{1}{\beta_1}\exp\left(-\beta_1^2\right)\right]\{\lim_{z\to\infty}[\operatorname{Re}(\alpha\beta_1 z)=\lim_{z\to\infty}\operatorname{Re}(\alpha\beta_2 z)=+\infty\}.$$

15. $$\int_{\infty(T_1)}^{\infty(T_2)} z\exp\left(-\alpha^2 z^2+\beta z\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=\frac{\sqrt{\pi}\,\beta}{2\alpha^3}\exp\left(\frac{\beta^2}{4\alpha^2}\right)\times$$

$$\times\left[A_1\operatorname{erf}\left(\frac{2\alpha^2\beta_1+\alpha_1\beta}{2\alpha\sqrt{\alpha^2+\alpha_1^2}}\right)\mp A_2\operatorname{erf}\left(\frac{2\alpha^2\beta_2+\alpha_2\beta}{2\alpha\sqrt{\alpha^2+\alpha_2^2}}\right)\right]+$$

$$+\frac{1}{\alpha^2}\left[\frac{A_1\alpha_1}{\sqrt{\alpha^2+\alpha_1^2}}\exp\left(\frac{\beta^2-4\alpha^2\beta_1^2-4\alpha_1\beta\beta_1}{4\alpha^2+4\alpha_1^2}\right)\mp\right.$$

$$\left.\mp\frac{A_2\alpha_2}{\sqrt{\alpha^2+\alpha_2^2}}\exp\left(\frac{\beta^2-4\alpha^2\beta_2^2-4\alpha_2\beta\beta_2}{4\alpha^2+4\alpha_2^2}\right)\right]$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2+2\alpha_1\beta_1 z-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_1^2z^2+2\alpha_1\beta_1 z-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2z+\alpha_2^2z^2+2\alpha_2\beta_2 z-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha_2^2z^2+2\alpha_2\beta_2 z-\beta z\right)+\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=-\infty,$$

$$A_k=0 \text{ if } \lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)\cdot\lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_k^2}\,z\right)=+\infty;$$

$$\pm\operatorname{Re}(\alpha_1 z+\beta_1)\cdot\operatorname{Re}(\alpha_2 z+\beta_2)>0 \text{ if } z\to\infty(T_1) \text{ and } z\to\infty(T_2)\}.$$

2.17.3.

1. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(-a^2z^2\right)[1-\operatorname{erf}(az+\beta)]dz =$$

$$= -\int\limits_{-\infty}^{0} z^{2m}\exp\left(-a^2z^2\right)\left[-1-\operatorname{erf}(az-\beta)\right]dz = W_1^{(17)}(a,\beta,1) \qquad \{a>0\}.$$

2. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(-a^2z^2\right)[1-\operatorname{erf}(a_1z)]dz = -\int\limits_{-\infty}^{0} z^{2m}\exp\left(-a^2z^2\right)[-1-\operatorname{erf}(a_1z)]dz =$$

$$= \frac{(2m)!}{m!\sqrt{\pi}}\left[\frac{1}{4^m a^{2m+1}}\operatorname{arctg}\frac{a}{a_1} - a_1\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!(2a)^{2m-2k}\left(a^2+a_1^2\right)^{k+1}}\right] \quad \{a_1>0\}.$$

3. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(a^2z^2\right)[1-\operatorname{erf}(a_1z)]dz = -\int\limits_{-\infty}^{0} z^{2m}\exp\left(a^2z^2\right)[-1-\operatorname{erf}(a_1z)]dz =$$

$$= \frac{(2m)!}{m!\sqrt{\pi}}\left[\frac{(-1)^m}{(2a)^{2m+1}}\ln\frac{a_1+a}{a_1-a} - a_1\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(-4a^2\right)^{m-k}\left(a_1^2-a^2\right)^{k+1}}\right] \quad \{a_1>a\}.$$

4. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(-a^2z^2\right)[\operatorname{erf}(az+\beta)-\operatorname{erf}(a_1z)]dz = \frac{(2m)!}{m!\sqrt{\pi}}\times$$

$$\times\left[\frac{1}{4^m a^{2m+1}}\operatorname{arctg}\frac{a}{a_1} - a_1\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!(2a)^{2m-2k}\left(a^2+a_1^2\right)^{k+1}}\right] - W_1^{(17)}(a,\beta,1)$$

$$\{a>0,\ a_1>0\}.$$

5. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(-a^2z^2\right)[\operatorname{erf}(a_1z)-\operatorname{erf}(a_2z)]\,dz = \frac{(2m)!}{4^m m!\sqrt{\pi}a^{2m+1}}\times$$

$$\times\left(\operatorname{arctg}\frac{a}{a_2} - \operatorname{arctg}\frac{a}{a_1}\right) + \frac{(2m)!}{m!\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(4a^2\right)^{m-k}}\left[\frac{a_1}{\left(a_1^2+a^2\right)^{k+1}} - \frac{a_2}{\left(a_2^2+a^2\right)^{k+1}}\right]$$

$$\{a_1>0,\ a_2>0\}.$$

6. $$\int\limits_{0}^{+\infty} z^{2m}\exp\left(a^2z^2\right)[\operatorname{erf}(a_1z)-\operatorname{erf}(a_2z)]dz = \frac{(-1)^m(2m)!}{m!\sqrt{\pi}(2a)^{2m+1}}\times$$

$$\times\left(\ln\frac{a_1-a}{a_1+a} - \ln\frac{a_2-a}{a_2+a}\right) + \frac{(2m)!}{m!\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(-4a^2\right)^{m-k}}\times$$

$$\times\left[\frac{a_1}{\left(a_1^2-a^2\right)^{k+1}}-\frac{a_2}{\left(a_2^2-a^2\right)^{k+1}}\right] \qquad \{a_1>a,\, a_2>a\}.$$

7. $$\int_0^{+\infty} z^{2m}\exp\left(-a^2z^2\right)[\operatorname{erf}(az+\beta_1)-\operatorname{erf}(az+\beta_2)]dz=$$

$$=W_1^{(17)}(a,\beta_2,1)-W_1^{(17)}(a,\beta_1,1) \qquad \{a>0\}.$$

8. $$\int_0^{+\infty} z^{2m+1}\exp\left(-a^2z^2\right)[1-\operatorname{erf}(a_1z+\beta)]dz=$$

$$=\int_{-\infty}^{0} z^{2m+1}\exp\left(-a^2z^2\right)[-1-\operatorname{erf}(a_1z-\beta)]dz=$$

$$=\frac{m!}{2\left(a^2\right)^{m+1}}\left[1-\operatorname{erf}(\beta)\right]-W_2^{(17)}\left(a^2,a_1,\beta,1\right) \qquad \{a_1>0\}.$$

9.

$$\int_0^{+\infty} z^{2m+1}\exp\left(a^2z^2\right)[1-\operatorname{erf}(a_1z+\beta)]dz=\int_{-\infty}^{0} z^{2m+1}\exp\left(a^2z^2\right)\left[-1-\operatorname{erf}\left(a_1z-\beta\right)\right]dz=$$

$$=\frac{m!}{2\left(-a^2\right)^{m+1}}[1-\operatorname{erf}(\beta)]-W_2^{(17)}\left(-a^2,a_1,\beta,1\right) \qquad \{a_1>a\}.$$

10. $$\int_0^{+\infty} z^{2m+1}\exp\left(a^2z^2\right)[1-\operatorname{erf}(az+b)]dz=\int_{-\infty}^{0} z^{2m+1}\exp\left(a^2z^2\right)[-1-\operatorname{erf}(az-b)]dz=$$

$$=\frac{m!}{2\left(-a^2\right)^{m+1}}\left[1-\operatorname{erf}(b)\right]+\frac{m!}{\sqrt{\pi}a}\exp\left(-b^2\right)\sum_{k=0}^{m}\frac{(2k)!}{k!\left(-a^2\right)^{m-k}(2ab)^{2k+1}}$$

$$\{a>0,\, b>0\}.$$

11. $$\int_0^{+\infty} z^{2m+1}\exp\left(-a^2z^2\right)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]\,dz=$$

$$=\frac{m!}{2(a^2)^{m+1}}[\operatorname{erf}(\beta_1)-\operatorname{erf}(\beta_2)]+$$

$$+W_2^{(17)}\left(a^2,a_1,\beta_1,1\right)-W_2^{(17)}\left(a^2,a_2,\beta_2,1\right) \qquad \{a_1>0,\, a_2>0\}.$$

12. $$\int_0^{+\infty} z^{2m+1}\exp\left(a^2z^2\right)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]dz=$$

$$=\frac{m!}{2\left(-a^2\right)^{m+1}}\left[\operatorname{erf}(\beta_1)-\operatorname{erf}(\beta_2)\right]+W_2^{(17)}\left(-a^2,a_1,\beta_1,1\right)-W_2^{(17)}\left(-a^2,a_2,\beta_2,1\right)$$

$$\{a_1>a,\ a_2>a\}.$$

13. $$\int_0^{+\infty} z^{2m+1}\exp\left(a^2z^2\right)[\operatorname{erf}(a_1z+\beta)-\operatorname{erf}(az+b)]dz=$$

$$=\int_{-\infty}^{0} z^{2m+1}\exp\left(a^2z^2\right)\left[\operatorname{erf}(a_1z-\beta)-\operatorname{erf}(az-b)\right]dz=$$

$$=\frac{m!}{2\left(-a^2\right)^{m+1}}\left[\operatorname{erf}(\beta)-\operatorname{erf}(b)\right]+W_2^{(17)}\left(-a^2,a_1,\beta,1\right)+$$

$$+\frac{m!}{\sqrt{\pi}a^{2m+2}}\exp\left(-b^2\right)\sum_{k=0}^{m}\frac{(-1)^{m-k}(2k)!}{k!(2b)^{2k+1}}\quad\{a>0,\ a_1>a,\ b>0\}.$$

14. $$\int_0^{+\infty} z^{2m+1}\exp\left(a^2z^2\right)[\operatorname{erf}(az+b_1)-\operatorname{erf}(az+b_2)]dz=$$

$$=\int_{-\infty}^{0} z^{2m+1}\exp\left(a^2z^2\right)[\operatorname{erf}(az-b_1)-\operatorname{erf}(az-b_2)]\,dz=$$

$$=\frac{m!}{2\left(-a^2\right)^{m+1}}\left[\operatorname{erf}(b_1)-\operatorname{erf}(b_2)\right]+\frac{m!}{\sqrt{\pi}a}\sum_{k=0}^{m}\frac{(2k)!}{k!\left(-a^2\right)^{m-k}}\times$$

$$\times\left[\frac{1}{(2ab_2)^{2k+1}}\exp\left(-b_2^2\right)-\frac{1}{(2ab_1)^{2k+1}}\exp\left(-b_1^2\right)\right]\quad\{a>0,\ b_1>0,\ b_2>0\}.$$

15. $$\int_{-\infty}^{+\infty} z^n\exp\left(-a^2z^2+\beta z\right)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]\,dz=W_3^{(17)}\left(n,a^2,a_1,\beta,\beta_1\right)-$$

$$-W_3^{(17)}\left(n,a^2,a_2,\beta,\beta_2\right)+W_4^{(17)}(n,a,a_1,\beta,\beta_1)-W_4^{(17)}(n,a,a_2,\beta,\beta_2)$$

$$\{a>0,\ a_1>0,\ a_2>0\}.$$

16. $$\int_{-\infty}^{+\infty} z^n\exp\left(a^2z^2+\beta z\right)[\operatorname{erf}(a_1z+\beta_1)-\operatorname{erf}(a_2z+\beta_2)]\,dz=W_3^{(17)}\left(n,-a^2,a_1,\beta,\beta_1\right)-$$

$$-W_3^{(17)}\left(n,-a^2,a_2,\beta,\beta_2\right)+W_5^{(17)}(n,a,a_1,\beta,\beta_1)-W_5^{(17)}(n,a,a_2,\beta,\beta_2)$$

$$\{a>0,\ a_1>a,\ a_2>a\}.$$

17. $$\int_0^{\infty} z^{2m}\exp\left(-\alpha^2z^2\right)[\pm1-\operatorname{erf}(\alpha z+\beta)]dz=W_1^{(17)}(\alpha,\beta,\pm1)$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta z\right)-(m-1)\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty \}.$$

18. $$\int_0^\infty z^{2m}\exp\left(-\alpha^2 z^2\right)[\pm 1-\operatorname{erf}(\alpha_1 z)]\,dz=\frac{(2m)!}{m!\sqrt{\pi}}\left[\frac{1}{4^m\alpha^{2m+1}}\operatorname{arctg}\frac{\alpha}{\alpha_1}-\right.$$

$$\left.-\alpha_1\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(4\alpha^2\right)^{m-k}\left(\alpha^2+\alpha_1^2\right)^{k+1}}\right]$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2\right)-(2m-2)\ln|z|\right]=+\infty \text{ (for } m>0\text{)};$$

$$\pm\operatorname{Re}(\alpha_1 z)>0 \text{ if } z\to\infty\}.$$

19. $$\int_0^\infty z^{2m}\exp\left(-\alpha^2 z^2\right)[\operatorname{erf}(\alpha z+\beta)\mp\operatorname{erf}(\alpha_1 z)]dz=\pm\frac{(2m)!}{m!\sqrt{\pi}}\left[\frac{1}{4^m\alpha^{2m+1}}\operatorname{arctg}\frac{\alpha}{\alpha_1}-\right.$$

$$\left.-\alpha_1\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(4\alpha^2\right)^{m-k}\left(\alpha^2+\alpha_1^2\right)^{k+1}}\right]-W_1^{(17)}(\alpha,\beta,A)$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta z\right)-(m-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2\right)-(2m-2)\ln|z|\right]=+\infty \text{ (for } m>0\text{)};$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha z)=-\infty;$$

$$\pm\operatorname{Re}(\alpha z+\beta)\cdot\operatorname{Re}(\alpha_1 z)>0 \text{ if } z\to\infty\}.$$

20. $$\int_0^\infty z^{2m}\exp\left(-\alpha^2 z^2\right)\left[\operatorname{erf}(\alpha_1 z)\mp\operatorname{erf}(\alpha_2 z)\right]dz=-\frac{(2m)!}{4^m\, m!\sqrt{\pi}\,\alpha^{2m+1}}\times$$

$$\times\left(\operatorname{arctg}\frac{\alpha}{\alpha_1}\mp\operatorname{arctg}\frac{\alpha}{\alpha_2}\right)-\frac{(2m)!}{m!\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!\left(4\alpha^2\right)^{m-k}}\times$$

$$\times\left[\frac{\alpha_1}{\left(\alpha^2+\alpha_1^2\right)^{k+1}}\mp\frac{\alpha_2}{\left(\alpha^2+\alpha_2^2\right)^{k+1}}\right]$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2\right)-(2m-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2\right)-(2m-2)\ln|z|\right]=+\infty \text{ (for } m>0\text{)};$$

$$\pm\operatorname{Re}(\alpha_1 z)\cdot\operatorname{Re}(\alpha_2 z)>0 \text{ if } z\to\infty\}.$$

21. $\int_0^\infty z^{2m} \exp\left(-\alpha^2 z^2\right)[\mathrm{erf}(\alpha z+\beta_1)-\mathrm{erf}(\alpha z+\beta_2)]dz =$

$$=W_1^{(17)}(\alpha,\beta_2,\pm 1)-W_1^{(17)}(\alpha,\beta_1,\pm 1)$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2+\alpha\beta_1 z\right)-(m-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2+\alpha\beta_2 z\right)-(m-1)\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\mathrm{Re}(\alpha z)=\pm\infty\}.$$

22. $\int_0^\infty z^{2m+1} \exp\left(-\alpha^2 z^2\right)[\pm 1-\mathrm{erf}(\alpha_1 z+\beta)]dz =$

$$=\frac{m!}{2(\alpha^2)^{m+1}}[\pm 1-\mathrm{erf}(\beta)]-W_2^{(17)}\left(\alpha^2,\alpha_1,\beta,A\right)$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta z\right)-(2m-1)\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left(\sqrt{\alpha^2+\alpha_1^2}\, z\right)=+\infty,$$

$$A=-1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left(\sqrt{\alpha^2+\alpha_1^2}\, z\right)=-\infty;\ \pm\mathrm{Re}(\alpha_1 z+\beta)>0 \text{ if } z\to\infty\}.$$

23. $\int_0^\infty z^{2m+1}\exp\left(\alpha^2 z^2\right)\left[\pm 1-\mathrm{erf}(\alpha z+\beta)\right]dz=\frac{(-1)^m m!}{2\alpha^{2m+2}}\left[\mathrm{erf}(\beta)\mp 1+\right.$

$$\left.+\frac{1}{\sqrt{\pi}}\exp\left(-\beta^2\right)\sum_{k=0}^{m}\frac{(2k)!}{(-4)^k k!\beta^{2k+1}}\right]$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}(\alpha\beta z)-m\ln|z|\right]=+\infty, \quad \pm\mathrm{Re}(\alpha z+\beta)>0 \text{ if } z\to\infty\}.$$

24. $\int_0^\infty z^{2m+1} \exp\left(-\alpha^2 z^2\right)[\mathrm{erf}(\alpha_1 z+\beta_1)\mp\mathrm{erf}(\alpha_2 z+\beta_2)]dz =$

$$=\frac{m!}{2\left(\alpha^2\right)^{m+1}}[\mathrm{erf}(\beta_1)\mp\mathrm{erf}(\beta_2)]+W_2^{(17)}\left(\alpha^2,\alpha_1,\beta_1,A_1\right)\mp W_2^{(17)}\left(\alpha^2,\alpha_2,\beta_2,A_2\right)$$

$$\{\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)-(2m-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)-(2m-1)\ln|z|\right]=+\infty,\ A_s=1 \text{ if}$$

$$\lim_{z\to\infty}\mathrm{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right)=+\infty,\ A_s=-1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right)=-\infty;$$

$$\pm\operatorname{Re}(\alpha_1 z+\beta_1)\cdot\operatorname{Re}(\alpha_2 z+\beta_2)>0 \text{ if } z\to\infty\}.$$

25. $$\int_0^\infty z^{2m+1}\exp\left(\alpha^2 z^2\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha z+\beta_2)\right]dz=$$

$$=\frac{m!}{2\left(-\alpha^2\right)^{m+1}}\left[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)\right]+W_2^{(17)}\left(-\alpha^2,\alpha_1,\beta_1,A\right)\mp$$

$$\mp\frac{m!}{2\sqrt{\pi}\left(-\alpha^2\right)^{m+1}}\exp\left(-\beta_2^2\right)\sum_{k=0}^{m}\frac{(2k)!}{(-4)^k\,k!\beta_2^{2k+1}}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2-\alpha^2 z^2+2\alpha_1\beta_1 z\right)-(2m-1)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta_2 z)-m\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2-\alpha^2}\,z\right)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2-\alpha^2}\,z\right)=-\infty;$$

$$\pm\operatorname{Re}(\alpha z+\beta_2)\cdot\operatorname{Re}(\alpha_1 z+\beta_1)>0 \text{ if } z\to\infty\}.$$

26. $$\int_0^\infty z^{2m+1}\exp\left(\alpha^2 z^2\right)\left[\operatorname{erf}(\alpha z+\beta)-\operatorname{erf}(\alpha z+\beta_1)\right]dz=\frac{(-1)^m m!}{2\alpha^{2m+2}}\left\{\operatorname{erf}(\beta_1)-\operatorname{erf}(\beta)+\right.$$

$$\left.+\frac{1}{\sqrt{\pi}}\sum_{k=0}^{m}\frac{(2k)!}{(-4)^k\,k!}\left[\frac{1}{\beta_1^{2k+1}}\exp\left(-\beta_1^2\right)-\frac{1}{\beta^{2k+1}}\exp\left(-\beta^2\right)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta z)-m\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta_1 z)-m\ln|z|\right]=+\infty\}.$$

27. $$\int_{\infty(T_1)}^{\infty(T_2)} z^n\exp\left(-\alpha^2 z^2+\beta z\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=$$

$$=A_1\left[W_3^{(17)}\left(n,\alpha^2,\alpha_1,\beta,\beta_1\right)+W_4^{(17)}(n,\alpha,\alpha_1,\beta,\beta_1)\right]\mp$$

$$\mp A_2\left[W_3^{(17)}\left(n,\alpha^2,\alpha_2,\beta,\beta_2\right)+W_4^{(17)}(n,\alpha,\alpha_2,\beta,\beta_2)\right]$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_1^2 z^2+2\alpha_1\beta_1 z-\beta z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2+2\alpha_2\beta_2 z-\beta z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha^2 z^2+\alpha_2^2 z^2+2\alpha_2\beta_2 z-\beta z\right)-(n-2)\ln|z|\right]=+\infty;$$

$$A_s=1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\,z\right)=+\infty,$$

$$A_s = -1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right) = -\infty,$$

$$A_s = 0 \text{ if } \lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right)\cdot \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha^2+\alpha_s^2}\, z\right) = +\infty;$$

$$\pm\operatorname{Re}(\alpha_1 z+\beta_1)\cdot\operatorname{Re}(\alpha_2 z+\beta_2) > 0 \text{ if } z\to\infty(T_1) \text{ and } z\to\infty(T_2)\}.$$

Introduced notations:

1) $$W_1^{(17)}(\alpha,\beta,A) = \frac{(2m)!\sqrt{\pi}}{4^{m+1}\, m!\,\alpha^{2m+1}}\left[A-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 + \frac{(2m)!}{m!(2\alpha)^{2m+1}}\sum_{k=0}^{m-1} k!\times$$

$$\times\left\{\frac{1}{\sqrt{2}}\exp\left(-\frac{\beta^2}{2}\right)\left[A-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]\sum_{l=0}^{k}\frac{\beta^{2k+1-2l}}{2^l\, l!(2k+1-2l)!} + \frac{\exp(-\beta^2)}{\sqrt{\pi}}\times\right.$$

$$\left.\times\left[\sum_{l=1}^{k}\frac{1}{l!(2k+1-2l)!}\sum_{r=1}^{l}\frac{r!\beta^{2k-2l+2r}}{2^{l-r}(2r)!} - \sum_{l=0}^{k}\frac{l!}{(2l+1)!(2k-2l)!}\sum_{r=0}^{l}\frac{2^{l-r}\beta^{2k-2l+2r}}{r!}\right]\right\},$$

$$W_1^{(17)}(\alpha,0,A) = \frac{(2m)!}{m!\alpha^{2m+1}}\left[\frac{\sqrt{\pi}}{4^{m+1}} - \frac{1}{\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{(k!)^2}{2^{2m+1-k}(2k+1)!}\right];$$

2) $$W_2^{(17)}\left(\alpha^2,\alpha_1,\beta,A\right) = \frac{m!\alpha_1}{2\sqrt{\alpha^2+\alpha_1^2}}\exp\left(-\frac{\alpha^2\beta^2}{\alpha^2+\alpha_1^2}\right)\left[A-\operatorname{erf}\left(\frac{\alpha_1\beta}{\sqrt{\alpha^2+\alpha_1^2}}\right)\right]\times$$

$$\times\sum_{k=0}^{m}\frac{(2k)!}{k!\left(\alpha^2\right)^{m+1-k}}\sum_{l=0}^{k}\frac{(\alpha_1\beta)^{2k-2l}}{4^l\, l!(2k-2l)!\left(\alpha^2+\alpha_1^2\right)^{2k-l}} + \frac{m!\alpha_1}{2\sqrt{\pi}}\exp\left(-\beta^2\right)\times$$

$$\times\sum_{k=1}^{m}\frac{(2k)!}{k!\left(\alpha^2\right)^{m+1-k}}\left[\sum_{l=1}^{k}\frac{1}{l!(2k-2l)!}\sum_{r=1}^{l}\frac{r!(\alpha_1\beta)^{2k-1-2l+2r}}{4^{l-r}(2r)!\left(\alpha^2+\alpha_1^2\right)^{2k-l+r}} - \right.$$

$$\left.-\sum_{l=1}^{k}\frac{(l-1)!}{(2l-1)!(2k+1-2l)!}\sum_{r=1}^{l}\frac{(\alpha_1\beta)^{2k-1-2l+2r}}{(r-1)!\left(\alpha^2+\alpha_1^2\right)^{2k-l+r}}\right],$$

$$W_2^{(17)}\left(\alpha^2,\alpha_1,0,A\right) = A\frac{m!\alpha_1}{2\sqrt{\alpha^2+\alpha_1^2}}\sum_{k=0}^{m}\frac{(2k)!}{4^k(k!)^2\left(\alpha^2\right)^{m+1-k}\left(\alpha^2+\alpha_1^2\right)^k};$$

$$3)\ W_3^{(17)}\left(n,\alpha^2,\alpha_1,\beta,\beta_1\right)=\frac{n!}{\left(2\alpha^2\right)^n\sqrt{\alpha^2+\alpha_1^2}}\exp\left(\frac{\beta^2-4\alpha^2\beta_1^2-4\alpha_1\beta\beta_1}{4\alpha^2+4\alpha_1^2}\right)\times$$

$$\times\left[\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=1}^{k}\frac{4^{k+1-l}(2l-2)!}{(l-1)!}\times\right.$$

$$\times\sum_{r=1}^{l}\frac{\left(\alpha^2\right)^{k-2-l+2r}\alpha_1^{2l+1-2r}\left(2\alpha^2\beta_1+\alpha_1\beta\right)^{2l-2r}}{(r-1)!(2l-2r)!\left(\alpha^2+\alpha_1^2\right)^{2l-1-r}}-\sum_{k=1}^{E(n/2)}\frac{\beta^{n-2k}}{k!(n-2k)!}\times$$

$$\left.\times\sum_{l=1}^{k}(l-1)!\sum_{r=0}^{l-1}\frac{\left(\alpha^2\right)^{k-l+2r}\alpha_1^{2l-2r}\left(2\alpha^2\beta_1+\alpha_1\beta\right)^{2l-1-2r}}{r!\,(2l-1-2r)!\left(\alpha^2+\alpha_1^2\right)^{2l-1-r}}\right],$$

$$W_3^{(17)}(n,\alpha^2,\alpha_1,\beta,-\frac{\alpha_1\beta}{2\alpha^2})=\frac{n!\alpha_1}{\left(2\alpha^2\right)^n\sqrt{\alpha^2+\alpha_1^2}}\exp\left(\frac{\beta^2}{4\alpha^2}\right)\times$$

$$\times\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{4^{k-l}(2l)!\left(\alpha^2\right)^{k-1+l}}{(l!)^2\left(\alpha^2+\alpha_1^2\right)^l},$$

$$W_3^{(17)}\left(2m,\alpha^2,\alpha_1,0,\beta_1\right)=-\frac{(2m)!}{4^m m!\sqrt{\alpha^2+\alpha_1^2}}\exp\left(-\frac{\alpha^2\beta_1^2}{\alpha^2+\alpha_1^2}\right)\times$$

$$\times\sum_{k=1}^{m}\frac{(k-1)!}{\left(\alpha^2\right)^{m+1-k}}\sum_{l=0}^{k-1}\frac{\alpha_1^{2k-2l}(2\beta_1)^{2k-1-2l}}{l!(2k-1-2l)!\left(\alpha^2+\alpha_1^2\right)^{2k-1-l}},$$

$$W_3^{(17)}\left(2m+1,\alpha^2,\alpha_1,0,\beta_1\right)=\frac{m!}{\sqrt{\alpha^2+\alpha_1^2}}\exp\left(-\frac{\alpha^2\beta_1^2}{\alpha^2+\alpha_1^2}\right)\times$$

$$\times\sum_{k=0}^{m}\frac{(2k)!}{k!\left(\alpha^2\right)^{m+1-k}}\sum_{l=0}^{k}\frac{\alpha_1^{2k+1-2l}\beta_1^{2k-2l}}{4^l\,l!(2k-2l)!\left(\alpha^2+\alpha_1^2\right)^{2k-1}};$$

$$4)\ W_4^{(17)}(n,\alpha,\alpha_1,\beta,\beta_1)=\frac{n!\sqrt{\pi}}{2^n\alpha}\exp\left(\frac{\beta^2}{4\alpha^2}\right)\operatorname{erf}\left(\frac{2\alpha^2\beta_1+\alpha_1\beta}{2\alpha\sqrt{\alpha^2+\alpha_1^2}}\right)\times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{\beta^{n-2k}}{k!(n-2k)!\alpha^{2n-2k}},$$

$$W_4^{(17)}(2m,\alpha,\alpha_1,0,\beta_1)=\frac{(2m)!\sqrt{\pi}}{4^m\, m!\alpha^{2m+1}}\operatorname{erf}\left(\frac{\alpha\beta_1}{\sqrt{\alpha^2+\alpha_1^2}}\right),$$

$$W_4^{(17)}(2m+1,\alpha,\alpha_1,0,\beta_1)=0;$$

5) $$W_5^{(17)}(n,\alpha,\alpha_1,\beta,\beta_1)=\frac{n!\sqrt{\pi}}{2^n\,\alpha}\exp\left(-\frac{\beta^2}{4\alpha^2}\right)\operatorname{erfi}\left(\frac{2\alpha^2\beta_1-\alpha_1\beta}{2\alpha\sqrt{\alpha_1^2-\alpha^2}}\right)\times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{(-1)^{n-k}\beta^{n-2k}}{k!(n-2k)!\alpha^{2n-2k}},$$

$$W_5^{(17)}(2m,\alpha,\alpha_1,0,\beta_1)=\frac{(2m)!\sqrt{\pi}}{(-4)^m\, m!\alpha^{2m+1}}\operatorname{erfi}\left(\frac{\alpha\beta_1}{\sqrt{\alpha_1^2-\alpha^2}}\right),$$

$$W_5^{(17)}(2m+1,\alpha,\alpha_1,0,\beta_1)=0.$$

2.18. Integrals of $z^n\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]$, $z^n\left[\pm1-\operatorname{erf}(\alpha z+\beta_1)\right]\exp\left[-(\alpha z+\beta_2)^2\right]$, $z^n\exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)$, $z^n\exp(\beta z)\left[1-\operatorname{erf}^2(\alpha z+\beta_1)\right]$, $z^n\exp(\beta z)\left[\operatorname{erf}^2(\alpha_1 z+\beta_1)-\operatorname{erf}^2(\alpha_2 z+\beta_2)\right]$

2.18.1.

1. $$\int_{-\beta_1/a}^{+\infty}\operatorname{erf}(az+\beta_1)\exp\left[-(az+\beta_2)^2\right]dz=\frac{\sqrt{\pi}}{4a}\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

2. $$\int_0^{+\infty}\operatorname{erf}(az+\beta)\exp\left(-a^2z^2\right)dz=\frac{\sqrt{\pi}}{2a}-\frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

3. $$\int_0^{+\infty}\operatorname{erf}(az)\exp\left[-(az+\beta)^2\right]dz=\frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

4. $$\int_{-\beta_1/a}^{+\infty}[1-\operatorname{erf}(az+\beta_1)]\exp\left[-(az+\beta_2)^2\right]dz=$$

$$= -\int_{-\infty}^{\beta_1/a} \left[-1-\operatorname{erf}\left(az-\beta_1\right)\right]\exp\left[-\left(az-\beta_2\right)^2\right] dz =$$

$$= \frac{\sqrt{\pi}}{2a}[1+\operatorname{erf}(\beta_1-\beta_2)] - \frac{\sqrt{\pi}}{4a}\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]^2 \quad \{a>0\}.$$

5. $$\int_0^{+\infty} \left[1-\operatorname{erf}\left(az+\beta\right)\right]\exp\left[-\left(az+\beta\right)^2\right] dz =$$

$$= -\int_{-\infty}^{0} \left[-1-\operatorname{erf}\left(az-\beta\right)\right]\exp\left[-\left(az-\beta\right)^2\right] dz = \frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}(\beta)\right]^2 \qquad \{a>0\}.$$

6. $$\int_0^{+\infty} \left[1-\operatorname{erf}\left(az\right)\right]\exp\left[-\left(az+\beta\right)^2\right] dz = -\int_{-\infty}^{0} \left[-1-\operatorname{erf}\left(az\right)\right]\exp\left[-\left(az-\beta\right)^2\right] dz =$$

$$= \frac{\sqrt{\pi}}{2a}[1-\operatorname{erf}(\beta)] - \frac{\sqrt{\pi}}{4a}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \quad \{a>0\}.$$

7. $$\int_{-\infty}^{+\infty} \operatorname{erf}(az+\beta_1)\exp\left[-(az+\beta_2)^2\right] dz = \frac{\sqrt{\pi}}{a}\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right) \qquad \{a>0\}.$$

8. $$\int_{-\beta_1/\alpha}^{\infty} \operatorname{erf}\left(\alpha z+\beta_1\right)\exp\left[-\left(\alpha z+\beta_2\right)^2\right] dz = \frac{\sqrt{\pi}}{4\alpha}\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\pm 1\right]^2$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + \alpha\beta_1 z + \alpha\beta_2 z\right) + \ln|z|\right] =$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + 2\alpha\beta_2 z\right) + \ln|z|\right] = +\infty, \quad \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right) = \pm\infty \}.$$

9. $$\int_0^{\infty} \operatorname{erf}\left(\alpha z+\beta\right)\exp\left(-\alpha^2 z^2\right) dz = \frac{\sqrt{\pi}}{2\alpha} - \frac{\sqrt{\pi}}{4\alpha}\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]^2$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right) + \ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + \alpha\beta z\right) + \ln|z|\right] = +\infty,$$

$$\lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right) = \pm\infty \}.$$

10. $$\int_0^{\infty} \operatorname{erf}(\alpha z)\exp\left[-(\alpha z+\beta)^2\right] dz = \frac{\sqrt{\pi}}{4\alpha}\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]^2$$

$$\{ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + \alpha\beta z\right) + \ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2 + 2\alpha\beta z\right) + \ln|z|\right] = +\infty,$$

$$\lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right) = \pm\infty \}.$$

11. $$\int_{-\beta_1/\alpha}^{\infty} \left[\pm 1-\operatorname{erf}\left(\alpha z+\beta_1\right)\right]\exp\left[-\left(\alpha z+\beta_2\right)^2\right] dz =$$

$$=\frac{\sqrt{\pi}}{2\alpha}\left[1\pm\operatorname{erf}(\beta_1-\beta_2)\right]-\frac{\sqrt{\pi}}{4\alpha}\left[1\pm\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha\beta_1z+\alpha\beta_2z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

12. $$\int_0^{\infty}[\pm1-\operatorname{erf}(\alpha z+\beta)]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{4\alpha}[\operatorname{erf}(\beta)\mp1]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

13. $$\int_0^{\infty}[\pm1-\operatorname{erf}(\alpha z)]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{2\alpha}[1\mp\operatorname{erf}(\beta)]-\frac{\sqrt{\pi}}{4\alpha}\left[1\mp\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

14. $$\int_{-\beta_1/\alpha}^{-\beta_2/\alpha}\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=\frac{\sqrt{\pi}}{2\alpha}\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right).$$

15. $$\int_{-\beta_1/\alpha}^{-(\beta_1+\beta_2)/(2\alpha)}\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=$$

$$=\frac{\sqrt{\pi}}{4\alpha}\left[\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)-\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{2}\right)\right].$$

2.18.2.

1. $$\int_{-\beta/a}^{+\infty}\exp(-bz)\operatorname{erf}^2(az+\beta)dz=\int_{-\infty}^{\beta/a}\exp(bz)\operatorname{erf}^2(az-\beta)dz=$$

$$=\frac{1}{b}\exp\left(\frac{b^2+4ab\beta}{4a^2}\right)\left[1-\operatorname{erf}\left(\frac{\sqrt{2}b}{4a}\right)\right]^2\quad\{a>0,b>0\}.$$

2. $$\int_{-\beta_1/a}^{+\infty}\exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz=\frac{1}{\beta}\exp\left(\frac{\beta^2-4a\beta\beta_1}{4a^2}\right)\times$$

$$\times\left[1+\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a}\right)\right]^2-\frac{1}{\beta}\exp\left(-\frac{\beta\beta_1}{a}\right)\quad\{a>0\}.$$

3. $$\int_0^{+\infty}\exp(\beta z)\left[\operatorname{erf}^2(a_1z)-\operatorname{erf}^2(a_2z)\right]dz=\frac{1}{\beta}\exp\left(\frac{\beta^2}{4a_2^2}\right)\left[1+\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_2}\right)\right]^2-$$

$$-\frac{1}{\beta}\exp\left(\frac{\beta^2}{4a_1^2}\right)\left[1+\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_1}\right)\right]^2 \qquad \{a_1>0,\, a_2>0\}.$$

4. $$\int_{-\infty}^{+\infty}\exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz=\frac{4}{\beta}\exp\left(\frac{\beta^2-4a\beta\beta_1}{4a^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a}\right) \qquad \{a>0\}.$$

5. $$\int_{-\infty}^{+\infty}\exp(\beta z)\left[\operatorname{erf}^2(a_1z+\beta_1)-\operatorname{erf}^2(a_2z+\beta_2)\right]dz=\frac{4}{\beta}\exp\left(\frac{\beta^2-4a_2\beta\beta_2}{4a_2^2}\right)\times$$

$$\times\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_2}\right)-\frac{4}{\beta}\exp\left(\frac{\beta^2-4a_1\beta\beta_1}{4a_1^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_1}\right) \qquad \{a_1>0,\, a_2>0\}.$$

6. $$\int_{-\beta_1/\alpha}^{\infty}\exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)\,dz=-\frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm1\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2z^2+4\alpha\beta_1z-\beta z\right)+3\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}\operatorname{Re}(\beta z)=-\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

7. $$\int_{-\beta_1/\alpha}^{\infty}\exp(\beta z)\left[1-\operatorname{erf}^2(\alpha z+\beta_1)\right]dz=\frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm1\right]^2-\frac{1}{\beta}\exp\left(-\frac{\beta\beta_1}{\alpha}\right) \quad \{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1z-\beta z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2z^2+4\alpha\beta_1z-\beta z\right)+3\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $$\int_0^{\infty}\exp(\beta z)\left[\operatorname{erf}^2(\alpha_1z)-\operatorname{erf}^2(\alpha_2z)\right]dz=\frac{1}{\beta}\exp\left(\frac{\beta^2}{4\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_2}\right)\pm1\right]^2-$$

$$-\frac{1}{\beta}\exp\left(\frac{\beta^2}{4\alpha_1^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_1}\right)+A\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2-\beta z\right)+2\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2z^2-\beta z\right)+2\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha_1^2z^2-\beta z\right)+3\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha_2^2z^2-\beta z\right)+3\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_1z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_1z)=-\infty;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_2z)=\pm\infty\}.$$

9. $$\int_{-\beta_1/\alpha}^{(\beta-2\alpha\beta_1)/(2\alpha^2)} \exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)dz = \frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\left[\exp\left(\frac{\beta^2}{4\alpha^2}\right)\operatorname{erf}^2\left(\frac{\beta}{2\alpha}\right)-2\operatorname{erf}^2\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\right].$$

10. $$\int_{-\beta_1/\alpha}^{(\beta-4\alpha\beta_1)/(4\alpha^2)} \exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)dz = \frac{1}{\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\left[2\operatorname{erf}^2\left(\frac{\beta}{4\alpha}\right)-\operatorname{erf}^2\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\right].$$

2.18.3.

1. $$\int_{-\beta_1/a}^{+\infty} z\operatorname{erf}(az+\beta_1)\exp\left[-(az+\beta_2)^2\right]dz = \frac{\sqrt{2}}{4a^2}\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]\times$$

$$\times\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]-\frac{\sqrt{\pi}\beta_2}{4a^2}\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

2. $$\int_0^{+\infty} z\operatorname{erf}(az)\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{2}}{4a^2}\exp\left(-\frac{\beta^2}{2}\right)\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]-$$

$$-\frac{\sqrt{\pi}\beta}{4a^2}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 \qquad \{a>0\}.$$

3. $$\int_{-\beta_1/a}^{+\infty} z\left[1-\operatorname{erf}(az+\beta_1)\right]\exp\left[-(az+\beta_2)^2\right]dz =$$

$$=\int_{-\infty}^{\beta_1/a} z\left[-1-\operatorname{erf}(az-\beta_1)\right]\exp\left[-(az-\beta_2)^2\right]dz = \frac{\sqrt{\pi}\beta_2}{4a^2}\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]^2-$$

$$-\frac{\sqrt{2}}{4a^2}\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]\left[1+\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]-\frac{\sqrt{\pi}\beta_2}{2a^2}\left[1+\operatorname{erf}(\beta_1-\beta_2)\right]+$$

$$+\frac{1}{2a^2}\exp\left[-(\beta_1-\beta_2)^2\right] \qquad \{a>0\}.$$

4. $\int\limits_0^{+\infty} z\left[1-\operatorname{erf}(az+\beta)\right]\exp\left[-(az+\beta)^2\right]dz = \int\limits_{-\infty}^{0} z\left[-1-\operatorname{erf}(az-\beta)\right]\exp\left[-(az-\beta)\right]^2 dz =$

$$= \frac{1}{2a^2}\operatorname{erf}\left(-\beta^2\right)[1-\operatorname{erf}(\beta)] - \frac{\sqrt{\pi}\beta}{4a^2}[1-\operatorname{erf}(\beta)]^2 - \frac{\sqrt{2}}{4a^2}\left[1-\operatorname{erf}\left(\sqrt{2}\beta\right)\right] \ \{a>0\}.$$

5. $\int\limits_0^{+\infty} z[1-\operatorname{erf}(az)]\exp\left[-(az+\beta)^2\right]dz = \int\limits_{-\infty}^{0} z[-1-\operatorname{erf}(az)]\exp\left[-(az-\beta)^2\right]dz =$

$$= \frac{\sqrt{\pi}\beta}{4a^2}\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right]^2 - \frac{\sqrt{2}}{4a^2}\exp\left(-\frac{\beta^2}{2}\right)\left[1-\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\right] - \frac{\sqrt{\pi}\beta}{2a^2}\times$$

$$\times[1-\operatorname{erf}(\beta)] + \frac{1}{2a^2}\exp\left(-\beta^2\right)\{a>0\}.$$

6. $\int\limits_{-\infty}^{+\infty} z\operatorname{erf}(az+\beta_1)\exp\left[-(az+\beta_2)^2\right]dz = \frac{\sqrt{\pi}\beta_2}{a^2}\operatorname{erf}\left(\frac{\beta_2-\beta_1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}a^2}\times$

$$\times\exp\left[-\frac{(\beta_2-\beta_1)^2}{2}\right] \qquad \{a>0\}.$$

7. $\int\limits_{-\beta_1/\alpha}^{\infty} z\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz = \frac{1}{4\alpha^2}\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\pm 1\right]\times$

$$\times\left\{\sqrt{2}\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right] - \sqrt{\pi}\beta_2\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\pm 1\right]\right\}$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta_1 z+\alpha\beta_2 z\right)+\ln|z|\right] =$$

$$= \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_2 z\right) = +\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

8. $\int\limits_0^{\infty} z\operatorname{erf}(\alpha z)\exp\left[-(\alpha z+\beta)^2\right]dz = -\frac{1}{4\alpha^2}\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]\times$

$$\times\left\{\sqrt{2}\exp\left(-\frac{\beta^2}{2}\right)+\sqrt{\pi}\,\beta\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]\right\}$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta z\right)+\ln|z|\right] = \lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right) = +\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

9. $\int\limits_{-\beta_1/\alpha}^{\infty} z\left[\pm 1-\operatorname{erf}(\alpha z+\beta_1)\right]\exp\left[-(\alpha z+\beta_2)^2\right]dz = \frac{1}{4\alpha^2}\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\pm 1\right]\times$

$$\times\left\{\sqrt{\pi}\beta_2\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\pm 1\right]-\sqrt{2}\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]\right\}-$$

$$-\frac{\sqrt{\pi}\beta_2}{2\alpha^2}[1\pm\operatorname{erf}(\beta_1-\beta_2)]\pm\frac{1}{2\alpha^2}\exp\left[-(\beta_1-\beta_2)^2\right]$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta_1 z+\alpha\beta_2 z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

10. $$\int_0^\infty z\left[\pm1-\operatorname{erf}(\alpha z+\beta)\right]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{2}}{4\alpha^2}\left[\operatorname{erf}\left(\sqrt{2}\beta\right)\mp 1\right]-$$

$$-\frac{\sqrt{\pi}\beta}{4\alpha^2}\left[\operatorname{erf}(\beta)\mp 1\right]^2-\frac{1}{2\alpha^2}\exp\left(-\beta^2\right)\left[\operatorname{erf}(\beta)\mp 1\right]$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

11. $$\int_0^\infty z[\pm1-\operatorname{erf}(\alpha z)]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{1}{4\alpha^2}\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]\left\{\sqrt{2}\exp\left(-\frac{\beta^2}{2}\right)+\right.$$

$$\left.+\sqrt{\pi}\beta\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)\mp 1\right]\right\}-\frac{\sqrt{\pi}\beta}{2\alpha^2}[1\mp\operatorname{erf}(\beta)]\pm\frac{1}{2\alpha^2}\exp\left(-\beta^2\right)$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

12. $$\int_{-\beta_1/\alpha}^{-\beta_2/\alpha} z\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=\frac{\sqrt{2}}{2\alpha^2}\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\times$$

$$\times\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]-\frac{\sqrt{\pi}\beta_2}{2\alpha^2}\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)-\frac{1}{2\alpha^2}\operatorname{erf}(\beta_1-\beta_2).$$

13. $$\int_{-\beta_1/\alpha}^{-(\beta_1+\beta_2)/(2\alpha)} z\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=\frac{\sqrt{2}}{4\alpha^2}\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\times$$

$$\times\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]+\frac{\sqrt{\pi}\beta_2}{4\alpha^2}\left[\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{2}\right)-\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\right]-$$

$$-\frac{1}{2\alpha^2}\operatorname{erf}\left(\frac{\beta_1-\beta_2}{2}\right)\exp\left[-\frac{(\beta_1-\beta_2)^2}{4}\right].$$

2.18.4.

1. $$\int_{-\beta/a}^{+\infty} z\exp(-bz)\operatorname{erf}^2(az+\beta)\,dz = -\int_{-\infty}^{\beta/a} z\exp(bz)\operatorname{erf}^2(az-\beta)\,dz =$$

$$=\frac{2a^2-b^2-2ab\beta}{2a^2b^2}\exp\left(\frac{b^2+4ab\beta}{4a^2}\right)\left[1-\operatorname{erf}\left(\frac{\sqrt{2}b}{4a}\right)\right]^2+.$$

$$+\frac{2}{\sqrt{2\pi}ab}\exp\left(\frac{b^2+8ab\beta}{8a^2}\right)\left[1-\operatorname{erf}\left(\frac{\sqrt{2}b}{4a}\right)\right] \quad \{a>0,\, b>0\}.$$

2. $$\int_{-\beta_1/a}^{+\infty} z\exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz = \frac{a+\beta\beta_1}{a\beta^2}\exp\left(-\frac{\beta\beta_1}{a}\right)+\frac{2}{\sqrt{2\pi}a\beta}\times$$

$$\times\exp\left(\frac{\beta^2-8a\beta\beta_1}{8a^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a}\right)+1\right]+\frac{\beta^2-2a^2-2a\beta\beta_1}{2a^2\beta^2}\exp\left(\frac{\beta^2-4a\beta\beta_1}{4a^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a}\right)+1\right]^2 \qquad \{a>0\}.$$

3. $$\int_0^{+\infty} z\exp(\beta z)\left[\operatorname{erf}^2(a_1z)-\operatorname{erf}^2(a_2z)\right]dz = \frac{2}{\sqrt{2\pi}a_2\beta}\exp\left(\frac{\beta^2}{8a_2^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_2}\right)+1\right]+$$

$$+\frac{\beta^2-2a_2^2}{2a_2^2\beta^2}\exp\left(\frac{\beta^2}{4a_2^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_2}\right)+1\right]^2-\frac{2}{\sqrt{2\pi}a_1\beta}\exp\left(\frac{\beta^2}{8a_1^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_1}\right)+1\right]+\frac{2a_1^2-\beta^2}{2a_1^2\beta^2}\exp\left(\frac{\beta^2}{4a_1^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_1}\right)+1\right]^2 \quad \{a_1>0,\, a_2>0\}.$$

4. $$\int_{-\infty}^{+\infty} z\exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz = \frac{2}{a\beta}\exp\left(\frac{\beta^2-8a\beta\beta_1}{8a^2}\right)\left[\frac{2}{\sqrt{2\pi}}+\right.$$

$$\left.+\frac{\beta^2-2a^2-2a\beta\beta_1}{a\beta}\exp\left(\frac{\beta^2}{8a^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a}\right)\right] \qquad \{a>0\}.$$

5. $$\int_{-\infty}^{+\infty} z\exp(\beta z)\left[\operatorname{erf}^2(a_1z+\beta_1)-\operatorname{erf}^2(a_2z+\beta_2)\right]dz = \frac{2}{a_2\beta}\exp\left(\frac{\beta^2-8a_2\beta\beta_2}{8a_2^2}\right)\times$$

$$\times\left[\frac{2}{\sqrt{2\pi}}+\frac{\beta^2-2a_2^2-2a_2\beta\beta_2}{\alpha_2\beta}\exp\left(\frac{\beta^2}{8a_2^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_2}\right)\right]-\frac{2}{a_1\beta}\exp\left(\frac{\beta^2-8a_1\beta\beta_1}{8a_1^2}\right)\times$$

$$\times\left[\frac{2}{\sqrt{2\pi}}+\frac{\beta^2-2a_1^2-2a_1\beta\beta_1}{\alpha_1\beta}\exp\left(\frac{\beta^2}{8a_1^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4a_1}\right)\right]\quad\{a_1>0,\, a_2>0\}.$$

6. $$\int_{-\beta_1/\alpha}^{\infty} z\exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)\,dz=\frac{2\alpha^2+2\alpha\beta\beta_1-\beta^2}{2\alpha^2\beta^2}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm 1\right]^2-\frac{2}{\sqrt{2\pi}\alpha\beta}\exp\left(\frac{\beta^2-8\alpha\beta\beta_1}{8\alpha^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm 1\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2z^2+4\alpha\beta_1 z-\beta z\right)+2\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}(\beta z)+\ln|z|\right]=-\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

7. $$\int_{-\beta_1/\alpha}^{\infty} z\exp(\beta z)\left[1-\operatorname{erf}^2(\alpha z+\beta_1)\right]dz=\frac{2}{\sqrt{2\pi}\alpha\beta}\exp\left(\frac{\beta^2-8\alpha\beta\beta_1}{8\alpha^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm 1\right]+$$

$$+\frac{\beta^2-2\alpha^2-2\alpha\beta\beta_1}{2\alpha^2\beta^2}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right)\pm 1\right]^2+\frac{\alpha+\beta\beta_1}{\alpha\beta^2}\exp\left(-\frac{\beta\beta_1}{\alpha}\right)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1 z-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2z^2+4\alpha\beta_1 z-\beta z\right)+2\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

8. $$\int_0^{\infty} z\exp(\beta z)\left[\operatorname{erf}^2(\alpha_1 z)-\operatorname{erf}^2(\alpha_2 z)\right]dz=\frac{2}{\sqrt{2\pi}\alpha_2\beta}\exp\left(\frac{\beta^2}{8\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_2}\right)\pm 1\right]+$$

$$+\frac{\beta^2-2\alpha_2^2}{2\alpha_2^2\beta^2}\exp\left(\frac{\beta^2}{4\alpha_2^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_2}\right)\pm 1\right]^2-\frac{2}{\sqrt{2\pi}\alpha_1\beta}\exp\left(\frac{\beta^2}{8\alpha_1^2}\right)\times$$

$$\times\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_1}\right)+A\right]+\frac{2\alpha_1^2-\beta^2}{2\alpha_1^2\beta^2}\exp\left(\frac{\beta^2}{4\alpha_1^2}\right)\left[\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha_1}\right)+A\right]^2$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2-\beta z\right)+\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2z^2-\beta z\right)+\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha_1^2z^2-\beta z\right)+2\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha_2^2z^2-\beta z\right)+2\ln|z|\right]=+\infty;$$

$$A=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=+\infty,\ A=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)=-\infty;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\}.$$

9. $$\int_{-\beta_1/\alpha}^{(\beta-2\alpha\beta_1)/(2\alpha^2)} z\exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)\,dz = \frac{\beta^2-2\alpha^2-2\alpha\beta\beta_1}{2\alpha^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2-2\alpha\beta\beta_1}{2\alpha^2}\right)\operatorname{erf}^2\left(\frac{\beta}{2\alpha}\right)+\frac{2}{\sqrt{\pi}\,\alpha\beta}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\operatorname{erf}\left(\frac{\beta}{2\alpha}\right)+$$

$$+\frac{2\alpha^2+2\alpha\beta\beta_1-\beta^2}{\alpha^2\beta^2}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\operatorname{erf}^2\left(\frac{\sqrt{2}\beta}{4\alpha}\right)-\frac{4}{\sqrt{2\pi}\,\alpha\beta}\times$$

$$\times\exp\left(\frac{\beta^2-8\alpha\beta\beta_1}{8\alpha^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right).$$

10. $$\int_{-\beta_1/\alpha}^{(\beta-4\alpha\beta_1)/(4\alpha^2)} z\exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)\,dz = \frac{3\beta^2-8\alpha^2-8\alpha\beta\beta_1}{4\alpha^2\beta^2}\times$$

$$\times\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\operatorname{erf}^2\left(\frac{\beta}{4\alpha}\right)+\frac{2}{\sqrt{\pi}\,\alpha\beta}\exp\left(\frac{3\beta^2-16\alpha\beta\beta_1}{16\alpha^2}\right)\operatorname{erf}\left(\frac{\beta}{4\alpha}\right)+$$

$$+\frac{2\alpha^2+2\alpha\beta\beta_1-\beta^2}{2\alpha^2\beta^2}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\operatorname{erf}^2\left(\frac{\sqrt{2}\beta}{4\alpha}\right)-\frac{2}{\sqrt{2\pi}\,\alpha\beta}\times$$

$$\times\exp\left(\frac{\beta^2-8\alpha\beta\beta_1}{8\alpha^2}\right)\operatorname{erf}\left(\frac{\sqrt{2}\beta}{4\alpha}\right).$$

2.18.5.

1. $$\int_{-\beta_1/a}^{+\infty} z^n\operatorname{erf}(az+\beta_1)\exp\left[-(az+\beta_2)^2\right]dz = W_1^{(18)}(n,a,\beta_1,\beta_2,1) \qquad \{a>0\}.$$

2. $$\int_0^{+\infty} z^{2m}\operatorname{erf}(az+\beta)\exp\left(-a^2z^2\right)dz = W_1^{(18)}(2m,a,\beta,0,1)-W_2^{(18)}(2m,a,\beta,0)$$

$$\{a>0\}.$$

3. $$\int_0^{+\infty} z^n\operatorname{erf}(az)\exp\left[-(az+\beta)^2\right]dz = W_1^{(18)}(n,a,0,\beta,1) \qquad \{a>0\}.$$

4. $$\int_{-\beta_1/a}^{+\infty} z^n\left[1-\operatorname{erf}(az+\beta_1)\right]\exp\left[-(az+\beta_2)^2\right]dz = (-1)^{n+1}\times$$

$$\times\int_{-\infty}^{\beta_1/a} z^n\left[-1-\operatorname{erf}(az-\beta_1)\right]\exp\left[-(az-\beta_2)^2\right]dz =$$

$$=W_3^{(18)}\left(n,a,\beta_1,\beta_2,1\right)-W_1^{(18)}\left(n,a,\beta_1,\beta_2,1\right)\quad \{a>0\}.$$

5. $$\int_0^{+\infty} z^n\left[1-\operatorname{erf}\left(az+\beta\right)\right]\exp\left[-\left(az+\beta\right)^2\right]dz=\left(-1\right)^{n+1}\times$$

$$\times\int_{-\infty}^{0} z^n\left[-1-\operatorname{erf}\left(az-\beta\right)\right]\exp\left[-\left(az-\beta\right)^2\right]dz=\frac{n!}{\left(-a\right)^{n+1}}W_4^{(18)}\left(n,\beta,1\right)\quad \{a>0\}.$$

6. $$\int_0^{+\infty} z^n\left[1-\operatorname{erf}\left(a\,z\right)\right]\exp\left[-\left(az+\beta\right)^2\right]dz=\left(-1\right)^{n+1}\times$$

$$\times\int_{-\infty}^{0} z^n\left[-1-\operatorname{erf}\left(a\,z\right)\right]\exp\left[-\left(az-\beta\right)^2\right]dz=W_3^{(18)}\left(n,a,0,\beta,1\right)-W_1^{(18)}\left(n,a,0,\beta,1\right)\quad \{a>0\}.$$

7. $$\int_{-\infty}^{+\infty} z^n\operatorname{erf}\left(az+\beta_1\right)\exp\left[-\left(az+\beta_2\right)^2\right]dz=W_5^{(18)}\left(n,a,\beta_1,\beta_2\right)\qquad \{a>0\}.$$

8. $$\int_{-\beta_1/\alpha}^{\infty} z^n\operatorname{erf}\left(\alpha z+\beta_1\right)\exp\left[-\left(\alpha z+\beta_2\right)^2\right]dz=W_1^{(18)}\left(n,\alpha,\beta_1,\beta_2,\pm 1\right)$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta_1 z+\alpha\beta_2 z\right)-\left(n-2\right)\ln\left|z\right|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_2 z\right)-\left(n-1\right)\ln\left|z\right|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty\ \}.$$

9. $$\int_0^{\infty} z^{2m}\operatorname{erf}\left(\alpha z+\beta\right)\exp\left(-\alpha^2z^2\right)dz=W_1^{(18)}\left(2m,\alpha,\beta,0,\pm 1\right)-W_2^{(18)}\left(2m,\alpha,\beta,0\right)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)-\left(m-1\right)\ln\left|z\right|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2\right)-\left(2m-1\right)\ln\left|z\right|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty\ \}.$$

10. $$\int_0^{\infty} z^n\operatorname{erf}\left(\alpha z\right)\exp\left[-\left(\alpha z+\beta\right)^2\right]dz=W_1^{(18)}\left(n,\alpha,0,\beta,\pm 1\right)$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)-\left(n-1\right)\ln\left|z\right|\right]=$$

$$=\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)-\left(n-2\right)\ln\left|z\right|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}\left(\alpha z\right)=\pm\infty\ \}.$$

11. $$\int_{-\beta_1/\alpha}^{\infty} z^n\left[\pm 1-\operatorname{erf}\left(\alpha z+\beta_1\right)\right]\exp\left[-\left(\alpha z+\beta_2\right)^2\right]dz=W_3^{(18)}\left(n,\alpha,\beta_1,\beta_2,\pm 1\right)-$$

$$-W_1^{(18)}(n,\alpha,\beta_1,\beta_2,\pm 1)$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta_1 z+\alpha\beta_2 z\right)-(n-2)\ln|z|\right]=+\infty,\ \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

12. $$\int_0^\infty z^n\left[\pm 1-\operatorname{erf}(\alpha z+\beta)\right]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{n!}{(-\alpha)^{n+1}}W_4^{(18)}(n,\beta,\pm 1)$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta z\right)-(n-2)\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

13. $$\int_0^\infty z^n\left[\pm 1-\operatorname{erf}(\alpha z)\right]\exp\left[-(\alpha z+\beta)^2\right]dz=W_3^{(18)}(n,\alpha,0,\beta,\pm 1)-$$

$$-W_1^{(18)}(n,\alpha,0,\beta,\pm 1)$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2+\alpha\beta z\right)-(n-2)\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

14. $$\int_{-\beta_1/\alpha}^{-\beta_2/\alpha} z^n\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=W_2^{(18)}(n,\alpha,\beta_1,\beta_2).$$

15. $$\int_{-\beta_1/\alpha}^{-(\beta_1+\beta_2)/(2\alpha)} z^n\operatorname{erf}(\alpha z+\beta_1)\exp\left[-(\alpha z+\beta_2)^2\right]dz=W_6^{(18)}(n,\alpha,\beta_1,\beta_2).$$

2.18.6.

1. $$\int_{-\beta/a}^{+\infty} z^n\exp(-bz)\operatorname{erf}^2(az+\beta)dz=(-1)^n\int_{-\infty}^{\beta/a} z^n\exp(bz)\operatorname{erf}^2(az-\beta)dz=$$

$$=\frac{4a}{\sqrt{\pi}}\exp\left(\frac{b^2+4ab\beta}{4a^2}\right)\sum_{m=0}^{n}\frac{n!}{m!b^{n+1-m}}W_1^{(18)}(m,a,\beta,\frac{b}{2a}+\beta,1)$$

$$\{a>0,\,b>0\}.$$

2. $$\int_{-\beta_1/a}^{+\infty} z^n\exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz=\exp\left(-\frac{\beta\beta_1}{a}\right)\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n+1-m}}\times$$

$$\times\left[\left(-\frac{\beta_1}{a}\right)^m-\frac{4a}{\sqrt{\pi}}\exp\left(\frac{\beta^2}{4a^2}\right)W_1^{(18)}(m,a,\beta_1,\ \beta_1-\frac{\beta}{2a},1)\right]\ \{a>0\}.$$

3. $$\int_0^{+\infty} z^n\exp(\beta z)\left[\operatorname{erf}^2(a_1 z)-\operatorname{erf}^2(a_2 z)\right]dz=\frac{4}{\sqrt{\pi}\beta}\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n-m}}\times$$

$$\times\left[a_2\exp\left(\frac{\beta^2}{4a_2^2}\right)W_1^{(18)}\left(m,a_2,0,-\frac{\beta}{2a_2},1\right)-a_1\exp\left(\frac{\beta^2}{4a_1^2}\right)\times\right.$$

$$\times W_1^{(18)}(m, a_1, 0, -\frac{\beta}{2a_1}, 1)\Bigg] \qquad \{a_1 > 0, a_2 > 0\}.$$

4. $$\int_{-\infty}^{+\infty} z^n \exp(\beta z)\left[1-\operatorname{erf}^2(az+\beta_1)\right]dz = \frac{4a}{\sqrt{\pi}\beta}\exp\left(\frac{\beta^2-4a\beta\beta_1}{4a^2}\right)\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n-m}}\times$$

$$\times W_5^{(18)}(m, a, \beta_1, \beta_1 - \frac{\beta}{2a}) \qquad \{a > 0\}.$$

5. $$\int_{-\infty}^{+\infty} z^n \exp(\beta z)\left[\operatorname{erf}^2(a_1 z+\beta_1)-\operatorname{erf}^2(a_2 z+\beta_2)\right]dz = \frac{4}{\sqrt{\pi}\beta}\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n-m}}\times$$

$$\times\Bigg[a_2\exp\left(\frac{\beta^2-4a_2\beta\beta_2}{4a_2^2}\right)W_5^{(18)}(m, a_2, \beta_2, \beta_2-\frac{\beta}{2a_2}) - a_1\times$$

$$\times\exp\left(\frac{\beta^2-4a_1\beta\beta_1}{4a_1^2}\right)W_5^{(18)}(m, a_1, \beta_1, \beta_1-\frac{\beta}{2a_1})\Bigg] \quad \{a_1 > 0, a_2 > 0\}.$$

6. $$\int_{-\beta_1/\alpha}^{\infty} z^n \exp(\beta z)\operatorname{erf}^2(\alpha z+\beta_1)dz = \frac{4\alpha}{\sqrt{\pi}}\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n+1-m}}\times$$

$$\times W_1^{(18)}(m, \alpha, \beta_1, \beta_1-\frac{\beta}{2\alpha}, \pm 1)\ \{\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2+4\alpha\beta_1 z-\beta z\right)-(n-3)\ln|z|\right]=+\infty,$$

$$\lim_{z\to\infty}\left[\operatorname{Re}(\beta z)+n\ln|z|\right]=-\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

7. $$\int_{-\beta_1/\alpha}^{\infty} z^n \exp(\beta z)\left[1-\operatorname{erf}^2(\alpha z+\beta_1)\right]dz = \exp\left(-\frac{\beta\beta_1}{\alpha}\right)\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n+1-m}}\left[\left(-\frac{\beta_1}{\alpha}\right)^m-\right.$$

$$\left.-\frac{4\alpha}{\sqrt{\pi}}\exp\left(\frac{\beta^2}{4\alpha^2}\right)W_1^{(18)}(m, \alpha, \beta_1, \beta_1-\frac{\beta}{2\alpha}, \pm 1)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2+2\alpha\beta_1 z-\beta z\right)-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(2\alpha^2 z^2+4\alpha\beta_1 z-\beta z\right)-(n-3)\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\ \}.$$

8. $$\int_0^{\infty} z^n \exp(\beta z)\left[\operatorname{erf}^2(\alpha_1 z)-\operatorname{erf}^2(\alpha_2 z)\right]dz = \frac{4}{\sqrt{\pi}}\sum_{m=0}^{n}\frac{n!}{m!(-\beta)^{n+1-m}}\times$$

$$\times\Bigg[\alpha_1\exp\left(\frac{\beta^2}{4\alpha_1^2}\right)W_1^{(18)}(m, \alpha_1, 0, -\frac{\beta}{2\alpha_1}, \pm 1)-\alpha_2\exp\left(\frac{\beta^2}{4\alpha_2^2}\right)\times$$

$$\times W_1^{(18)}(m, \alpha_2, 0, -\frac{\beta}{2\alpha_2}, A) \Big]$$

$$\{ \lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha_1^2 z^2-\beta z\right)-\left(n-2\right)\ln|z|\right]=\lim_{z\to\infty}\left[\mathrm{Re}\left(\alpha_2^2 z^2-\beta z\right)-\left(n-2\right)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\mathrm{Re}\left(2\alpha_1^2 z^2-\beta z\right)-\left(n-3\right)\ln|z|\right]=\lim_{z\to\infty}\left[\mathrm{Re}\left(2\alpha_2^2 z^2-\beta z\right)-\left(n-3\right)\ln|z|\right]=+\infty\,;$$

$$A=1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left(\alpha_2 z\right)=+\infty\,;\; A=-1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\left(\alpha_2 z\right)=-\infty\,;$$

$$\lim_{z\to\infty}\mathrm{Re}\left(\alpha_1 z\right)=\pm\infty\,\}.$$

9. $$\int_{-\beta_1/\alpha}^{(\beta-2\alpha\beta_1)/\left(2\alpha^2\right)} z^n \exp(\beta z)\mathrm{erf}^2(\alpha z+\beta_1)dz=\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\sum_{m=0}^{n}\frac{n!}{m!\left(-\beta\right)^{n+1-m}}\left[\frac{4\alpha}{\sqrt{\pi}}W_2^{(18)}(m,\alpha,\beta_1,\beta_1-\frac{\beta}{2\alpha})-\left(\frac{\beta-2\alpha\beta_1}{2\alpha^2}\right)^m\times\right.$$

$$\left.\times\exp\left(\frac{\beta^2}{4\alpha^2}\right)\mathrm{erf}^2\left(\frac{\beta}{2\alpha}\right)\right].$$

10. $$\int_{-\beta_1/\alpha}^{(\beta-4\alpha\beta_1)/\left(4\alpha^2\right)} z^n \exp(\beta z)\mathrm{erf}^2(\alpha z+\beta_1)dz=\exp\left(\frac{\beta^2-4\alpha\beta\beta_1}{4\alpha^2}\right)\times$$

$$\times\sum_{m=0}^{n}\frac{n!}{m!\left(-\beta\right)^{n+1-m}}\left[\frac{4\alpha}{\sqrt{\pi}}W_6^{(18)}(m,\alpha,\beta_1,\beta_1-\frac{\beta}{2\alpha})-\left(\frac{\beta-4\alpha\beta_1}{4\alpha^2}\right)^m\mathrm{erf}^2\left(\frac{\beta}{4\alpha}\right)\right].$$

Introduced notations:

1) $$W_1^{(18)}(n,\alpha,\beta_1,\beta_2,A)=\frac{\sqrt{\pi}\,n!}{4\alpha^{n+1}}\left[\mathrm{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)+A\right]^2\sum_{k=0}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{4^k\,k!(n-2k)!}+$$

$$+2\frac{n!}{\left(-\alpha\right)^{n+1}}\left[\sum_{k=1}^{n-E(n/2)}\frac{k!\beta_2^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{1}{l!}\,W_7^{(18)}\left(2l,\beta_1,\beta_2,A\right)-\right.$$

$$\left.-\sum_{k=1}^{E(n/2)}\frac{\beta_2^{n-2k}}{k!(n-2k)!}\sum_{l=0}^{k-1}\frac{l!}{4^{k-l}\left(2l+1\right)!}W_7^{(18)}\left(2l+1,\beta_1,\beta_2,A\right)\right],$$

$$W_1^{(18)}(n,\alpha,\beta,\beta,A)=\frac{\sqrt{\pi}n!}{4\alpha^{n+1}}\sum_{k=0}^{E(n/2)}\frac{(-\beta)^{n-2k}}{4^k\,k!(n-2k)!}+\frac{n!}{(-\alpha)^{n+1}}\times$$

$$\times\left[\frac{A}{\sqrt{2}}\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{(2l)!}{8^l\,(l!)^2}-\right.$$

$$\left.-\frac{1}{\sqrt{\pi}}\sum_{k=1}^{E(n/2)}\frac{\beta^{n-2k}}{k!(n-2k)!}\sum_{l=0}^{k-1}\frac{(l!)^2}{2^{2k+1-l}\,(2l+1)!}\right],$$

$$W_1^{(18)}(2n_1,\alpha,\beta,0,A)=\frac{(2n_1)!\sqrt{\pi}}{4^{n_1+1}n_1!\alpha^{2n_1+1}}\left[\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)+A\right]^2+$$

$$+2\frac{(2n_1)!}{n_1!\alpha^{2n_1+1}}\sum_{k=0}^{n_1-1}\frac{k!}{4^{n_1-k}\,(2k+1)!}W_7^{(18)}(2k+1,\beta,0,A),$$

$$W_1^{(18)}(2n_1+1,\alpha,\beta,0,A)=\frac{n_1!}{\alpha^{2n_1+2}}\sum_{k=0}^{n_1}\frac{1}{k!}W_7^{(18)}(2k,\beta,0,A);$$

2) $$W_2^{(18)}(n,\alpha,\beta_1,\beta_2)=\frac{n!}{(-\alpha)^{n+1}}\operatorname{erf}(\beta_2-\beta_1)\sum_{k=1}^{n-E(n/2)}\frac{k!\beta_2^{n+1-2k}}{(2k)!(n+1-2k)!}+$$

$$+\frac{n!\sqrt{\pi}}{2\alpha^{n+1}}\operatorname{erf}^2\left(\frac{\beta_2-\beta_1}{\sqrt{2}}\right)\sum_{k=0}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{4^k\,k!(n-2k)!}+2\frac{n!}{(-\alpha)^{n+1}}\times$$

$$\times\left[\sum_{k=1}^{E(n/2)}\frac{\beta_2^{n-2k}}{k!(n-2k)!}\sum_{l=0}^{k-1}\frac{l!}{4^{k-l}\,(2l+1)!}\times\right.$$

$$\left.\times W_8^{(18)}(2l+1,\beta_1,\beta_2)-\sum_{k=1}^{n-E(n/2)}\frac{k!\beta_2^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{1}{l!}W_8^{(18)}(2l,\beta_1,\beta_2)\right],$$

$$W_2^{(18)}(2n_1,\alpha,\beta,0)=\frac{(2n_1)!\sqrt{\pi}}{n_1!(2\alpha)^{2n_1+1}}\operatorname{erf}^2\left(\frac{\beta}{\sqrt{2}}\right)-2\frac{(2n_1)!}{n_1!\alpha^{2n_1+1}}\times$$

$$\times\sum_{k=0}^{n_1-1}\frac{k!}{4^{n_1-k}\,(2k+1)!}W_8^{(18)}(2k+1,\beta,0),$$

$$W_2^{(18)}(2n_1+1,\alpha,\beta,0)=-\frac{n_1!}{\alpha^{2n_1+2}}\left[\frac{\operatorname{erf}(\beta)}{2}-\sum_{k=0}^{n_1}\frac{1}{k!}W_8^{(18)}(2k,\beta,0)\right];$$

3) $$W_3^{(18)}(n,\alpha,\beta_1,\beta_2,A)=\frac{\sqrt{\pi}n!}{2\alpha^{n+1}}\left[1+A\operatorname{erf}(\beta_1-\beta_2)\right]\sum_{k=0}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{4^k\,k!(n-2k)!}+$$

$$+A\frac{n!}{(-\alpha)^{n+1}}\exp\left[-(\beta_1-\beta_2)^2\right]\left[\sum_{k=1}^{E(n/2)}\frac{\beta_2^{n-2k}}{k!(n-2k)!}\sum_{l=0}^{k-1}\frac{l!(\beta_1-\beta_2)^{2l+1}}{4^{k-l}(2l+1)!}+\right.$$

$$\left.+\sum_{k=1}^{n-E(n/2)}\frac{k!\beta_2^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{(\beta_1-\beta_2)^{2l}}{l!}\right],$$

$$W_3^{(18)}(n,\alpha,\beta,\beta,A)=\frac{\sqrt{\pi}n!}{2\alpha^{n+1}}\sum_{k=0}^{E(n/2)}\frac{(-\beta)^{n-2k}}{4^k\,k!(n-2k)!}+A\frac{n!}{(-\alpha)^{n+1}}\times$$

$$\times\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!},$$

$$W_3^{(18)}(2n_1,\alpha,\beta,0,A)=\frac{\sqrt{\pi}(2n_1)!}{n_1!(2\alpha)^{2n_1+1}}\left[1+A\,\mathrm{erf}(\beta)\right]-A\frac{(2n_1)!}{n_1!\alpha^{2n_1+1}}\times$$

$$\times\exp\left(-\beta^2\right)\sum_{k=0}^{n_1-1}\frac{k!\beta^{2k+1}}{4^{n_1-k}(2k+1)!},$$

$$W_3^{(18)}(2n_1+1,\alpha,\beta,0,A)=A\frac{n_1!\exp\left(-\beta^2\right)}{2\alpha^{2n_1+2}}\sum_{k=0}^{n_1}\frac{\beta^{2k}}{k!};$$

4) $$W_4^{(18)}(n,\beta,A)=\exp\left(-\beta^2\right)\left[\mathrm{erf}(\beta)-A\right]\left[\sum_{k=1}^{E(n/2)}\frac{1}{k!(n-2k)!}\sum_{l=0}^{k-1}\frac{l!\,\beta^{n+1-2k+2l}}{4^{k-l}(2l+1)!}-\right.$$

$$\left.-\sum_{k=1}^{n-E(n/2)}\frac{k!}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{\beta^{n+1-2k+2l}}{l!}\right]-\sqrt{\pi}\left[\mathrm{erf}(\beta)-A\right]^2\times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{\beta^{n-2k}}{4^{k+1}k!(n-2k)!}+\frac{\exp\left(-2\beta^2\right)}{2\sqrt{\pi}}\left[\sum_{k=1}^{E(n/2)}\frac{\beta^{n-2k}}{4^k\,k!(n-2k)!}\sum_{l=0}^{k-1}\frac{(l!)^2}{(2l+1)!}\times\right.$$

$$\left.\times\sum_{r=0}^{l}\frac{2^{l+r}\beta^{2r}}{r!}-\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=1}^{k-1}\frac{(2l)!}{(l!)^2}\sum_{r=0}^{l-1}\frac{r!\beta^{2r+1}}{2^{3l-2-3r}(2r+1)!}\right]+$$

$$+\frac{\mathrm{erf}\left(\sqrt{2}\beta\right)-A}{\sqrt{2}}\sum_{k=1}^{n-E(n/2)}\frac{k!\beta^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{(2l)!}{8^l\,(l!)^2},$$

$$W_4^{(18)}(2n_1,0,A)=-\frac{\sqrt{\pi}}{4^{n_1+1}n_1!}+\frac{1}{2^{2n_1+1}n_1!\sqrt{\pi}}\sum_{k=0}^{n_1-1}\frac{2^k\,(k!)^2}{(2k+1)!},$$

$$W_4^{(18)}\left(2n_1+1,0,A\right)=-A\frac{n_1!}{2\sqrt{2}\left(2n_1+1\right)!}\sum_{k=0}^{n_1}\frac{\left(2k\right)!}{8^k\left(k!\right)^2};$$

$$5)\ W_5^{(18)}(n,\alpha,\beta_1,\beta_2)=\frac{n!\sqrt{\pi}}{\alpha^{n+1}}\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)\sum_{k=0}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{4^k\,k!(n-2k)!}+\frac{n!\sqrt{2}}{\alpha^{n+1}}\times$$

$$\times\exp\left[-\frac{\left(\beta_1-\beta_2\right)^2}{2}\right]\left[\sum_{k=1}^{n-E(n/2)}\frac{k!\left(-\beta_2\right)^{n+1-2k}}{(2k)!\left(n+1-2k\right)!}\sum_{l=0}^{k-1}\frac{(2l!)}{4^l\,l!}\sum_{r=0}^{l}\frac{\left(\beta_1-\beta_2\right)^{2l-2r}}{2^r\,r!\left(2l-2r\right)!}-\right.$$

$$\left.-\sum_{k=1}^{E(n/2)}\frac{\left(-\beta_2\right)^{n-2k}}{2^{2k+1}k!\left(n-2k\right)!}\sum_{l=0}^{k-1}l!\ \sum_{r=0}^{l}\frac{\left(\beta_1-\beta_2\right)^{2l+1-2r}}{2^r\,r!\left(2l+1-2r\right)!}\right],$$

$$W_5^{(18)}\left(2n_1,\alpha,\beta,0\right)=\frac{\left(2n_1\right)!\sqrt{\pi}}{4^{n_1}\ n_1!\alpha^{2n_1+1}}\operatorname{erf}\left(\frac{\beta}{\sqrt{2}}\right)-\frac{\left(2n_1\right)!\sqrt{2}}{2^{2n_1+1}\,n_1!\,\alpha^{2n_1+1}}\times$$
$$\times\exp\left(-\frac{\beta^2}{2}\right)\sum_{k=0}^{n_1-1}k!\ \sum_{l=0}^{k}\frac{\beta^{2k+1-2l}}{2^l\ l!(2k+1-2l)!},$$

$$W_5^{(18)}\left(2n_1+1,\alpha,\beta,0\right)=\frac{n_1!}{\sqrt{2}\alpha^{2n_1+2}}\exp\left(-\frac{\beta^2}{2}\right)\sum_{k=0}^{n_1}\frac{(2k)!}{4^k\,k!}\sum_{l=0}^{k}\frac{\beta^{2k-2l}}{2^l\,l!\left(2k-2l\right)!};$$

$$6)\ W_6^{(18)}\left(n,\alpha,\beta_1,\beta_2\right)=\frac{n!\sqrt{\pi}}{4\alpha^{n+1}}\left[\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)-\operatorname{erf}^2\left(\frac{\beta_1-\beta_2}{2}\right)\right]\times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{4^k\,k!(n-2k)!}+\frac{n!}{\alpha^{n+1}}\exp\left[-\frac{(\beta_1-\beta_2)^2}{4}\right]\operatorname{erf}\left(\frac{\beta_1-\beta_2}{2}\right)\times$$

$$\times\left[\sum_{k=1}^{E(n/2)}\frac{(-\beta_2)^{n-2k}}{2^{2k+1}\,k!(n-2k)!}\sum_{l=0}^{k-1}\frac{l!(\beta_1-\beta_2)^{2l+1}}{(2l+1)!}-\right.$$

$$\left.-\sum_{k=1}^{n-E(n/2)}\frac{k!\left(-\beta_2\right)^{n+1-2k}}{\left(2k\right)!\left(n+1-2k\right)!}\sum_{l=0}^{k-1}\frac{\left(\beta_1-\beta_2\right)^{2l}}{4^l\,l!}\right]-2\frac{n!}{\alpha^{n+1}}\left[\sum_{k=1}^{E(n/2)}\frac{\left(-\beta_2\right)^{n-2k}}{k!\left(n-2k\right)!}\times\right.$$

$$\times\sum_{l=0}^{k-1}\frac{l!}{4^{k-l}\left(2l+1\right)!}W_9^{(18)}\left(2l+1,\beta_1,\beta_2\right)+$$

$$\left.+\sum_{k=1}^{n-E(n/2)}\frac{k!\left(-\beta_2\right)^{n+1-2k}}{\left(2k\right)!\left(n+1-2k\right)!}\sum_{l=0}^{k-1}\frac{1}{l!}W_9^{(18)}\left(2l,\beta_1,\beta_2\right)\right],$$

$$W_6^{(18)}(2n_1,\alpha,\beta,0)=\frac{(2n_1)!\sqrt{\pi}}{4^{n_1+1}n_1!\alpha^{2n_1+1}}\left[\operatorname{erf}^2\left(\frac{\beta}{\sqrt{2}}\right)-\operatorname{erf}^2\left(\frac{\beta}{2}\right)\right]+$$

$$+\frac{(2n_1)!}{n_1!(2\alpha)^{2n_1+1}}\exp\left(-\frac{\beta^2}{4}\right)\operatorname{erf}\left(\frac{\beta}{2}\right)\times$$

$$\times\sum_{k=0}^{n_1-1}\frac{k!\,\beta^{2k+1}}{(2k+1)!}-2\frac{(2n_1)!}{n_1!\,\alpha^{2n_1+1}}\sum_{k=0}^{n_1-1}\frac{k!}{4^{n_1-k}(2k+1)!}W_9^{(18)}(2k+1,\beta,0),$$

$$W_6^{(18)}(2n_1+1,\alpha,\beta,0)=-\frac{n_1!}{2\alpha^{2n_1+2}}\exp\left(-\frac{\beta^2}{4}\right)\operatorname{erf}\left(\frac{\beta}{2}\right)\sum_{k=0}^{n_1}\frac{\beta^{2k}}{4^k k!}-\frac{n_1!}{\alpha^{2n_1+2}}\times$$

$$\times\sum_{k=0}^{n_1}\frac{W_9^{(18)}(2k,\beta,0)}{k!};$$

7) $$W_7^{(18)}(m_1,\beta_1,\beta_2,A)=\frac{m_1!}{2^{m_1+1}\sqrt{2}}\exp\left[-\frac{(\beta_1-\beta_2)^2}{2}\right]\left[\operatorname{erf}\left(\frac{\beta_1-\beta_2}{\sqrt{2}}\right)+A\right]\times$$

$$\times\sum_{r=0}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{2^r r!(m_1-2r)!}+\frac{m_1!}{2^{m_1+1}\sqrt{\pi}}\exp\left[-(\beta_1-\beta_2)^2\right]\times$$

$$\times\left[\sum_{r=1}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{r!(m_1-2r)!}\sum_{q=0}^{r-1}\frac{q!(\beta_2-\beta_1)^{2q+1}}{2^{r-q}(2q+1)!}+\right.$$

$$\left.+\sum_{r=1}^{m_1-E(m_1/2)}\frac{r!(\beta_2-\beta_1)^{m_1+1-2r}}{(2r)!(m_1+1-2r)!}\sum_{q=0}^{r-1}\frac{2^{r-q}(\beta_2-\beta_1)^{2q}}{q!}\right];$$

8) $$W_8^{(18)}(m_1,\beta_1,\beta_2)=\frac{m_1!}{2^{m_1}\sqrt{2}}\exp\left[-\frac{(\beta_2-\beta_1)^2}{2}\right]\operatorname{erf}\left(\frac{\beta_2-\beta_1}{\sqrt{2}}\right)\times$$

$$\times\sum_{r=0}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{2^r r!(m_1-2r)!}-$$

$$-\frac{m_1!}{2^{m_1}\sqrt{\pi}}\exp\left[-(\beta_2-\beta_1)^2\right]\sum_{r=1}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{r!(m_1-2r)!}\sum_{q=0}^{r-1}\frac{q!(\beta_2-\beta_1)^{2q+1}}{2^{r-q}(2q+1)!};$$

9) $$W_9^{(18)}(m_1,\beta_1,\beta_2)=\frac{m_1!}{2^{m_1+1}\sqrt{2}}\exp\left[-\frac{(\beta_2-\beta_1)^2}{2}\right]\operatorname{erf}\left(\frac{\beta_2-\beta_1}{\sqrt{2}}\right)\times$$

$$\times\sum_{r=0}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{2^r\,r!(m_1-2r)!}+\frac{m_1!}{2^{m_1+1}\sqrt{\pi}}\exp\left[-\frac{(\beta_2-\beta_1)^2}{2}\right]\times$$

$$\times\sum_{r=1}^{m_1-E(m_1/2)}\frac{2^r\,r!(\beta_2-\beta_1)^{m_1+1-2r}}{(2r)!(m_1+1-2r)!}-\frac{m_1!}{2^{m_1+1}\sqrt{\pi}}\exp\left[-(\beta_2-\beta_1)^2\right]\times$$

$$\times\left[\sum_{r=1}^{E(m_1/2)}\frac{(\beta_2-\beta_1)^{m_1-2r}}{r!(m_1-2r)!}\sum_{q=0}^{r-1}\frac{q!(\beta_2-\beta_1)^{2q+1}}{2^{r-q}(2q+1)!}+\right.$$

$$\left.+\sum_{r=1}^{m_1-E(m_1/2)}\frac{r!(\beta_2-\beta_1)^{m_1+1-2r}}{(2r)!(m_1+1-2r)!}\sum_{q=0}^{r-1}\frac{2^{r-q}(\beta_2-\beta_1)^{2q}}{q!}\right].$$

2.19. Integrals of $\left[(\pm 1)^n-\operatorname{erf}^n(\alpha z+\beta)\right]\exp\left[-(\alpha z+\beta)^2\right]$, $z^{2m+1}[\pm 1-\operatorname{erf}(\alpha z)]^3$, $z^{2m+1}\left[\pm 1-\operatorname{erf}^3(\alpha z)\right]$, $z^{2m+1}\left[\operatorname{erf}^3(\alpha_1 z)\mp\operatorname{erf}^3(\alpha_2 z)\right]$, $z^{2m}\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2 z^2\right)$, $z^{2m}\left[1-\operatorname{erf}^2(\alpha z)\right]\exp\left(-\alpha^2 z^2\right)$

2.19.1.

1. $$\int_0^{+\infty}\left[1-\operatorname{erf}^2(az+\beta)\right]\exp\left[-(az+\beta)^2\right]dz=\frac{\sqrt{\pi}}{2a}[1-\operatorname{erf}(\beta)]-\frac{\sqrt{\pi}}{6a}\left[1-\operatorname{erf}^3(\beta)\right]$$

$\{a>0\}$.

2. $$\int_{-\infty}^{+\infty}\left[1-\operatorname{erf}^2(az+\beta)\right]\exp\left[-(az+\beta)^2\right]dz=\frac{2\sqrt{\pi}}{3a}\qquad\{a>0\}.$$

3. $$\int_0^{\infty}\left[1-\operatorname{erf}^2(\alpha z+\beta)\right]\exp\left[-(\alpha z+\beta)^2\right]dz=\frac{\sqrt{\pi}}{6\alpha}[\operatorname{erf}^3(\beta)\mp 1]-\frac{\sqrt{\pi}}{2\alpha}[\operatorname{erf}(\beta)\mp 1]$$

$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}$.

2.19.2.

1. $$\int_0^{+\infty}z[1-\operatorname{erf}(az)]^3\,dz=\int_{-\infty}^{0}z[-1-\operatorname{erf}(az)]^3\,dz=\frac{1}{2a^2}\left(\frac{1}{2}-\frac{3-\sqrt{3}}{\pi}\right)\qquad\{a>0\}.$$

2. $\int_0^{+\infty} z\left[1-\operatorname{erf}^3(az)\right]dz = \int_{-\infty}^{0} z\left[-1-\operatorname{erf}^3(az)\right]dz = \frac{1}{2a^2}\left(\frac{1}{2}+\frac{\sqrt{3}}{\pi}\right)$ $\{a>0\}$.

3. $\int_0^{+\infty} z\left[\operatorname{erf}^3(a_1 z)-\operatorname{erf}^3(a_2 z)\right]dz = \frac{1}{2}\int_{-\infty}^{+\infty} z\left[\operatorname{erf}^3(a_1 z)-\operatorname{erf}^3(a_2 z)\right]dz =$

$$=\left(\frac{1}{4}+\frac{\sqrt{3}}{2\pi}\right)\left(\frac{1}{a_2^2}-\frac{1}{a_1^2}\right) \qquad \{a_1>0,\, a_2>0\}.$$

4. $\int_0^{\infty} z\left[\pm 1-\operatorname{erf}(\alpha z)\right]^3 dz = \pm\frac{1}{2\alpha^2}\left(\frac{1}{2}-\frac{3-\sqrt{3}}{\pi}\right)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right)+\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

5. $\int_0^{\infty} z\left[\pm 1-\operatorname{erf}^3(\alpha z)\right]dz = \pm\frac{1}{2\alpha^2}\left(\frac{1}{2}+\frac{\sqrt{3}}{\pi}\right)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right)+\ln|z|\right]=+\infty, \quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

6. $\int_0^{\infty} z\left[\operatorname{erf}^3(\alpha_1 z)\mp\operatorname{erf}^3(\alpha_2 z)\right]dz = -\left(\frac{1}{4}+\frac{\sqrt{3}}{2\pi}\right)\left(\frac{A_1}{\alpha_1^2}\mp\frac{A_2}{\alpha_2^2}\right)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2\right)+\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2\right)+\ln|z|\right]=+\infty;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=+\infty, \quad A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=-\infty;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\}.$$

2.19.3.

1. $\int_0^{+\infty} z^2\operatorname{erf}^2(az)\exp\left(-a^2z^2\right)dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^2\operatorname{erf}^2(az)\exp\left(-a^2z^2\right)dz = \frac{1}{2a^3}\left(\frac{\sqrt{\pi}}{6}+\frac{1}{\sqrt{3\pi}}\right)$

$$\{a>0\}.$$

2. $\int_0^{+\infty} z^2\left[1-\operatorname{erf}^2(az)\right]\exp\left(-a^2z^2\right)dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^2\left[1-\operatorname{erf}^2(az)\right]\exp\left(-a^2z^2\right)dz =$

$$=\frac{1}{2a^3}\left(\frac{\sqrt{\pi}}{3}-\frac{1}{\sqrt{3\pi}}\right) \quad \{a>0\}.$$

3. $\int_0^{\infty} z^2\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2z^2\right)dz = \pm\frac{1}{2\alpha^3}\left(\frac{\sqrt{\pi}}{6}+\frac{1}{\sqrt{3\pi}}\right)$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right)-\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

4. $$\int_0^{\infty} z^2\left[1-\operatorname{erf}^2(\alpha z)\right]\exp\left(-\alpha^2 z^2\right)dz=\pm\frac{1}{2\alpha^3}\left(\frac{\sqrt{\pi}}{3}-\frac{1}{\sqrt{3\pi}}\right)$$

$$\{\lim_{z\to\infty}\operatorname{Re}\left(\alpha^2 z^2\right)=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.19.4.

1. $$\int_0^{+\infty} z^{2m}\operatorname{erf}^2(az)\exp\left(-a^2z^2\right)dz=\frac{1}{2}\int_{-\infty}^{+\infty} z^{2m}\operatorname{erf}^2(az)\exp\left(-a^2z^2\right)dz=$$

$$=\frac{(2m)!}{m!}\left[\frac{\sqrt{\pi}}{3(2a)^{2m+1}}+\frac{1}{\sqrt{3\pi}\;a^{2m+1}}W_1^{(19)}(m)\right]\quad\{a>0\}.$$

2. $$\int_0^{+\infty} z^{2m}\left[1-\operatorname{erf}^2(az)\right]\exp\left(-a^2z^2\right)dz=\frac{1}{2}\int_{-\infty}^{+\infty} z^{2m}\left[1-\operatorname{erf}^2(az)\right]\exp\left(-a^2z^2\right)dz=$$

$$=\frac{(2m)!}{m!}\left[\frac{2\sqrt{\pi}}{3(2a)^{2m+1}}-\frac{1}{\sqrt{3\pi}\;a^{2m+1}}W_1^{(19)}(m)\right]\quad\{a>0\}.$$

3. $$\int_{-\infty}^{+\infty}\left[1-\operatorname{erf}^{2m}(az+\beta)\right]\exp\left[-(az+\beta)^2\right]dz=\frac{2m}{2m+1}\frac{\sqrt{\pi}}{a}\qquad\{a>0\}.$$

4. $$\int_0^{\infty} z^{2m}\operatorname{erf}^2(\alpha z)\exp\left(-\alpha^2z^2\right)dz=\pm\frac{(2m)!}{m!}\left[\frac{\sqrt{\pi}}{3(2\alpha)^{2m+1}}+\frac{1}{\sqrt{3\pi}\alpha^{2m+1}}W_1^{(19)}(m)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right)-(2m-1)\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

5. $$\int_0^{\infty} z^{2m}\left[1-\operatorname{erf}^2(\alpha z)\right]\exp\left(-\alpha^2z^2\right)dz=\pm\frac{(2m)!}{m!}\left[\frac{2\sqrt{\pi}}{3(2\alpha)^{2m+1}}-\frac{1}{\sqrt{3\pi}\alpha^{2m+1}}W_1^{(19)}(m)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right)-(m-1)\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

2.19.5.

1. $$\int_0^{+\infty} z^{2m+1}[1-\operatorname{erf}(az)]^3\,dz=\int_{-\infty}^{0} z^{2m+1}[-1-\operatorname{erf}(az)]^3\,dz=W_2^{(19)}(a)\qquad\{a>0\}.$$

2. $$\int_0^{+\infty} z^{2m+1}\left[1-\operatorname{erf}^3(az)\right]dz = \int_{-\infty}^{0} z^{2m+1}\left[-1-\operatorname{erf}^3(az)\right]dz =$$

$$= \frac{(2m+1)!}{(m+1)!}\left[\frac{1}{(2a)^{2m+2}} + \frac{2\sqrt{3}}{\pi a^{2m+2}} W_1^{(19)}(m+1)\right] \qquad \{a>0\}.$$

3. $$\int_0^{+\infty} z^{2m+1}\left[\operatorname{erf}^3(a_1 z) - \operatorname{erf}^3(a_2 z)\right]dz = \frac{1}{2}\int_{-\infty}^{+\infty} z^{2m+1}\left[\operatorname{erf}^3(a_1 z) - \operatorname{erf}^3(a_2 z)\right]dz =$$

$$= \frac{(2m+1)!}{(m+1)!}\left[\frac{1}{(2a_2)^{2m+2}} - \frac{1}{(2a_1)^{2m+2}} + \frac{2\sqrt{3}}{\pi}\left(\frac{1}{a_2^{2m+2}} - \frac{1}{a_1^{2m+2}}\right) W_1^{(19)}(m+1)\right]$$

$$\{a_1>0,\, a_2>0\}.$$

4. $$\int_0^{+\infty}\left[1-\operatorname{erf}^n(az+\beta)\right]\exp\left[-(az+\beta)^2\right]dz = \frac{\sqrt{\pi}}{2a}\left[1-\operatorname{erf}(\beta)\right] -$$

$$-\frac{\sqrt{\pi}}{(2n+2)a}\left[1-\operatorname{erf}^{n+1}(\beta)\right] \qquad \{a>0\}.$$

5. $$\int_0^{\infty} z^{2m+1}\left[\pm 1-\operatorname{erf}(\alpha z)\right]^3 dz = \pm W_2^{(19)}(\alpha)$$

$$\{\lim_{z\to\infty}\left[3\operatorname{Re}\left(\alpha^2 z^2\right) - (2m-3)\ln|z|\right] = +\infty;\ \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

6. $$\int_0^{\infty} z^{2m+1}\left[\pm 1-\operatorname{erf}^3(\alpha z)\right]dz = \pm\frac{(2m+1)!}{(m+1)!}\left[\frac{1}{(2\alpha)^{2m+2}} + \frac{2\sqrt{3}}{\pi\alpha^{2m+2}} W_1^{(19)}(m+1)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2 z^2\right) - (2m-1)\ln|z|\right] = +\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z) = \pm\infty\}.$$

7. $$\int_0^{\infty} z^{2m+1}\left[\operatorname{erf}^3(\alpha_1 z) \mp \operatorname{erf}^3(\alpha_2 z)\right]dz = \pm\frac{(2m+1)!}{(m+1)!}\left[\frac{A_2}{(2\alpha_2)^{2m+2}} \mp \frac{A_1}{(2\alpha_1)^{2m+2}} +\right.$$

$$\left. + \frac{2\sqrt{3}}{\pi}\left(\frac{A_2}{\alpha_2^{2m+2}} \mp \frac{A_1}{\alpha_1^{2m+2}}\right) W_1^{(19)}(m+1)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2\right) - (2m-1)\ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2\right) - (2m-1)\ln|z|\right] = +\infty;$$

$$A_r = 1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_r z) = +\infty,\ A_r = -1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_r z) = -\infty;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z) = \pm\infty\}.$$

8. $\int_0^{\infty}\left[(\pm 1)^n-\operatorname{erf}^n(\alpha z+\beta)\right]\exp\left[-(\alpha z+\beta)^2\right]dz=(\pm 1)^n\frac{\sqrt{\pi}}{2\alpha}\left[\pm 1-\operatorname{erf}(\beta)\right]-$

$$-\frac{\sqrt{\pi}}{(2n+2)\alpha}\left[(\pm 1)^{n+1}-\operatorname{erf}^{n+1}(\beta)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta z\right)+\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

Introduced notations:

1) $W_1^{(19)}(m)=\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!}\sum_{l=0}^{k}\frac{(2l)!}{2^{2m-k+l}3^l(l!)^2}$;

2) $W_2^{(19)}(\alpha)=\frac{(2m+1)!}{(m+1)!\alpha^{2m+2}}\left\{\frac{1}{4^{m+1}}-\frac{\sqrt{3}}{2\pi}\sum_{k=0}^{m}\frac{(k!)^2}{(2k+1)!2^{2m-k}}\left[\sqrt{3}-\sum_{l=0}^{k}\frac{(2l)!}{6^l(l!)^2}\right]\right\}$.

2.20. Integrals of $z^n\sin(\beta z+\gamma)[\pm 1-\operatorname{erf}(\alpha z+\beta_1)]$, $z^n\sin(\beta z+\gamma)\left[\operatorname{erf}(\alpha_1z+\beta_1)\mp\operatorname{erf}(\alpha_2z+\beta_2)\right]$, $z^n\sin\left(\alpha z^2+\beta z+\gamma\right)[\pm 1-\operatorname{erf}(\alpha_1z+\beta_1)]$, $z^n\sin\left(\alpha z^2+\beta z+\gamma\right)[\operatorname{erf}(\alpha_1z+\beta_1)\mp\operatorname{erf}(\alpha_2z+\beta_2)]$

2.20.1.

1. $\int_0^{+\infty}\sin(bz+\gamma)[1-\operatorname{erf}(az+b_1)]\,dz=\int_{-\infty}^{0}\sin(bz-\gamma)[-1-\operatorname{erf}(az-b_1)]dz=$

$$=\frac{\cos\gamma}{b}[1-\operatorname{erf}(b_1)]-\frac{1}{b}\exp\left(-\frac{b^2}{4a^2}\right)\left\{\cos\left(\frac{bb_1}{a}-\gamma\right)\left[1-\operatorname{Re}\ \operatorname{erf}\left(\frac{2ab_1+ib}{2a}\right)\right]+\right.$$

$$\left.+\sin\left(\frac{bb_1}{a}-\gamma\right)\operatorname{Im}\ \operatorname{erf}\left(\frac{2ab_1+ib}{2a}\right)\right\}\quad\{a>0\}.$$

2. $\int_0^{+\infty}\sin(bz+\gamma)\,[\operatorname{erf}(a_1z+b_1)-\operatorname{erf}(a_2z+b_2)]\,dz=\frac{\cos\gamma}{b}[\operatorname{erf}(b_1)-\operatorname{erf}(b_2)]+$

$$+\frac{1}{b}\exp\left(-\frac{b^2}{4a_1^2}\right)\left\{\cos\left(\frac{bb_1}{a_1}-\gamma\right)\left[1-\operatorname{Re}\ \operatorname{erf}\left(\frac{2a_1b_1+ib}{2a_1}\right)\right]+\right.$$

$$+\sin\left(\frac{bb_1}{a_1}-\gamma\right)\operatorname{Im}\ \operatorname{erf}\left(\frac{2a_1b_1+ib}{2a_1}\right)\Bigg\}-\frac{1}{b}\exp\left(-\frac{b^2}{4a_2^2}\right)\times$$

$$\times\left\{\cos\left(\frac{bb_2}{a_2}-\gamma\right)\left[1-\operatorname{Re}\ \operatorname{erf}\left(\frac{2a_2b_2+ib}{2a_2}\right)\right]+\sin\left(\frac{bb_2}{a_2}-\gamma\right)\operatorname{Im}\ \operatorname{erf}\left(\frac{2a_2b_2+ib}{2a_2}\right)\right\}$$

$$\{a_1>0,\ a_2>0\}.$$

3. $$\int_{-\infty}^{+\infty}\sin(bz+\gamma)\left[\operatorname{erf}(a_1z+b_1)-\operatorname{erf}(a_2z+b_2)\right]dz=$$

$$=\frac{2}{b}\left[\exp\left(-\frac{b^2}{4a_1^2}\right)\cos\left(\frac{bb_1}{a_1}-\gamma\right)-\exp\left(-\frac{b^2}{4a_2^2}\right)\cos\left(\frac{bb_2}{a_2}-\gamma\right)\right]\quad\{a_1>0,\ a_2>0\}.$$

4. $$\int_0^{\infty}\sin(\beta z+\gamma)[\pm 1-\operatorname{erf}(\alpha z+\beta_1)]\,dz=\frac{1}{2\beta}\exp\left(-\frac{\beta^2}{4\alpha^2}\right)\left\{\exp\left[i\left(\gamma-\frac{\beta\beta_1}{\alpha}\right)\right]\times\right.$$

$$\left.\times\left[\operatorname{erf}\left(\frac{2\alpha\beta_1-i\beta}{2\alpha}\right)\mp 1\right]+\exp\left[i\left(\frac{\beta\beta_1}{\alpha}-\gamma\right)\right]\left[\operatorname{erf}\left(\frac{2\alpha\beta_1+i\beta}{2\alpha}\right)\mp 1\right]\right\}-$$

$$-\frac{\cos\gamma}{\beta}[\operatorname{erf}(\beta_1)\mp 1]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha^2z^2+2\alpha\beta_1 z\right)-\left|\operatorname{Im}(\beta z)\right|+2\ln|z|\right]=+\infty,\quad \lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

5. $$\int_0^{\infty}\sin(\beta z+\gamma)\left[\operatorname{erf}(\alpha_1z+\beta_1)\mp\operatorname{erf}(\alpha_2z+\beta_2)\right]dz=\frac{\cos\gamma}{\beta}[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)]+$$

$$+\frac{1}{2\beta}\left\{\exp\left(i\gamma-\frac{\beta^2+4i\alpha_1\beta\beta_1}{4\alpha_1^2}\right)\left[A_1-\operatorname{erf}\left(\frac{2\alpha_1\beta_1-i\beta}{2\alpha_1}\right)\right]\mp\right.$$

$$\mp\exp\left(i\gamma-\frac{\beta^2+4i\alpha_2\beta\beta_2}{4\alpha_2^2}\right)\left[A_2-\operatorname{erf}\left(\frac{2\alpha_2\beta_2-i\beta}{2\alpha_2}\right)\right]+$$

$$+\exp\left(-i\gamma-\frac{\beta^2-4i\alpha_1\beta\beta_1}{4\alpha_1^2}\right)\left[A_1-\operatorname{erf}\left(\frac{2\alpha_1\beta_1+i\beta}{2\alpha_1}\right)\right]\mp$$

$$\left.\mp\exp\left(-i\gamma-\frac{\beta^2-4i\alpha_2\beta\beta_2}{4\alpha_2^2}\right)\left[A_2-\operatorname{erf}\left(\frac{2\alpha_2\beta_2+i\beta}{2\alpha_2}\right)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1 z\right)-\left|\operatorname{Im}(\beta z)\right|+2\ln|z|\right]=$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2 + 2\alpha_2\beta_2 z\right) - \left|\operatorname{Im}(\beta z)\right| + 2\ln|z|\right] = +\infty;$$

$$A_k = 1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z) = +\infty,\ A_k = -1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z) = -\infty;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z) = \pm\infty\ \}.$$

2.20.2.

1. $$\int_0^{+\infty}\sin\left(a^2z^2+\gamma\right)\left[1-\operatorname{erf}(a_1 z)\right]dz = -\int_{-\infty}^{0}\sin\left(a^2z^2+\gamma\right)\left[-1-\operatorname{erf}(a_1 z)\right]dz =$$

$$= \frac{1}{2\sqrt{2\pi}a}\left[(\sin\gamma-\cos\gamma)\operatorname{Re}\ln\frac{\sqrt{2}a_1+(1+i)a}{\sqrt{2}a_1-(1+i)a} + (\sin\gamma+\cos\gamma)\operatorname{Im}\ln\frac{\sqrt{2}a_1+(1+i)a}{\sqrt{2}a_1-(1+i)a}\right]$$

$$\{a_1>0\}.$$

2. $$\int_0^{+\infty}\sin\left(a^2z^2+\gamma\right)\left[\operatorname{erf}(a_1 z)-\operatorname{erf}(a_2 z)\right]dz = \frac{1}{2\sqrt{2\pi}a}\left\{(\cos\gamma-\sin\gamma)\left[\operatorname{Re}\ln\frac{\sqrt{2}a_1+(1+i)a}{\sqrt{2}a_1-(1+i)a} -\right.\right.$$

$$\left.\left. - \operatorname{Re}\ln\frac{\sqrt{2}a_2+(1+i)a}{\sqrt{2}a_2-(1+i)a}\right] + (\sin\gamma+\cos\gamma)\left[\operatorname{Im}\ln\frac{\sqrt{2}a_2+(1+i)a}{\sqrt{2}a_2-(1+i)a} - \operatorname{Im}\ln\frac{\sqrt{2}a_1+(1+i)a}{\sqrt{2}a_1-(1+i)a}\right]\right.$$

$$\{a_1>0,\ a_2>0\}.$$

3.

$$\int_{-\infty}^{+\infty}\sin\left(a^2z^2+bz+\gamma\right)\left[\operatorname{erf}(a_1z+b_1)-\operatorname{erf}(a_2z+b_2)\right]dz = \frac{\sqrt{2\pi}}{2a}\left[\cos\left(\frac{b^2}{4a^2}-\gamma\right) - \right.$$

$$\left. - \sin\left(\frac{b^2}{4a^2}-\gamma\right)\right]\left\{\operatorname{Re}\operatorname{erf}\left[\frac{\left(2a^2b_1-a_1b\right)(1+i)}{\sqrt{8}a\sqrt{a_1^2+ia^2}}\right] - \operatorname{Re}\operatorname{erf}\left[\frac{\left(2a^2b_2-a_2b\right)(1+i)}{\sqrt{8}a\sqrt{a_2^2+ia^2}}\right]\right\} +$$

$$+ \frac{\sqrt{2\pi}}{2a}\left[\sin\left(\frac{b^2}{4a^2}-\gamma\right) + \cos\left(\frac{b^2}{4a^2}-\gamma\right)\right]\left\{\operatorname{Im}\operatorname{erf}\left[\frac{\left(2a^2b_2-a_2b\right)(1+i)}{\sqrt{8}a\sqrt{a_2^2+ia^2}}\right] - \right.$$

$$\left. - \operatorname{Im}\operatorname{erf}\left[\frac{\left(2a^2b_1-a_1b\right)(1+i)}{\sqrt{8}a\sqrt{a_1^2+ia^2}}\right]\right\} \qquad \{a_1>0,\ a_2>0\}.$$

4. $$\int_0^{\infty}\sin\left(\alpha z^2+\gamma\right)\left[\pm 1-\operatorname{erf}(\alpha_1 z)\right]dz = \frac{1+i}{4\sqrt{2\pi}\sqrt{\alpha}}\exp(i\gamma)\ln\frac{\sqrt{2}\alpha_1-(1+i)\sqrt{\alpha}}{\sqrt{2}\alpha_1+(1+i)\sqrt{\alpha}} +$$

$$+ \frac{1-i}{4\sqrt{2\pi}\sqrt{\alpha}}\exp(-i\gamma)\ln\frac{\sqrt{2}\alpha_1-(1-i)\sqrt{\alpha}}{\sqrt{2}\alpha_1+(1-i)\sqrt{\alpha}}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2\right)-\left|\operatorname{Im}\left(\alpha z^2\right)\right|+2\ln|z|\right]=+\infty,$$

$$\alpha_1^2\neq i\alpha,\ \alpha_1^2\neq -i\alpha;\ \pm\operatorname{Re}(\alpha_1 z)>0 \text{ if } z\to\infty\}.$$

5. $$\int_0^\infty \sin\left(\alpha z^2+\gamma\right)\left[\operatorname{erf}(\alpha_1 z)\mp\operatorname{erf}(\alpha_2 z)\right]dz=\frac{1+i}{4\sqrt{2\pi}\sqrt{\alpha}}\exp(i\gamma)\left[\ln\frac{\sqrt{2}\alpha_1+(1+i)\sqrt{\alpha}}{\sqrt{2}\alpha_1-(1+i)\sqrt{\alpha}}\mp\right.$$

$$\left.\mp\ln\frac{\sqrt{2}\alpha_2+(1+i)\sqrt{\alpha}}{\sqrt{2}\alpha_2-(1+i)\sqrt{\alpha}}\right]+\frac{1-i}{4\sqrt{2\pi}\sqrt{\alpha}}\exp(-i\gamma)\times$$

$$\times\left[\ln\frac{\sqrt{2}\alpha_1+(1-i)\sqrt{\alpha}}{\sqrt{2}\alpha_1-(1-i)\sqrt{\alpha}}\mp\ln\frac{\sqrt{2}\alpha_2+(1-i)\sqrt{\alpha}}{\sqrt{2}\alpha_2-(1-i)\sqrt{\alpha}}\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2\right)-\left|\operatorname{Im}\left(\alpha z^2\right)\right|+2\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2\right)-\left|\operatorname{Im}\left(\alpha z^2\right)\right|+2\ln|z|\right]=+\infty,$$

$$\alpha_1^2\neq i\alpha,\ \alpha_2^2\neq i\alpha,\ \alpha_1^2\neq -i\alpha,\ \alpha_2^2\neq -i\alpha;\ \pm\operatorname{Re}(\alpha_1 z)\cdot\operatorname{Re}(\alpha_2 z)>0 \text{ if } z\to\infty\}.$$

6. $$\int_{\infty(T_1)}^{\infty(T_2)}\sin\left(\alpha z^2+\beta z+\gamma\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=\frac{\sqrt{2\pi}(1-i)}{4\sqrt{\alpha}}\times$$

$$\times\exp\left[i\left(\gamma-\frac{\beta^2}{4\alpha}\right)\right]\left\{A_1\operatorname{erf}\left[\frac{(2\alpha\beta_1-\alpha_1\beta)(1-i)}{\sqrt{8}\sqrt{\alpha_1^2-i\alpha}\sqrt{\alpha}}\right]\mp A_2\operatorname{erf}\left[\frac{(2\alpha\beta_2-\alpha_2\beta)(1-i)}{\sqrt{8}\sqrt{\alpha_2^2-i\alpha}\sqrt{\alpha}}\right]\right\}+$$

$$+\frac{\sqrt{2\pi}(1+i)}{4\sqrt{\alpha}}\exp\left[i\left(\frac{\beta^2}{4\alpha}-\gamma\right)\right]\times$$

$$\times\left\{B_1\operatorname{erf}\left[\frac{(2\alpha\beta_1-\alpha_1\beta)(1+i)}{\sqrt{8}\sqrt{\alpha_1^2+i\alpha}\sqrt{\alpha}}\right]\mp B_2\operatorname{erf}\left[\frac{(2\alpha\beta_2-\alpha_2\beta)(1+i)}{\sqrt{8}\sqrt{\alpha_2^2+i\alpha}\sqrt{\alpha}}\right]\right\}$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_1^2 z^2+2\alpha_1\beta_1 z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|+2\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_2^2 z^2+2\alpha_2\beta_2 z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|+2\ln|z|\right]=+\infty;$$

$$A_k = 1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) = +\infty,$$

$$A_k = -1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) = -\infty,$$

$$A_k = 0 \text{ if } \lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) \cdot \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 - i\alpha z}\right) = +\infty,$$

$$B_k = 1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = +\infty,$$

$$B_k = -1 \text{ if } \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = -\lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = -\infty,$$

$$B_k = 0 \text{ if } \lim_{z\to\infty(T_1)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) \cdot \lim_{z\to\infty(T_2)} \operatorname{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = +\infty;$$

$$\pm \operatorname{Re}(\alpha_1 z + \beta_1) \cdot \operatorname{Re}(\alpha_2 z + \beta_2) > 0 \text{ if } z \to \infty(T_1) \text{ and } z \to \infty(T_2)\}.$$

7. $$\int_{-\nu}^{\nu} \sin\left(\alpha z^2\right)\left[\operatorname{erf}(\alpha_1 z) \mp \operatorname{erf}(\alpha_2 z)\right] dz = 0.$$

2.20.3.

1. $$\int_0^{+\infty} z^{2m} \sin(bz+\gamma)[1-\operatorname{erf}(az+b_1)]dz = \int_{-\infty}^{0} z^{2m} \sin(bz-\gamma)[-1-\operatorname{erf}(az-b_1)]dz =$$
$$= (-1)^m \frac{(2m)!}{b^{2m+1}} \cos\gamma[1-\operatorname{erf}(b_1)] - (2m)!\left[\sin\gamma \cdot \operatorname{Re} W_1^{(20)}(2m, a, b, b_1, 1) + \right.$$
$$\left. + \cos\gamma \cdot \operatorname{Im} W_1^{(20)}(2m, a, b, b_1, 1)\right] \quad \{a>0\}.$$

2. $$\int_0^{+\infty} z^{2m+1} \sin(bz+\gamma)[1-\operatorname{erf}(az+b_1)]dz = -\int_{-\infty}^{0} z^{2m+1} \sin(bz-\gamma)[-1-\operatorname{erf}(az-b_1)]\,dz =$$
$$= (-1)^m \frac{(2m+1)!}{b^{2m+2}} \sin\gamma[\operatorname{erf}(b_1)-1] + (2m+1)!\left[\sin\gamma \cdot \operatorname{Re} W_1^{(20)}(2m+1, a, b, b_1, 1) + \right.$$
$$\left. + \cos\gamma \cdot \operatorname{Im} W_1^{(20)}(2m+1, a, b, b_1, 1)\right] \quad \{a>0\}.$$

3. $$\int_0^{+\infty} z^{2m} \sin(bz+\gamma)\ [\operatorname{erf}(a_1 z + b_1) - \operatorname{erf}(a_2 z + b_2)]\,dz =$$

$$= (-1)^m \frac{(2m)!}{b^{2m+1}} \cos\gamma \, [\mathrm{erf}\,(b_1) - \mathrm{erf}\,(b_2)] +$$

$$+ (2m)! \Big\{ \sin\gamma \Big[\mathrm{Re}\, W_1^{(20)}(2m, a_1, b, b_1, 1) - \mathrm{Re}\, W_1^{(20)}(2m, a_2, b, b_2, 1) \Big] +$$

$$+ \cos\gamma \Big[\mathrm{Im}\, W_1^{(20)}(2m, a_1, b, b_1, 1) - \mathrm{Im}\, W_1^{(20)}(2m, a_2, b, b_2, 1) \Big] \Big\} \quad \{a_1 > 0,\, a_2 > 0\}.$$

4. $$\int_0^{+\infty} z^{2m+1} \sin(bz + \gamma) [\mathrm{erf}\,(a_1 z + b_1) - \mathrm{erf}\,(a_2 z + b_2)]\, dz =$$

$$= (-1)^{m+1} \frac{(2m+1)!}{b^{2m+2}} \sin\gamma \big[\mathrm{erf}\,(b_1) - \mathrm{erf}\,(b_2) \big] + (2m+1)! \times$$

$$\times \Big\{ \sin\gamma \Big[\mathrm{Re}\, W_1^{(20)}(2m+1, a_2, b, b_2, 1) - \mathrm{Re}\, W_1^{(20)}(2m+1, a_1, b, b_1, 1) \Big] +$$

$$+ \cos\gamma \Big[\mathrm{Im}\, W_1^{(20)}(2m+1, a_2, b, b_2, 1) - \mathrm{Im}\, W_1^{(20)}(2m+1, a_1, b, b_1, 1) \Big] \Big\}$$

$$\{a_1 > 0,\, a_2 > 0\}.$$

5. $$\int_{-\infty}^{+\infty} z^n \sin(bz + \gamma) \big[\mathrm{erf}\,(a_1 z + b_1) - \mathrm{erf}\,(a_2 z + b_2) \big] dz = (-1)^n\, n! \times$$

$$\times \Big[\sin\gamma \cdot \mathrm{Re}\, W_2^{(20)}(a_1, b, b_1) + \cos\gamma \cdot \mathrm{Im}\, W_2^{(20)}(a_1, b, b_1) -$$

$$- \sin\gamma \cdot \mathrm{Re}\, W_2^{(20)}(a_2, b, b_2) - \cos\gamma \cdot \mathrm{Im}\, W_2^{(20)}(a_2, b, b_2) \Big] \quad \{a_1 > 0,\, a_2 > 0\}.$$

6. $$\int_0^{\infty} z^n \sin(\beta z + \gamma) [\pm 1 - \mathrm{erf}\,(\alpha z + \beta_1)] dz = (-1)^n\, n! \frac{\exp(i\gamma)}{2i} \Big\{ \frac{1}{(i\beta)^{n+1}} [\mathrm{erf}\,(\beta_1) \mp 1] -$$

$$- W_1^{(20)}(n, \alpha, \beta, \beta_1, \pm 1) \Big\} + (-1)^{n+1}\, n! \frac{\exp(-i\gamma)}{2i} \times$$

$$\times \Big\{ \Big(\frac{i}{\beta} \Big)^{n+1} \big[\mathrm{erf}\,(\beta_1) \mp 1 \big] - W_1^{(20)}(n, \alpha, -\beta, \beta_1, \pm 1) \Big\}$$

$$\{ \lim_{z\to\infty} \Big[\mathrm{Re}\big(\alpha^2 z^2 + 2\alpha\beta_1 z \big) - \big| \mathrm{Im}(\beta z) \big| - (n-2) \ln|z| \Big] = +\infty ,\ \lim_{z\to\infty} \mathrm{Re}(\alpha z) = \pm\infty \}.$$

7. $$\int_0^{\infty} z^n \sin(\beta z + \gamma) [\mathrm{erf}\,(\alpha_1 z + \beta_1) \mp \mathrm{erf}\,(\alpha_2 z + \beta_2)]\, dz = (-1)^{n+1}\, n! \frac{\exp(i\gamma)}{2i} \times$$

$$\times \Big\{ \frac{1}{(i\beta)^{n+1}} [\mathrm{erf}\,(\beta_1) \mp \mathrm{erf}\,(\beta_2)] -$$

$$-W_1^{(20)}(n,\alpha_1,\beta,\beta_1,A_1)\pm W_1^{(20)}(n,\alpha_2,\beta,\beta_2,A_2)\Big\}+$$

$$+(-1)^n n!\frac{\exp(-i\gamma)}{2i}\left\{\left(\frac{i}{\beta}\right)^{n+1}\left[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)\right]-\right.$$

$$\left.-W_1^{(20)}(n,\alpha_1,-\beta,\beta_1,A_1)\pm W_1^{(20)}(n,\alpha_2,-\beta,\beta_2,A_2)\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1 z\right)-\left|\operatorname{Im}(\beta z)\right|-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2 z\right)-\left|\operatorname{Im}(\beta z)\right|-(n-2)\ln|z|\right]=+\infty\ ;$$

$$A_k=1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=+\infty,\ A_k=-1 \text{ if } \lim_{z\to\infty}\operatorname{Re}(\alpha_k z)=-\infty\ ;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha_1 z)\cdot\lim_{z\to\infty}\operatorname{Re}(\alpha_2 z)=\pm\infty\ \}.$$

2.20.4.

1. $$\int_0^{+\infty} z^{2m}\sin\left(a^2z^2+\gamma\right)\left[1-\operatorname{erf}(a_1z)\right]dz=-\int_{-\infty}^{0} z^{2m}\sin\left(a^2z^2+\gamma\right)\left[-1-\operatorname{erf}(a_1z)\right]dz=$$

$$=\frac{(2m)!}{m!\sqrt{\pi}}\left[\sin\gamma\cdot\operatorname{Re}W_3^{(20)}\left(a^2,a_1\right)-\cos\gamma\cdot\operatorname{Im}W_3^{(20)}\left(a^2,a_1\right)\right]\ \{a>0,\ a_1>0\}.$$

2. $$\int_0^{+\infty} z^{2m}\sin\left(a^2z^2+\gamma\right)\left[\operatorname{erf}(a_1z)-\operatorname{erf}(a_2z)\right]dz=$$

$$=\frac{(2m)!}{m!\sqrt{\pi}}\left\{\sin\gamma\left[\operatorname{Re}W_3^{(20)}\left(a^2,a_2\right)-\operatorname{Re}W_3^{(20)}\left(a^2,a_1\right)\right]+\right.$$

$$\left.+\cos\gamma\left[\operatorname{Im}W_3^{(20)}\left(a^2,a_1\right)-\operatorname{Im}W_3^{(20)}\left(a^2,a_2\right)\right]\right\}\quad\{a>0,a_1>0,a_2>0\}.$$

3. $$\int_0^{+\infty} z^{2m+1}\sin\left(a^2z^2+\gamma\right)\left[1-\operatorname{erf}(a_1z+b)\right]dz=$$

$$=\int_{-\infty}^{0} z^{2m+1}\sin\left(a^2z^2+\gamma\right)\left[-1-\operatorname{erf}(a_1z-b)\right]dz=$$

$$=\frac{m!}{2a^{2m+2}}\left[1-\operatorname{erf}(b)\right]\left[\cos\gamma\cdot\operatorname{Re}\left(i^m\right)-\sin\gamma\cdot\operatorname{Im}\left(i^m\right)\right]+\frac{m!}{2}\times$$

$$\times\left[\cos\gamma\cdot\operatorname{Im}W_4^{(20)}\left(a^2,a_1,b,1\right)-\sin\gamma\cdot\operatorname{Re}W_4^{(20)}\left(a^2,a_1,b,1\right)\right]\quad\{a_1>0\}.$$

4. $$\int_0^{+\infty} z^{2m+1}\sin\left(a^2z^2+\gamma\right)[\operatorname{erf}(a_1z+b_1)-\operatorname{erf}(a_2z+b_2)]dz=$$

$$=\frac{m!}{2a^{2m+2}}\left[\operatorname{erf}(b_1)-\operatorname{erf}(b_2)\right]\left[\cos\gamma\cdot\operatorname{Re}\left(i^m\right)-\sin\gamma\cdot\operatorname{Im}\left(i^m\right)\right]+$$

$$+\frac{m!}{2}\sin\gamma\left[\operatorname{Re}W_4^{(20)}\left(a^2,a_1,b_1,1\right)-\operatorname{Re}W_4^{(20)}\left(a^2,a_2,b_2,1\right)\right]+$$

$$+\frac{m!}{2}\cos\gamma\left[\operatorname{Im}W_4^{(20)}\left(a^2,a_2,b_2,1\right)-\operatorname{Im}W_4^{(20)}\left(a^2,a_1,b_1,1\right)\right]\quad\{a_1>0,\,a_2>0\}.$$

5. $$\int_{-\infty}^{+\infty} z^{n}\sin\left(a^2z^2+bz+\gamma\right)[\operatorname{erf}(a_1z+b_1)-\operatorname{erf}(a_2z+b_2)]\,dz=$$

$$=\sin\gamma\left[\operatorname{Re}W_5^{(20)}\left(n,a^2,a_1,b,b_1\right)-\operatorname{Re}W_5^{(20)}\left(n,a^2,a_2,b,b_2\right)\right]+$$

$$+\cos\gamma\left[\operatorname{Im}W_5^{(20)}\left(n,a^2,a_2,b,b_2\right)-\operatorname{Im}W_5^{(20)}\left(n,a^2,a_1,b,b_1\right)\right]$$

$$\{a>0,a_1>0,a_2>0\}.$$

6. $$\int_0^{\infty} z^{2m}\sin\left(\alpha z^2+\gamma\right)[\pm1-\operatorname{erf}(\alpha_1z)]dz=\frac{(2m)!}{m!\sqrt{\pi}}\times$$

$$\times\left[\frac{\exp(i\gamma)}{2i}W_3^{(20)}(-\alpha,\alpha_1)-\frac{\exp(-i\gamma)}{2i}W_3^{(20)}(\alpha,\alpha_1)\right]$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2\right)-\left|\operatorname{Im}\left(\alpha z^2\right)\right|-(2m-2)\ln|z|\right]=+\infty,\ \alpha\neq0,\ \alpha_1^2+i\alpha\neq0,\ \alpha_1^2-i\alpha\neq0;$$

$$\pm\operatorname{Re}(\alpha_1z)>0\ \text{ if }\ z\to\infty\}.$$

7. $$\int_0^{\infty} z^{2m}\sin\left(\alpha z^2+\gamma\right)[\operatorname{erf}(\alpha_1z)\mp\operatorname{erf}(\alpha_2z)]\,dz=\frac{(2m)!}{m!\sqrt{\pi}}\times$$

$$\times\left\{\frac{\exp(-i\gamma)}{2i}\left[W_3^{(20)}(\alpha,\alpha_1)\mp W_3^{(20)}(\alpha,\alpha_2)\right]-\right.$$

$$\left.-\frac{\exp(i\gamma)}{2i}\left[W_3^{(20)}(-\alpha,\alpha_1)\mp W_3^{(20)}(-\alpha,\alpha_2)\right]\right\}$$

$$\{\lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2z^2\right)-\left|\operatorname{Im}\left(\alpha z^2\right)\right|-(2m-2)\ln|z|\right]=$$

$$= \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_2^2 z^2\right) - \left|\operatorname{Im}\left(\alpha z^2\right)\right| - (2m-2)\ln|z|\right] = +\infty\,,$$

$$\alpha \neq 0,\ \alpha_1^2 + i\alpha \neq 0,\ \alpha_1^2 - i\alpha \neq 0,\ \alpha_2^2 + i\alpha \neq 0,\ \alpha_2^2 - i\alpha \neq 0;$$

$$\pm\operatorname{Re}(\alpha_1 z)\cdot\operatorname{Re}(\alpha_2 z) > 0 \text{ if } z\to\infty\}.$$

8. $$\int_0^\infty z^{2m+1}\sin\left(\alpha z^2 + \gamma\right)[\pm 1 - \operatorname{erf}(\alpha_1 z + \beta)]\,dz = \frac{m!}{4i}\exp(-i\gamma)\left[W_4^{(20)}(\alpha, \alpha_1, \beta, A_1) + \right.$$

$$\left. + \frac{\operatorname{erf}(\beta)\mp 1}{(i\alpha)^{m+1}}\right] - \frac{m!}{4i}\exp(i\gamma)\left[W_4^{(20)}(-\alpha, \alpha_1, \beta, A_2) + \frac{\operatorname{erf}(\beta)\mp 1}{(-i\alpha)^{m+1}}\right]$$

$$\{\ \lim_{z\to\infty}\left[\operatorname{Re}\left(\alpha_1^2 z^2 + 2\alpha_1\beta z\right) - \left|\operatorname{Im}\left(\alpha z^2\right)\right| - (2m-1)\ln|z|\right] = +\infty\,;$$

$$A_1 = 1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2 + i\alpha}\, z\right) = +\infty,\ A_1 = -1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2 + i\alpha}\, z\right) = -\infty,$$

$$A_2 = 1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2 - i\alpha}\, z\right) = +\infty,\ A_2 = -1 \text{ if } \lim_{z\to\infty}\operatorname{Re}\left(\sqrt{\alpha_1^2 - i\alpha}\, z\right) = -\infty;$$

$$\pm\operatorname{Re}\left(\alpha_1 z + \beta\right) > 0 \text{ if } z\to\infty\}.$$

9. $$\int_0^\infty z^{2m+1}\sin\left(Ai\alpha^2 z^2 + \gamma\right)[\pm 1 - \operatorname{erf}(\alpha z + \beta)]dz = A\frac{m!}{\left(-\alpha^2\right)^{m+1}}\cdot\frac{\exp(-Ai\gamma)}{4i}\times$$

$$\times\left[\operatorname{erf}(\beta)\mp 1 + W_6^{(20)}(\beta)\right] - Am!\frac{\exp(Ai\gamma)}{4i}\left[\frac{\operatorname{erf}(\beta)\mp 1}{\alpha^{2m+2}} + W_4^{(20)}\left(-i\alpha^2, \alpha, \beta, \pm 1\right)\right]$$

$$\{\ \lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2 z^2 + \alpha\beta z\right) - (2m-1)\ln|z|\right] = \lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta z) - m\ln|z|\right] = +\infty\,;$$

$$A = 1 \text{ or } A = -1;\ \pm\operatorname{Re}\left(\alpha z + \beta\right) > 0 \text{ if } z\to\infty\}.$$

10. $$\int_0^\infty z^{2m+1}\sin\left(\alpha z^2 + \gamma\right)\left[\operatorname{erf}(\alpha_1 z + \beta_1)\mp\operatorname{erf}(\alpha_2 z + \beta_2)\right]dz = \frac{m!}{4i}\left[\operatorname{erf}(\beta_1)\mp\operatorname{erf}(\beta_2)\right]\times$$

$$\times\left[\frac{\exp(i\gamma)}{(-i\alpha)^{m+1}} - \frac{\exp(-i\gamma)}{(i\alpha)^{m+1}}\right] + \frac{m!}{4i}\Big\{\exp(i\gamma)\times$$

$$\times\left[W_4^{(20)}(-\alpha, \alpha_1, \beta_1, A_1)\mp W_4^{(20)}(-\alpha, \alpha_2, \beta_2, A_2)\right] -$$

$$-\exp(-i\gamma)\Big[W_4^{(20)}(\alpha, \alpha_1, \beta_1, B_1) \mp W_4^{(20)}(\alpha, \alpha_2, \beta_2, B_2)\Big]\Big\}$$

$$\{\lim_{z\to\infty}\Big[\mathrm{Re}\Big(\alpha_1^2 z^2 + 2\alpha_1\beta_1 z\Big) - \Big|\mathrm{Im}\Big(\alpha z^2\Big)\Big| - (2m-1)\ln|z|\Big] =$$

$$= \lim_{z\to\infty}\Big[\mathrm{Re}\Big(\alpha_2^2 z^2 + 2\alpha_2\beta_2 z\Big) - \Big|\mathrm{Im}\Big(\alpha z^2\Big)\Big| - (2m-1)\ln|z|\Big] = +\infty;$$

$$A_k = 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_k^2 - i\alpha}\, z\Big) = +\infty,\ A_k = -1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_k^2 - i\alpha}\, z\Big) = -\infty,$$

$$B_k = 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_k^2 + i\alpha}\, z\Big) = +\infty,\ B_k = -1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_k^2 + i\alpha}\, z\Big) = -\infty;$$

$$\pm\mathrm{Re}(\alpha_1 z + \beta_1)\cdot\mathrm{Re}(\alpha_2 z + \beta_2) > 0 \text{ if } z \to \infty\}.$$

11. $$\int_0^\infty z^{2m+1}\sin\Big(Ai\alpha^2 z^2 + \gamma\Big)[\mathrm{erf}(\alpha z + \beta) \mp \mathrm{erf}(\alpha_1 z + \beta_1)]dz =$$

$$= A\frac{m!}{4i}\Big\{\exp(Ai\gamma)\Big[\frac{\mathrm{erf}(\beta) \mp \mathrm{erf}(\beta_1)}{\alpha^{2m+2}} + W_4^{(20)}\Big(-i\alpha^2, \alpha, \beta, B\Big) \mp W_4^{(20)}\Big(-i\alpha^2, \alpha_1, \beta_1, B_1\Big)\Big] +$$

$$+(-1)^m\frac{\exp(-Ai\gamma)}{\alpha^{2m+2}}\Big[\mathrm{erf}(\beta) \mp \mathrm{erf}(\beta_1) + W_6^{(20)}(\beta)\Big] \pm$$

$$\pm\exp(-Ai\gamma)W_4^{(20)}\Big(i\alpha^2, \alpha_1, \beta_1, A_1\Big)\Big\}$$

$$\{\lim_{z\to\infty}\Big[2\,\mathrm{Re}\Big(\alpha^2 z^2 + \alpha\beta z\Big) - (2m-1)\ln|z|\Big] =$$

$$= \lim_{z\to\infty}\Big[\mathrm{Re}\Big(\alpha_1^2 z^2 + 2\alpha_1\beta_1 z\Big) - \Big|\mathrm{Re}\Big(\alpha^2 z^2\Big)\Big| - (2m-1)\ln|z|\Big] =$$

$$= \lim_{z\to\infty}\Big[\mathrm{Re}(\alpha\beta z) - m\ln|z|\Big] = +\infty;$$

$$A = 1 \text{ or } A = -1;\ A_1 = 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_1^2 - \alpha^2}\, z\Big) = +\infty,$$

$$A_1 = -1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_1^2 - \alpha^2}\, z\Big) = -\infty,\ B = 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}(\alpha z) = +\infty,$$

$$B = -1 \text{ if } \lim_{z\to\infty}\mathrm{Re}(\alpha z) = -\infty,\ B_1 = 1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_1^2 + \alpha^2}\, z\Big) = +\infty,$$

$$B_1 = -1 \text{ if } \lim_{z\to\infty}\mathrm{Re}\Big(\sqrt{\alpha_1^2 + \alpha^2}\, z\Big) = -\infty;\ \pm\mathrm{Re}(\alpha z + \beta)\mathrm{Re}(\alpha_1 z + \beta_1) > 0 \text{ if } z \to \infty\}.$$

12. $$\int_0^{\infty} z^{2m+1}\sin\left(Ai\alpha^2 z^2+\gamma\right)\left[\operatorname{erf}(\alpha z+\beta)-\operatorname{erf}(\alpha z+\beta_1)\right]dz=$$

$$=\frac{Am!}{4i\alpha^{2m+2}}\Big\{\exp(Ai\gamma)\left[\operatorname{erf}(\beta)-\operatorname{erf}(\beta_1)\right]+(-1)^m\exp(-Ai\gamma)\times$$

$$\times\left[\operatorname{erf}(\beta)-\operatorname{erf}(\beta_1)+W_6^{(20)}(\beta)-W_6^{(20)}(\beta_1)\right]\Big\}+\frac{Am!}{4i}\exp(Ai\gamma)\times$$

$$\times\left[W_4^{(20)}\left(-i\alpha^2,\alpha,\beta,\pm1\right)-W_4^{(20)}\left(-i\alpha^2,\alpha,\beta_1,\pm1\right)\right]$$

$$\{\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta z\right)-(2m-1)\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[2\operatorname{Re}\left(\alpha^2z^2+\alpha\beta_1 z\right)-(2m-1)\ln|z|\right]=\lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta z)-m\ln|z|\right]=$$

$$=\lim_{z\to\infty}\left[\operatorname{Re}(\alpha\beta_1 z)-m\ln|z|\right]=+\infty;\ A=1 \text{ or } A=-1;$$

$$\lim_{z\to\infty}\operatorname{Re}(\alpha z)=\pm\infty\}.$$

13. $$\int_{\infty(T_1)}^{\infty(T_2)} z^n\sin\left(\alpha z^2+\beta z+\gamma\right)\left[\operatorname{erf}(\alpha_1 z+\beta_1)\mp\operatorname{erf}(\alpha_2 z+\beta_2)\right]dz=\frac{\exp(i\gamma)}{2i}\times$$

$$\times\left[A_1W_5^{(20)}(n,-\alpha,\alpha_1,-\beta,\beta_1)\mp A_2W_5^{(20)}(n,-\alpha,\alpha_2,-\beta,\beta_2)\right]-\frac{\exp(-i\gamma)}{2i}\times$$

$$\times\left[B_1W_5^{(20)}(n,\alpha,\alpha_1,\beta,\beta_1)\mp B_2W_5^{(20)}(n,\alpha,\alpha_2,\beta,\beta_2)\right]$$

$$\{\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_1^2z^2+2\alpha_1\beta_1z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_1)}\left[\operatorname{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|-(n-2)\ln|z|\right]=$$

$$=\lim_{z\to\infty(T_2)}\left[\operatorname{Re}\left(\alpha_2^2z^2+2\alpha_2\beta_2z\right)-\left|\operatorname{Im}\left(\alpha z^2+\beta z\right)\right|-(n-2)\ln|z|\right]=+\infty,\ \alpha\neq0;$$

$$A_k=1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha_k^2-i\alpha}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha_k^2-i\alpha}\,z\right)=+\infty,$$

$$A_k=-1 \text{ if } \lim_{z\to\infty(T_2)}\operatorname{Re}\left(\sqrt{\alpha_k^2-i\alpha}\,z\right)=-\lim_{z\to\infty(T_1)}\operatorname{Re}\left(\sqrt{\alpha_k^2-i\alpha}\,z\right)=-\infty,$$

$$A_k = 0 \text{ if } \lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha_k^2 - i\alpha\, z}\right)\cdot \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha_k^2 - i\alpha\, z}\right) = +\infty,$$

$$B_k = 1 \text{ if } \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha\, z}\right) = -\lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha\, z}\right) = +\infty,$$

$$B_k = -1 \text{ if } \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = -\lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha z}\right) = -\infty,$$

$$B_k = 0 \text{ if } \lim_{z\to\infty(T_1)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha\, z}\right)\cdot \lim_{z\to\infty(T_2)} \mathrm{Re}\left(\sqrt{\alpha_k^2 + i\alpha\, z}\right) = +\infty;$$

$$\pm \mathrm{Re}(\alpha_1 z + \beta_1)\cdot \mathrm{Re}(\alpha_2 z + \beta_2) > 0 \text{ if } z \to \infty(T_1) \text{ and } z \to \infty(T_2)\}.$$

14. $\int_{-\nu}^{\nu} z^{2m} \sin\left(\alpha z^2\right)[\mathrm{erf}(\alpha_1 z) \mp \mathrm{erf}(\alpha_2 z)]dz = 0.$

Introduced notations:

1) $$W_1^{(20)}(n, \alpha, \beta, \beta_1, A) = \exp\left(-\frac{\beta^2 + 4i\alpha\beta\beta_1}{4\alpha^2}\right)\left[\mathrm{erf}\left(\frac{2\alpha\beta_1 - i\beta}{2\alpha}\right) - A\right]\times$$

$$\times \sum_{k=0}^{n} \frac{1}{2^k (i\beta)^{n+1-k}} \sum_{l=0}^{E(k/2)} \frac{(2\alpha\beta_1 - i\beta)^{k-2l}}{l!(k-2l)!\alpha^{2k-2l}} + \frac{1}{\sqrt{\pi}} \exp\left(-\beta_1^2\right) \sum_{k=1}^{n} \frac{1}{2^k (i\beta)^{n+1-k}} \times$$

$$\times \left[\sum_{l=1}^{k-E(k/2)} \frac{l!}{(2l)!(k+1-2l)!} \sum_{r=1}^{l} \frac{4^{l+1-r}(2\alpha\beta_1 - i\beta)^{k-1-2l+2r}}{(r-1)!\alpha^{2k-1-2l+2r}} - \sum_{l=1}^{E(k/2)} \frac{1}{l!(k-2l)!}\times\right.$$

$$\left.\times \sum_{r=1}^{l} \frac{(r-1)!(2\alpha\beta_1 - i\beta)^{k-1-2l+2r}}{(2r-1)!\alpha^{2k-1-2l+2r}}\right],\; W_1^{(20)}\left(n, \alpha, \beta, \frac{i\beta}{2\alpha}, A\right) = \exp\left(\frac{\beta^2}{4\alpha^2}\right)\times$$

$$\times \left[\frac{2}{\sqrt{\pi}} \sum_{k=1}^{n-E(n/2)} \frac{k!}{(2k)!(i\beta)^{n+2-2k}\alpha^{2k-1}} - A \sum_{k=0}^{E(n/2)} \frac{1}{k!(i\beta)^{n+1-2k}(2\alpha)^{2k}}\right];$$

2) $$\mathrm{Re}\, W_2^{(20)}(a_k, b, b_k) = \mathrm{Re}\, W_1^{(20)}(n, a_k, b, b_k, 1) - \mathrm{Re}\, W_1^{(20)}(n, a_k, b, -b_k, 1),$$

$$\mathrm{Im}\, W_2^{(20)}(a_k, b, b_k) = \mathrm{Im}\, W_1^{(20)}(n, a_k, b, b_k, 1) + \mathrm{Im}\, W_1^{(20)}(n, a_k, b, -b_k, 1);$$

$$3)\ W_3^{(20)}(\alpha,\alpha_1)=\frac{1}{2^{2m+1}\,i(i\alpha)^m\sqrt{i\alpha}}\ln\frac{\sqrt{2}\alpha_1-(1-i)\sqrt{\alpha}}{\sqrt{2}\alpha_1+(1-i)\sqrt{\alpha}}-\alpha_1\times$$

$$\times\sum_{k=0}^{m-1}\frac{(k!)^2}{(2k+1)!(4i\alpha)^{m-k}\left(\alpha_1^2+i\alpha\right)^{k+1}};$$

$$4)\ W_4^{(20)}(\alpha,\alpha_1,\beta,A)=\frac{\alpha_1}{\sqrt{\alpha_1^2+i\alpha}}\exp\left(-\frac{i\alpha\beta^2}{\alpha_1^2+i\alpha}\right)\left[A-\operatorname{erf}\left(\frac{\alpha_1\beta}{\sqrt{\alpha_1^2+i\alpha}}\right)\right]\times$$

$$\times\sum_{k=0}^{m}\frac{(2k)!}{k!(i\alpha)^{m+1-k}}\sum_{l=0}^{k}\frac{(\alpha_1\beta)^{2k-2l}}{4^l\,l!(2k-2l)!\left(\alpha_1^2+i\alpha\right)^{2k-l}}+\frac{\alpha_1}{\sqrt{\pi}}\exp\left(-\beta^2\right)\times$$

$$\times\sum_{k=1}^{m}\frac{(2k)!}{k!(i\alpha)^{m+1-k}}\left[\sum_{l=1}^{k}\frac{1}{l!(2k-2l)!}\sum_{r=1}^{l}\frac{r!(\alpha_1\beta)^{2k-1-2l+2r}}{4^{l-r}(2r)!\left(\alpha_1^2+i\alpha\right)^{2k-l+r}}-\right.$$

$$\left.-\sum_{l=1}^{k}\frac{(l-1)!}{(2l-1)!(2k+1-2l)!}\sum_{r=1}^{l}\frac{(\alpha_1\beta)^{2k-1-2l+2r}}{(r-1)!\left(\alpha_1^2+i\alpha\right)^{2k-l+r}}\right],$$

$$W_4^{(20)}(\alpha,\alpha_1,0,A)=A\frac{\alpha_1}{\sqrt{\alpha_1^2+i\alpha}}\sum_{k=0}^{m}\frac{(2k)!}{4^k(k!)^2(i\alpha)^{m+1-k}\left(\alpha_1^2+i\alpha\right)^k};$$

$$5)\ W_5^{(20)}(n,\alpha,\alpha_1,\beta,\beta_1)=\frac{n!\sqrt{\pi}}{2^n\sqrt{i\alpha}}\exp\left(\frac{i\beta^2}{4\alpha}\right)\operatorname{erf}\left(\frac{2i\alpha\beta_1-i\alpha_1\beta}{2\sqrt{i\alpha}\sqrt{\alpha_1^2+i\alpha}}\right)\times$$

$$\times\sum_{k=0}^{E(n/2)}\frac{(-\beta)^{n-2k}}{k!(n-2k)!i^k\alpha^{n-k}}+\frac{n!}{(2\alpha)^n\sqrt{\alpha_1^2+i\alpha}}\exp\left(-\frac{\beta^2+4i\alpha\beta_1^2-4i\alpha_1\beta\beta_1}{4\alpha_1^2+4i\alpha}\right)\times$$

$$\times\left[\sum_{k=1}^{n-E(n/2)}\frac{k!(-\beta)^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=1}^{k}\frac{(-4i)^{k+1-l}(2l-2)!}{(l-1)!}\times\right.$$

$$\times\sum_{r=0}^{l-1}\frac{\alpha^{k-l+2r}\alpha_1^{2l-1-2r}(2\alpha\beta_1-\alpha_1\beta)^{2l-2-2r}}{r!(2l-2-2r)!\left(\alpha_1^2+i\alpha\right)^{2l-2-r}}+$$

$$+\sum_{k=1}^{E(n/2)}\frac{(-\beta)^{n-2k}}{k!(n-2k)!}\sum_{l=1}^{k}(l-1)!\,(-i)^{k-1-l}\sum_{r=0}^{l-1}\frac{\alpha^{k-l+2r}\alpha_1^{2l-2r}\,(2\alpha\beta_1-\alpha_1\beta)^{2l-1-2r}}{r!(2l-1-2r)!\left(\alpha_1^2+i\alpha\right)^{2l-1-r}}\Bigg],$$

$$W_5^{(20)}\,(2m,\alpha,\alpha_1,0,\beta_1)=\frac{(2m)!\sqrt{\pi}}{m!(4i\alpha)^m\,\sqrt{i\alpha}}\operatorname{erf}\left(\frac{i\alpha\beta_1}{\sqrt{i\alpha}\sqrt{\alpha_1^2+i\alpha}}\right)+$$

$$+\frac{(2m)!}{m!(2\alpha)^{2m}\sqrt{\alpha_1^2+i\alpha}}\exp\left(-\frac{i\alpha\beta_1^2}{\alpha_1^2+i\alpha}\right)\sum_{k=1}^{m}(k-1)!\,(-i)^{m-1-k}\times$$

$$\times\sum_{l=0}^{k-1}\frac{\alpha^{m-k+2l}\alpha_1^{2k-2l}\,(2\alpha\beta_1)^{2k-1-2l}}{l!(2k-1-2l)!\left(\alpha_1^2+i\alpha\right)^{2k-1-l}},$$

$$W_5^{(20)}\,(2m+1,\alpha,\alpha_1,0,\beta_1)=\frac{m!}{4^{m+1}\alpha^{2m+1}\sqrt{\alpha_1^2+i\alpha}}\exp\left(-\frac{i\alpha\beta_1^2}{\alpha_1^2+i\alpha}\right)\times$$

$$\times\sum_{k=0}^{m}\frac{(2k)!(-4i)^{m+1-k}}{k!}\sum_{l=0}^{k}\frac{\alpha^{m-k+2l}\alpha_1^{2k+1-2l}\,(2\alpha\beta_1)^{2k-2l}}{l!(2k-2l)!\left(\alpha_1^2+i\alpha\right)^{2k-l}},$$

$$W_5^{(20)}\,(n,\alpha,\alpha_1,\beta,\frac{\alpha_1}{2\alpha}\beta)=\frac{n!\alpha_1}{2^n\,\alpha^{n+1}\sqrt{\alpha_1^2+i\alpha}}\exp\left(\frac{i\beta^2}{4\alpha}\right)\times$$

$$\times\sum_{k=1}^{n-E(n/2)}\frac{k!(-\beta)^{n+1-2k}}{(2k)!(n+1-2k)!}\sum_{l=0}^{k-1}\frac{(2l)!(-4i)^{k-l}\,\alpha^{k+l}}{(l!)^2\left(\alpha_1^2+i\alpha\right)^{l}};$$

6) $W_6^{(20)}\,(\beta)=\dfrac{\exp\left(-\beta^2\right)}{\sqrt{\pi}}\displaystyle\sum_{k=0}^{m}\frac{(2k)!}{(-4)^k\,k!\beta^{2k+1}}.$

APPENDIX

Some useful formulas for obtaining other integrals

1. $\int_{\mu}^{\nu} f(z)\,\mathrm{erfi}\,(\alpha z+\beta)\,dz = \frac{1}{i}\int_{\mu}^{\nu} f(z)\,\mathrm{erf}\,(i\alpha z+i\beta)\,dz$.

2. $\int_{\mu}^{\nu} f(z)\,\mathrm{erfi}^{n}(\alpha z+\beta)\,dz = (-i)^{n}\int_{\mu}^{\nu} f(z)\,\mathrm{erf}^{n}\,(i\alpha z+i\beta)\,dz$.

3. $\int_{\mu}^{\nu} f(z)\cos\left(\alpha z^2+\beta z+\gamma\right)dz = \frac{\partial}{\partial\gamma}\left[\int_{\mu}^{\nu} f(z)\sin\left(\alpha z^2+\beta z+\gamma\right)dz\right]$.

4. $\int_{\mu}^{\nu} f(z)\cos\left(\alpha z^2+\beta z+\gamma\right)dz = \int_{\mu}^{\nu} f(z)\sin\left(\alpha z^2+\beta z+\gamma+\pi/2\right)dz$.

5. $\int_{\mu}^{\nu} f(z)\sinh^{m}\left(\alpha z^2+\beta z+\gamma\right)dz = (-i)^{m}\int_{\mu}^{\nu} f(z)\sin^{m}\left(i\alpha z^2+i\beta z+i\gamma\right)dz$.

6. $\int_{\mu}^{\nu} f(z)\cosh\left(\alpha z^2+\beta z+\gamma\right)dz = \int_{\mu}^{\nu} f(z)\sin\left(i\alpha z^2+i\beta z+i\gamma+\pi/2\right)dz$.

7. $\int_{\mu}^{\nu} f(z)\cosh^{m}\left(\alpha z^2+\beta z+\gamma\right)dz = \int_{\mu}^{\nu} f(z)\cos^{m}\left(i\alpha z^2+i\beta z+i\gamma\right)dz$.

8. $\int_{\mu}^{\nu} f(z)\sin^{2m}\left(\alpha z^2+\beta z+\gamma\right)dz = \frac{(2m)!}{4^{m}(m!)^2}\int_{\mu}^{\nu} f(z)\,dz +$

$$+\frac{(2m)!}{2^{2m-1}}\sum_{k=1}^{m}\frac{(-1)^{k}}{(m-k)!(m+k)!}\int_{\mu}^{\nu} f(z)\cos\left(2k\alpha z^2+2k\beta z+2k\gamma\right)dz .$$

9. $\int_{\mu}^{\nu} f(z)\sin^{2m+1}\left(\alpha z^2+\beta z+\gamma\right)dz = \frac{(2m+1)!}{4^{m}}\sum_{k=0}^{m}\frac{(-1)^{k}}{(m-k)!\,(m+1+k)!}\times$

$$\times\int_{\mu}^{\nu} f(z)\sin\left[(2k+1)\alpha z^2+(2k+1)\beta z+(2k+1)\gamma\right]dz .$$

10. $\int_{\mu}^{\nu} f(z)\cos^{2m}\left(\alpha z^2+\beta z+\gamma\right)dz = \frac{(2m)!}{4^{m}(m!)^2}\int_{\mu}^{\nu} f(z)\,dz+\frac{(2m)!}{2^{2m-1}}\times$

$$\times\sum_{k=1}^{m}\frac{1}{(m-k)!(m+k)!}\int_{\mu}^{\nu} f(z)\cos\left(2k\alpha z^2+2k\beta z+2k\gamma\right)dz\,.$$

11. $$\int_{\mu}^{\nu} f(z)\cos^{2m+1}\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{(2m+1)!}{4^m}\sum_{k=0}^{m}\frac{1}{(m-k)!\,(m+1+k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\cos\left[(2k+1)\alpha z^2+(2k+1)\beta z+(2k+1)\gamma\right]dz\,.$$

12. $$\int_{\mu}^{\nu} f(z)\sinh^{2m}\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{(2m)!}{(-4)^m(m!)^2}\int_{\mu}^{\nu} f(z)\,dz+\frac{(2m)!}{2^{2m-1}}\sum_{k=1}^{m}\frac{(-1)^{m-k}}{(m-k)!\,(m+k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\cosh\left(2k\alpha z^2+2k\beta z+2k\gamma\right)dz\,.$$

13. $$\int_{\mu}^{\nu} f(z)\sinh^{2m+1}\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{(2m+1)!}{4^m}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{(m-k)!\,(m+1+k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\sinh\left[(2k+1)\alpha z^2+(2k+1)\beta z+(2k+1)\gamma\right]dz\,.$$

14. $$\int_{\mu}^{\nu} f(z)\cosh^{2m}\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{(2m)!}{4^m(m!)^2}\int_{\mu}^{\nu} f(z)\,dz+\frac{(2m)!}{2^{2m-1}}\times$$

$$\times\sum_{k=1}^{m}\frac{1}{(m-k)!\,(m+k)!}\int_{\mu}^{\nu} f(z)\cosh\left(2k\alpha z^2+2k\beta z+2k\gamma\right)dz\,.$$

15. $$\int_{\mu}^{\nu} f(z)\cosh^{2m+1}\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{(2m+1)!}{4^m}\sum_{k=0}^{m}\frac{1}{(m-k)!\,(m+1+k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\cosh\left[(2k+1)\alpha z^2+(2k+1)\beta z+(2k+1)\gamma\right]dz\,.$$

16. $$\int_{\mu}^{\nu} f(z)\sin\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\sin\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)dz=$$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\cos\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)dz-\frac{1}{2}\int_{\mu}^{\nu} f(z)\cos\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)dz$$

$$\{\alpha_3=\alpha_1-\alpha_2,\ \beta_3=\beta_1-\beta_2,\ \gamma_3=\gamma_1-\gamma_2,$$

$$\alpha_4=\alpha_1+\alpha_2,\ \beta_4=\beta_1+\beta_2,\ \gamma_4=\gamma_1+\gamma_2\}\,.$$

17. $\int_{\mu}^{\nu} f(z)\,\sin\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\,\cos\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)\,dz=$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\sin\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)\,dz+\frac{1}{2}\int_{\mu}^{\nu} f(z)\sin\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)\,dz$$

$$\{\alpha_3=\alpha_1-\alpha_2,\,\beta_3=\beta_1-\beta_2,\,\gamma_3=\gamma_1-\gamma_2,$$

$$\alpha_4=\alpha_1+\alpha_2,\;\beta_4=\beta_1+\beta_2,\,\gamma_4=\gamma_1+\gamma_2\}.$$

18. $\int_{\mu}^{\nu} f(z)\,\cos\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\,\cos\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)\,dz=$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\cos\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)dz+\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\cos\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)dz$$

$$\{\alpha_3=\alpha_1-\alpha_2,\,\beta_3=\beta_1-\beta_2,\,\gamma_3=\gamma_1-\gamma_2,$$

$$\alpha_4=\alpha_1+\alpha_2,\,\beta_4=\beta_1+\beta_2,\,\gamma_4=\gamma_1+\gamma_2\}.$$

19. $\int_{\mu}^{\nu} f(z)\,\sinh\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\sinh\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)dz=$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\cosh\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)\,dz-\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\cosh\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)\,dz$$

$$\{\alpha_3=\alpha_1+\alpha_2,\,\beta_3=\beta_1+\beta_2,\,\gamma_3=\gamma_1+\gamma_2,$$

$$\alpha_4=\alpha_1-\alpha_2,\;\beta_4=\beta_1-\beta_2,\,\gamma_4=\gamma_1-\gamma_2\}.$$

20. $\int_{\mu}^{\nu} f(z)\,\sinh\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\,\cosh\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)\,dz=$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\sinh\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)\,dz+\frac{1}{2}\int_{\mu}^{\nu} f(z)\,\sinh\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)\,dz$$

$$\{\alpha_3=\alpha_1-\alpha_2,\,\beta_3=\beta_1-\beta_2,\,\gamma_3=\gamma_1-\gamma_2,$$

$$\alpha_4=\alpha_1+\alpha_2,\;\beta_4=\beta_1+\beta_2,\,\gamma_4=\gamma_1+\gamma_2\}.$$

21. $\int_{\mu}^{\nu} f(z)\,\cosh\left(\alpha_1 z^2+\beta_1 z+\gamma_1\right)\cosh\left(\alpha_2 z^2+\beta_2 z+\gamma_2\right)dz=$

$$=\frac{1}{2}\int_{\mu}^{\nu} f(z)\cosh\left(\alpha_3 z^2+\beta_3 z+\gamma_3\right)dz+\frac{1}{2}\int_{\mu}^{\nu} f(z)\cosh\left(\alpha_4 z^2+\beta_4 z+\gamma_4\right)dz$$

$$\{\alpha_3=\alpha_1-\alpha_2,\,\beta_3=\beta_1-\beta_2,\,\gamma_3=\gamma_1-\gamma_2,$$

$$\alpha_4=\alpha_1+\alpha_2,\,\beta_4=\beta_1+\beta_2,\,\gamma_4=\gamma_1+\gamma_2\}.$$

22. $$\int_{\mu}^{\nu} f(z)\sin^m\left(\alpha z^2+\beta z+\gamma\right)\cos\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{m+1}\times$$

$$\times\frac{\partial}{\partial\gamma}\left[\int_{\mu}^{\nu} f(z)\sin^{m+1}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

23. $$\int_{\mu}^{\nu} f(z)\cos^m\left(\alpha z^2+\beta z+\gamma\right)\sin\left(\alpha z^2+\beta z+\gamma\right)dz=-\frac{1}{m+1}\times$$

$$\times\frac{\partial}{\partial\gamma}\left[\int_{\mu}^{\nu} f(z)\cos^{m+1}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

24. $$\int_{\mu}^{\nu} f(z)\sinh^m\left(\alpha z^2+\beta z+\gamma\right)\cosh\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{m+1}\times$$

$$\times\frac{\partial}{\partial\gamma}\left[\int_{\mu}^{\nu} f(z)\sinh^{m+1}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

25. $$\int_{\mu}^{\nu} f(z)\cosh^m\left(\alpha z^2+\beta z+\gamma\right)\sinh\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{m+1}\times$$

$$\times\frac{\partial}{\partial\gamma}\left[\int_{\mu}^{\nu} f(z)\cosh^{m+1}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

26. $$\int_{\mu}^{\nu} f(z)\sin^m\left(\alpha z^2+\beta z+\gamma\right)\cos^2\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{m+1}\times$$

$$\times\int_{\mu}^{\nu} f(z)\sin^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz+\frac{1}{(m+1)(m+2)}\times$$

$$\times\frac{\partial^2}{\partial\gamma^2}\left[\int_{\mu}^{\nu} f(z)\sin^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

27. $$\int_{\mu}^{\nu} f(z)\cos^m\left(\alpha z^2+\beta z+\gamma\right)\sin^2\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{m+1}\times$$

$$\times\int_{\mu}^{\nu} f(z)\cos^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz+\frac{1}{(m+1)(m+2)}\times$$

$$\times\frac{\partial^2}{\partial\gamma^2}\left[\int_{\mu}^{\nu} f(z)\cos^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz\right].$$

28. $$\int_{\mu}^{\nu} f(z)\sinh^m\left(\alpha z^2+\beta z+\gamma\right)\cosh^2\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{(m+1)(m+2)}\times$$

$$\times\frac{\partial^2}{\partial\gamma^2}\left[\int_{\mu}^{\nu} f(z)\sinh^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz\right]-\frac{1}{m+1}\int_{\mu}^{\nu} f(z)\sinh^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz.$$

29. $$\int_{\mu}^{\nu} f(z)\cosh^m\left(\alpha z^2+\beta z+\gamma\right)\sinh^2\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{1}{(m+1)(m+2)}\times$$

$$\times\frac{\partial^2}{\partial\gamma^2}\left[\int_{\mu}^{\nu} f(z)\cosh^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz\right]-\frac{1}{m+1}\int_{\mu}^{\nu} f(z)\cosh^{m+2}\left(\alpha z^2+\beta z+\gamma\right)dz.$$

30. $$\int_{\mu}^{\nu} f(z)\sin^m\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{m!}{(2i)^m}\sum_{k=0}^{m}\frac{(-1)^k}{k!\,(m-k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\exp\left[(m-2k)i\alpha z^2+(m-2k)i\beta z+(m-2k)i\gamma\right]dz.$$

31. $$\int_{\mu}^{\nu} f(z)\cos^m\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{m!}{2^m}\sum_{k=0}^{m}\frac{1}{k!\,(m-k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\exp\left[(m-2k)i\alpha z^2+(m-2k)i\beta z+(m-2k)i\gamma\right]dz.$$

32. $$\int_{\mu}^{\nu} f(z)\sinh^m\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{m!}{2^m}\sum_{k=0}^{m}\frac{(-1)^k}{k!\,(m-k)!}\times$$

$$\times\int_{\mu}^{\nu} f(z)\exp\left[(m-2k)\alpha z^2+(m-2k)\beta z+(m-2k)\gamma\right]dz.$$

33. $\int_{\mu}^{\nu} f(z)\cosh^m\left(\alpha z^2+\beta z+\gamma\right)dz=\frac{m!}{2^m}\sum_{k=0}^{m}\frac{1}{k!\,(m-k)!}\times$

$$\times\int_{\mu}^{\nu} f(z)\exp\left[(m-2k)\alpha z^2+(m-2k)\beta z+(m-2k)\gamma\right]dz\ .$$

Note that the formulas in this Appendix can obviously be written also for improper and indefinite integrals. Also note that in formulas 3 and 22–29, $f(z)$ does not depend on γ.