Supplementary Information for "Rolling quantum dice with a superconducting qubit"

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ROTATIONAL GROUPS

The tetra-, octa-, and icosahedral rotational groups are shown in Tables. S1, S2 and S3.

PULSE CALIBRATION

The microwave pulses – the rotations around the X and Y axes – are created by generating envelopes using 1 Gsample/s digital to analog converters, and upconverting these envelopes to the qubit frequency using quadrature mixing (see Ref. [1] for the control and readout system). The room temperature electronics are calibrated by corrected the pulse from distortion using deconvolution techniques, and by correcting the quadrature mixer for gain and phase imbalances. The pulses for frequency control – the rotations around the Z axis – are generated by 1 Gsample/s digital to analog converters; non-idealities in the pulse shape from room temperature electronics are suppressed by deconvolution techniques. Non-idealities arising from stray inductance and reflections in the wiring of the cryostat are suppressed by using the qubit to measure the step response and by randomized benchmarking, see Refs. [1, 2] for details.

We then use the qubit to calibrate the pulse amplitudes, and the DRAG (derivative reduction for adiabatic gates) parameter for minimizing 2-state leakage [3, 4]. We do not calibrate the phase between a X and Y rotation using the qubit [5] as the quadrature mixer calibrations are sufficient. The pulse amplitudes for Z rotations are determined using quantum state tomography.

We use three parameters to generate the microwave pulses necessary for the tetrahedral and octahedral rotational groups (DRAG parameter, two pulse amplitudes) and 14 parameters (DRAG parameter, 13 pulse amplitudes) to generate the pulses for the icosahedral rotational group; see Table S4 for the generators.

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[2] Kelly, J. *et al.* Optimal quantum control using randomized benchmarking. arXiv:1403.0035.

[3] Lucero, E. *et al.* Reduced phase error through optimized control of a superconducting qubit. *Phys. Rev. A* **82**, 042339 (2010).

[4] Chow, J. M. *et al.* Implementing optimal control pulse shaping for improved single-qubit gates. *Phys. Rev. A* **82**, 040305 (2010).

[5] Gustavsson, S. *et al.* Improving Quantum Gate Fidelities by Using a Qubit to Measure Microwave Pulse Distortions. *Phys. Rev. Lett.* **110**, 040502 (2013).

TABLE S1: The tetrahedral rotational group, written in terms of the physical microwave gates applied in time. Negative angles are included through opposite rotational axes.

Paulis - π	$2\pi/3$
I	$X_{\pi/2}\ Y_{\pi/2}$
X_π	$X_{\pi/2}\ Y_{-\pi/2}$
Y_π	$X_{-\pi/2}\ Y_{\pi/2}$
$Y_\pi\ X_\pi$	$X_{-\pi/2}\ Y_{-\pi/2}$
	$Y_{\pi/2}\ X_{\pi/2}$
	$Y_{\pi/2}\ X_{-\pi/2}$
	$Y_{-\pi/2}\ X_{\pi/2}$
	$Y_{-\pi/2}\ X_{-\pi/2}$

TABLE S2: The octahedral rotational group – single qubit Cliffords. The Paulis and $2\pi/3$ rotations form the tetrahedral rotational group.

Paulis - π	$2\pi/3$	$\pi/2$	Hadamard-like - π
I	$X_{\pi/2}\ Y_{\pi/2}$	$X_{\pi/2}$	$X_\pi\ Y_{\pi/2}$
X_π	$X_{\pi/2}\ Y_{-\pi/2}$	$X_{-\pi/2}$	$X_\pi\ Y_{-\pi/2}$
Y_π	$X_{-\pi/2}\ Y_{\pi/2}$	$Y_{\pi/2}$	$Y_\pi\ X_{\pi/2}$
$Y_\pi\ X_\pi$	$X_{-\pi/2}\ Y_{-\pi/2}$	$Y_{-\pi/2}$	$Y_\pi\ X_{-\pi/2}$
	$Y_{\pi/2}\ X_{\pi/2}$	$X_{-\pi/2}\ Y_{\pi/2}\ X_{\pi/2}$	$X_{\pi/2}\ Y_{\pi/2}\ X_{\pi/2}$
	$Y_{\pi/2}\ X_{-\pi/2}$	$X_{-\pi/2}\ Y_{-\pi/2}\ X_{\pi/2}$	$X_{-\pi/2}\ Y_{\pi/2}\ X_{-\pi/2}$
	$Y_{-\pi/2}\ X_{\pi/2}$		
	$Y_{-\pi/2}\ X_{-\pi/2}$		

TABLE S3: The 60 icosahedral rotations, excluding the idle (I). The rotations are ordered based on their angles, and their points of intersection with the icosahedron. The edges and faces contain the Paulis and $2\pi/3$ rotations which overlap with the tetrahedral rotational group. For the edge rotations we have used $R_X(\phi)R_Y(\pi)R_X(-\phi) = R_X(2\phi)R_Y(\pi)$ and $R_X(\phi)R_Z(\pi)R_X(-\phi) = R_X(2\phi)R_Z(\pi)$ to reduce the gate count.

Vertices - $2\pi/5$	Faces - $2\pi/3$	Edges - π
$Y_\phi\ X_{2\pi/5}\ Y_{-\phi}$	$X_{-\pi/2}\ Y_{-\pi/2}$	X_π
$Y_\phi\ X_{-2\pi/5}\ Y_{-\phi}$	$Y_{\pi/2}\ X_{\pi/2}$	$X_\phi\ Z_{2\pi/5}\ X_\pi\ Z_{-2\pi/5}\ X_{-\phi}$
$Y_{-\phi}\ X_{2\pi/5}\ Y_\phi$	$X_\phi\ Z_{-2\pi/5}\ X_{-\phi}\ X_{-\pi/2}\ Y_{-\pi/2}\ X_\phi\ Z_{2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{-2\pi/5}\ X_\pi\ Z_{2\pi/5}\ X_{-\phi}$
$Y_{-\phi}\ X_{-2\pi/5}\ Y_\phi$	$X_\phi\ Z_{-2\pi/5}\ X_{-\phi}\ Y_{\pi/2}\ X_{\pi/2}\ X_\phi\ Z_{2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{4\pi/5}\ X_\pi\ Z_{-4\pi/5}\ X_{-\phi}$
$Z_\phi\ Y_{2\pi/5}\ Z_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ X_{-\phi}\ X_{-\pi/2}\ Y_{-\pi/2}\ X_\phi\ Z_{4\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ X_\pi\ Z_{4\pi/5}\ X_{-\phi}$
$Z_\phi\ Y_{-2\pi/5}\ Z_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ X_{-\phi}\ Y_{\pi/2}\ X_{\pi/2}\ X_\phi\ Z_{4\pi/5}\ X_{-\phi}$	Y_π
$Z_{-\phi}\ Y_{2\pi/5}\ Z_\phi$	$X_{-\pi/2}\ Y_{\pi/2}$	$X_\phi\ Z_{2\pi/5}\ Y_\pi\ X_{2\phi}\ Z_{-2\pi/5}\ X_{-\phi}$
$Z_{-\phi}\ Y_{-2\pi/5}\ Z_\phi$	$Y_{-\pi/2}\ X_{\pi/2}$	$X_\phi\ Z_{-2\pi/5}\ Y_\pi\ X_{2\phi}\ Z_{2\pi/5}\ X_{-\phi}$
$X_\phi\ Z_{2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{2\pi/5}\ X_{-\phi}\ X_{-\pi/2}\ Y_{-\pi/2}\ X_\phi\ Z_{-2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{4\pi/5}\ Y_\pi\ X_{2\phi}\ Z_{-4\pi/5}\ X_{-\phi}$
$X_\phi\ Z_{-2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{2\pi/5}\ X_{-\phi}\ Y_{\pi/2}\ X_{\pi/2}\ X_\phi\ Z_{-2\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ Y_\pi\ X_{2\phi}\ Z_{4\pi/5}\ X_{-\phi}$
$X_{-\phi}\ Z_{2\pi/5}\ X_\phi$	$X_{\pi/2}\ Y_{\pi/2}$	Z_π
$X_{-\phi}\ Z_{-2\pi/5}\ X_\phi$	$Y_{-\pi/2}\ X_{-\pi/2}$	$X_\phi\ Z_{2\pi/5}\ Z_\pi\ X_{2\phi}\ Z_{-2\pi/5}\ X_{-\phi}$
Vertices - $4\pi/5$	$X_\phi\ Z_{-4\pi/5}\ X_{-\phi}\ X_{\pi/2}\ Y_{\pi/2}\ X_\phi\ Z_{4\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{-2\pi/5}\ Z_\pi\ X_{2\phi}\ Z_{2\pi/5}\ X_{-\phi}$
$Y_\phi\ X_{4\pi/5}\ Y_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ X_{-\phi}\ Y_{-\pi/2}\ X_{-\pi/2}\ X_\phi\ Z_{4\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{4\pi/5}\ Z_\pi\ X_{2\phi}\ Z_{-4\pi/5}\ X_{-\phi}$
$Y_\phi\ X_{-4\pi/5}\ Y_{-\phi}$	$X_\phi\ Z_{4\pi/5}\ X_{-\phi}\ X_{\pi/2}\ Y_{\pi/2}\ X_\phi\ Z_{-4\pi/5}\ X_{-\phi}$	$X_\phi\ Z_{-4\pi/5}\ Z_\pi\ X_{2\phi}\ Z_{4\pi/5}\ X_{-\phi}$
$Y_{-\phi}\ X_{4\pi/5}\ Y_\phi$	$X_\phi\ Z_{4\pi/5}\ X_{-\phi}\ Y_{-\pi/2}\ X_{-\pi/2}\ X_\phi\ Z_{-4\pi/5}\ X_{-\phi}$	
$Y_{-\phi}\ X_{-4\pi/5}\ Y_\phi$	$X_\phi\ Z_{2\pi/5}\ X_{-\phi}\ X_{\pi/2}\ Y_{\pi/2}\ X_\phi\ Z_{-2\pi/5}\ X_{-\phi}$	
$Z_\phi\ Y_{4\pi/5}\ Z_{-\phi}$	$X_\phi\ Z_{2\pi/5}\ X_{-\phi}\ Y_{-\pi/2}\ X_{-\pi/2}\ X_\phi\ Z_{-2\pi/5}\ X_{-\phi}$	
$Z_\phi\ Y_{-4\pi/5}\ Z_{-\phi}$	$X_{\pi/2}\ Y_{-\pi/2}$	
$Z_{-\phi}\ Y_{4\pi/5}\ Z_\phi$	$Y_{\pi/2}\ X_{-\pi/2}$	
$Z_{-\phi}\ Y_{-4\pi/5}\ Z_\phi$		
$X_\phi\ Z_{4\pi/5}\ X_{-\phi}$		
$X_\phi\ Z_{-4\pi/5}\ X_{-\phi}$		
$X_{-\phi}\ Z_{4\pi/5}\ X_\phi$		
$X_{-\phi}\ Z_{-4\pi/5}\ X_\phi$		

TABLE S4: Generators of the tetra-, octa-, and icosahedral rotational groups, excluding the idle. The generators are listed in terms of shared pulse amplitude parameter. Including the DRAG parameter, we use a total of three parameters to generate the tetra-, and octahedral rotational group (DRAG parameter, pulse amplitude parameter for X_π and Y_π, pulse amplitude parameter for $X_{\pi/2}$, $X_{-\pi/2}$, $Y_{\pi/2}$, and $Y_{-\pi/2}$), and a total of 14 parameters to generate the icosahedral rotational group. The duration of the idle and each of the X and Y gates is 12 ns, and each Z gate is 10 ns.

Rotational group	Generators
Tetrahedral, octahedral, and icosahedral	X_π, Y_π $X_{\pi/2}$, $X_{-\pi/2}$, $Y_{\pi/2}$, $Y_{-\pi/2}$
Icosahedral	$X_{2\pi/5}$, $X_{-2\pi/5}$, $Y_{2\pi/5}$, $Y_{-2\pi/5}$ $X_{4\pi/5}$, $X_{-4\pi/5}$, $Y_{4\pi/5}$, $Y_{-4\pi/5}$ X_ϕ, Y_ϕ, $X_{-\phi}$, $Y_{-\phi}$ $X_{2\phi}$ $Z_{2\pi/5}$ $Z_{-2\pi/5}$ Z_ϕ $Z_{-\phi}$ $Z_{4\pi/5}$ $Z_{-4\pi/5}$ Z_π