Correlated quantum measurement of a solid-state qubit

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We propose a solid-state experiment to study the process of continuous quantum measurement of a qubit state. The experiment would verify that an individual qubit stays coherent during the process of measurement (in contrast to the gradual decoherence of the ensemble-averaged density matrix), thus confirming the possibility of qubit purification by continuous measurement. The experiment can be realized using quantum dots, single-electron transistors, or superconducting quantum interference devices.

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The impressive advantages promised by quantum computing[1] have revived interest in the fundamental quantum effects in two-level systems, which in this context are called qubits. In this paper we address the problem of the continuous measurement of a qubit state having in mind a solid-state realization of the setup. Among numerous proposals of quantum computers, the solid-state realizations (see, e.g., Refs. 2–5) look more promising because of better controllability of qubit parameters and interqubit couplings. However, the qubit measurement in this case is not as straightforward as in typical optical experiments where the single photon just "clicks" the detector. The reason is finite (and typically weak) coupling with a solid-state detector and finite intrinsic noise of the detector. As a result, the measurement cannot be done instantaneously, and so the collapse postulate of the "orthodox" quantum mechanics[6] cannot be applied directly. Instead, the quantum measurement should be considered as a continuous process, so that the rate of information acquisition (which defines the collapse time scale) can be comparable to the typical frequency of qubit evolution.

There are two main theoretical approaches to the continuous quantum measurements. One approach (which dominates in solid-state physics and so can be called "conventional") is based on the theory of the interaction with dissipative environment.[7,8] Taking the trace over the numerous degrees of freedom of the detector, it is possible to obtain the gradual evolution of the density matrix of the measured system from the pure initial state to the incoherent statistical mixture, thus describing the measurement process. Since the procedure implies averaging over the *ensemble*, the final equations of this formalism are deterministic and can be derived from the Schrödinger equation alone, without any notion of state collapse.

The other approach (see, e.g., Refs. 9–14) is closer to the collapse viewpoint and describes the stochastic evolution of an *individual* quantum system due to continuous measurement (note the close relation to the theory of imperfect measurement[15,16]). Such evolution obviously depends on a particular measurement result and is usually called selective or conditional quantum evolution. Depending on the details of the studied measurement setup and applied formalism, different authors[9–14] discuss quantum trajectories, quantum-state diffusion, stochastic evolution of the wave function, quantum jumps, stochastic Schrödinger equation, complex

Hamiltonian, method of restricted path integral, Bayesian formalism, etc. The theory of selective quantum evolution was only recently introduced into the context of solid-state mesoscopics.[14,17–20] In particular, it was shown that the continuous measurement of an individual qubit does not lead to gradual decoherence (in contrast to the conventional result for an ensemble); instead, the measurement can lead to gradual purification of the qubit density matrix.

Since the concept is still considered controversial, an experimental check is quite important. In this paper we propose an experiment which can be realized using three possible setups available for present-day technology: double-quantum-dot qubit measured by quantum point contact,[21] qubit based on single-Cooper-pair box measured by a single-electron transistor,[22] or superconducting quantum interference device (SQUID) based qubit measured by another SQUID.[23,24]

Let us start with reviewing the result of the conventional formalism for the continuous measurement (Fig. 1) of a qubit state (see, e.g., recent publications in Refs. 21 and 25–30). For the qubit characterized by the standard Hamiltonian $\mathcal{H}_{QB}=(\varepsilon/2)(c_1^\dagger c_1-c_2^\dagger c_2)+H(c_1^\dagger c_2+c_2^\dagger c_1)$ in the basis defined by coupling with the detector (here ε is the energy asymmetry and H is the tunneling strength), the evolution of qubit density matrix ρ is given by the equations

$$\dot{\rho}_{11}=-\dot{\rho}_{22}=-2H\,\mathrm{Im}\,\rho_{12}, \tag{1}$$

$$\dot{\rho}_{12}=\iota\varepsilon\rho_{12}+\iota H(\rho_{11}-\rho_{22})-\Gamma\rho_{12}, \tag{2}$$

where the continuous measurement is described by the dephasing rate Γ.

These equations do not depend on the detector output because they represent the result of ensemble averaging, including averaging over the measurement result. To study the evolution of an individual qubit[14,17–20] let us denote the noisy detector signal as $I(t)$ (assuming current for definiteness). Two "localized" qubit states $|1\rangle$ and $|2\rangle$ correspond to average detector currents I_1 and I_2 which by assumption do not differ much, $\Delta I\equiv I_1-I_2\ll I_0\equiv(I_1+I_2)/2$. The intrinsic noise
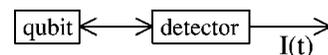
FIG. 1. Schematic of a qubit continuously measured by a detector with output signal $I(t)$.

of the detector signal is characterized by the spectral density S which is frequency independent in the range of interest. The noise determines the typical measurement time $t \sim S/(\Delta I)^2$ necessary to distinguish between states $|1\rangle$ and $|2\rangle$, and thus defines the time scale of the selective evolution of the qubit density matrix $\rho(t)$. Within the Bayesian formalism[14,17–20] the selective evolution is described by the equations

$$\dot{\rho}_{11} = -2H\,\mathrm{Im}\,\rho_{12} + (2\Delta I/S)\rho_{11}\rho_{22}[I(t)-I_0], \quad (3)$$

$$\dot{\rho}_{12} = \iota\varepsilon\rho_{12} + \iota H(\rho_{11}-\rho_{22}) - (\Delta I/S)(\rho_{11}-\rho_{22})$$
$$\times [I(t)-I_0]\rho_{12} - \gamma\rho_{12}, \quad (4)$$

where the dephasing $\gamma = \Gamma - (\Delta I)^2/4S \geq 0$ is now due to the contribution from the "pure environment" only. In particular, $\gamma = 0$ if the qubit is measured by symmetric quantum point contact, since in this case $\Gamma = (\Delta I)^2/4S$ (see Refs. 25,26,21 and 30). We will call such a detector an ideal detector, $\eta = 1$, where $\eta \equiv 1 - \gamma/\Gamma$ is the ideality factor. In contrast, the single-electron transistor in the operation point far outside the Coulomb blockade range is a significantly nonideal detector,[29] $\eta \ll 1$; however, η becomes comparable to unity when the current is mostly due to cotunneling processes.[19,31] The SQUID is an ideal detector when its sensitivity is quantum limited.[19,32,33]

Equations (3) and (4) allow us to calculate the evolution of qubit density matrix ρ if the detector output $I(t)$ is known from a particular experiment. To simulate the measurement we can use the replacement[14]

$$I(t)-I_0 = \Delta I(\rho_{11}-\rho_{22})/2 + \xi(t), \quad (5)$$

where the random process $\xi(t)$ has zero average and "white" spectral density $S_\xi = S$. One can check that averaging of Eqs. (3) and (4) over all possible measurement results [i.e., over the random contribution $\xi(t)$] reduces them to Eqs. (1) and (2). Notice that the stochastic equations are written in Stratonovich form which preserves the usual calculus rules, while averaging is more straightforward in Itô form.[34]

As follows from Eqs. (3) and (4), if a qubit with initially pure state, $|\rho_{12}(0)|^2 = \rho_{11}(0)\rho_{22}(0)$, is measured by an ideal detector, then its density matrix $\rho(t)$ stays pure during the measurement process. Even if the initial state is a statistical mixture, $\rho(t)$ is gradually purified during the measurement.[14]

The predictions of the Bayesian formalism can be checked experimentally; however, it is not simple at the present-day level of solid-state technology. The direct experiment was discussed in Ref. 14. The idea was to perform the measurement by a near-ideal detector during time τ, record the detector output $I(t)$, use Eqs. (3) and (4) to calculate $\rho(\tau)$, and then check the calculated value. This check can be done by changing qubit parameters ε and H in a way to ensure $\rho_{11} = 1$ at some specified moment of time, that can be verified by the detector switched on again. Since for coherent evolution the qubit can be placed with 100% certainty in the state $|1\rangle$ only if the wave function is known precisely, such a
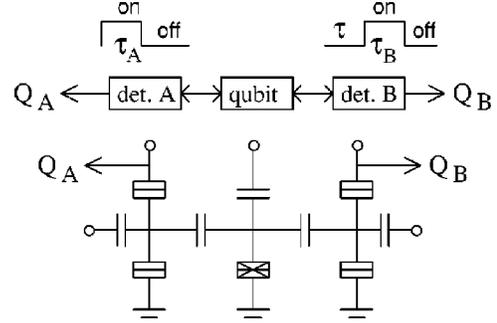
FIG. 2. Schematic of two-detector correlation experiment using Cooper-pair box and two single-electron transistors.

check (repeated many times) verifies that $\rho(\tau)$ is pure and coincides with the calculated value.

Unfortunately, this experiment would require very fast recording of $I(t)$. Since the expected coherence time is on the order of 10–100 ns at most (see, e.g., Ref. 22), the bandwidth of the detector signal coming out of the cryostat should be at least 1 GHz, which is very difficult experimentally. Another proposed experiment[18,30] is to measure the spectral density of the quantum coherent oscillations and check the predicted maximal peak-to-pedestal ratio of 4. Such an experiment may be easier to realize (because the basic spectral analysis can be done on-chip inside the cryostat); however, it would not prove unambiguously the Bayesian formalism, since an alternative interpretation of the result is possible.[18]

Here we propose an experiment which is even easier to realize and which can test the Bayesian formalism (3) and (4). The main idea is to use two detectors (A and B) connected to the same qubit (Fig. 2). The detectors are switched on for short periods of time by two shifted-in-time voltage pulses (one for each detector) with durations τ_A and τ_B, supplied from the outside. The output signal from the detector A is the total charge $Q_A = \int_0^{\tau_A} I_A(t)\,dt$ passed during the measurement period. Similarly, the output from detector B is $Q_B = \int_\tau^{\tau+\tau_B} I_B(t)\,dt$, where τ is the time shift between pulses. If the measurement by detector A changes the qubit density matrix, it will affect the result of measurement B. Repeating the experiment many times (with the same initial qubit state) we can obtain the probability distribution $P(Q_A,Q_B|\tau)$ of different outcomes, which contains information about the effect of the quantum measurement on the qubit density matrix. In comparison with previous suggestions, the advantage of this correlation experiment is that the wide signal bandwidth is required only for input pulses (that is relatively simple) while the outputs are essentially low-frequency signals. The experiment can be called "Bell type" because of some similarity with the famous proposal of Ref. 35. (In both experiments a quantum system is measured by two detectors so that one detector collapses the system and the other detector "feels" this collapse. However, in Bell's experiment the main point is the lonlocality of the collapse, while we check the effect of continuous collapse.)

Figure 2 shows a realization of the experiment using single-electron transistors as detectors. The transistors are

switched on by short pulses of the bias voltage (the use of gate voltage pulses is also possible; however, this would introduce an extra noise). Qubit is realized by the Cooper-pair box[36,22] so that the electric charge of the central island can be in coherent combination of two discrete charge states. Another similar setup is two quantum point contacts measuring the charge state of a double-quantum-dot qubit. One more setup is the 3-SQUID experiment in which the qubit is realized by one SQUID while two other SQUID's are in the detecting regime.

The conventional formalism (1) and (2) does not give any explicit predictions[37] for the resulting probability distribution $P(Q_A, Q_B | \tau)$. Since these equations cannot describe the correlations between $\rho(t)$ and $I(t)$, they imply, for example, that the average result of the second measurement $\bar{Q}_B(Q_A, \tau) \equiv \int Q_B P(Q_A, Q_B | \tau) dQ_B$ does not depend on Q_A. The Bayesian formalism (3) and (4) makes the different prediction: $\bar{Q}_B$ does depend on Q_A.

For simplicity let us assume symmetric qubit, $\varepsilon = 0$, which is initially in the ground state, $\rho_{11} = \rho_{22} = \rho_{12} = 0.5$, and also assume relatively strong coupling between the qubit and detectors, $(\Delta I_A)^2 / H S_A \gg 1$, $(\Delta I_B)^2 / H S_B \gg 1$ (subscripts A and B correspond to two detectors), so that we can neglect the qubit evolution due to finite H during the measurement periods τ_A and τ_B, which are assumed to be on the order of $S_{A,B} / (\Delta I_{A,B})^2$. Then from Eqs. (3) and (4) it follows that the first measurement localizes the qubit state only partially, and after obtaining the result Q_A from the first measurement the qubit density matrix is

$$2\rho_{11}(\tau_A) - 1 = \tanh \frac{(Q_A - \tau_A I_{2A})^2 - (Q_A - \tau_A I_{1A})^2}{2 S_A \tau_A}, \quad (6)$$

$$\rho_{12}(\tau_A) = [\rho_{11}(\tau_A)\rho_{22}(\tau_A)]^{1/2} \exp(-\gamma_A \tau_A). \quad (7)$$

Here Eq. (6) is just the Bayes formula, so this result can be called "quantum Bayes theorem."[16] [The probability to get Q_A has the distribution $P(Q_A) = (p_1 + p_2)/2$ where $p_i = (\pi S_A \tau_A)^{-1/2} \exp(-(Q_A - \tau_A I_{iA})^2 / S_A \tau_A)$.] The qubit performs the free evolution during the time $\tau - \tau_A$ between measurements (here we neglect $\tau_A \ll \tau$) and the average result of the second measurement $\bar{Q}_B = \tau_B [I_{2B} + \rho_{11}(\tau)\Delta I_B]$ depends on Q_A in the following way [Fig. 3(a)]:

$$\delta_B = \frac{1}{2}\tanh \frac{(Q_A - \tau_A I_{2A})^2 - (Q_A - \tau_A I_{1A})^2}{2 S_A \tau_A}$$

$$\times \frac{2H}{\Omega}\cos\left(\Omega \tau - \arcsin\frac{\gamma_f}{4H}\right)\exp(-\gamma_f \tau/2), \quad (8)$$

where $\delta_B \equiv (\bar{Q}_B - \tau_B I_{0B})/\tau_B \Delta I_B$, γ_f is the dephasing with both detectors switched off, and $\Omega = (4H^2 - \gamma_f^2/4)^{1/2}$ is the frequency of quantum oscillations (underdamped case is assumed). Notice that δ_B changes sign together with the sign of $Q_A - \tau_A I_{0A}$, while the phase of oscillations is a piece-constant function of Q_A.
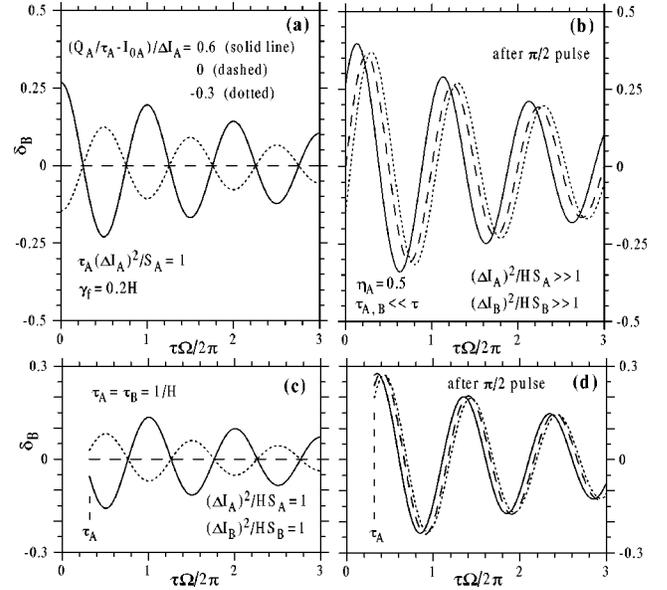
FIG. 3. The normalized average result δ_B of the second measurement for several selected results Q_A of the first measurement, as a function of the time τ between measurements. Panels (a) and (b) are for strong coupling and panels (c) and (d) for moderate coupling between the qubit and detectors (other parameters are the same). The calculations are done by Bayesian formalism while the conventional formalism does not predict any nontrivial dependence.

The dependence becomes quite different if the $\pi/2$ pulse is applied to the qubit immediately after the first measurement, which multiplies $\rho_{12}(\tau_A)$ given by Eq. (7) by (minus) the imaginary unit. In this case [Fig. 3(b)],

$$\delta_B = A\sin(\Omega \tau + \arcsin z/A)\exp(-\gamma_f \tau/2),$$

$$A = [(z^2 + y^2 - yz\gamma_f/2H)/(1 - \gamma_f^2/16H^2)]^{1/2}, \quad (9)$$

where $z \equiv \rho_{11}(\tau_A) - 1/2$ and $y \equiv \text{Im }\rho_{12}(\tau_A + 0) = \text{Re }\rho_{12}(\tau_A - 0)$ are given by Eqs. (6) and (7). This expression considerably simplifies for weak dephasing, $\gamma_A \tau_A \ll 1$ and $\gamma_f \ll H$; then,

$$\delta_B = \frac{1}{2}\sin[\Omega \tau + \arcsin(2\rho_{11}(\tau_A) - 1)]\exp\left(-\frac{\gamma_f \tau}{2}\right). \quad (10)$$

In contrast to Eq. (8), now the phase of oscillations $\delta_B(\tau)$ depends on the result Q_A of the first measurement, while the amplitude is maximum possible and independent of Q_A. This fact is very important since it *proves* that after the first measurement (by an ideal detector) the *qubit remains in a pure state* for any result Q_A. This new state depends on Q_A and is not one of the localized states as somebody could naively expect. [Notice that Eq. (8) can in principle be interpreted in terms of such "classical" localization, as indicated by its independence on η_A.] It is easy to check that the conventional equations (1) and (2) would lead to a prediction quite different from Eq. (10).

In a realistic experimental situation the assumption of strong coupling with detectors may be inapplicable. In this case the full probability distribution $P(Q_A, Q_B | \tau)$ as well as the dependence $\bar{Q}_B(Q_A, \tau)$ should be calculated numerically using Eqs. (3)–(5). The results of these calculations for $(\Delta I_A)^2/HS_A = (\Delta I_B)^2/HS_B = 1$ are shown in Figs. 3(c) and 3(d). Weak coupling as well as the nonideality of the detectors decreases the correlation between the results of the two measurements; however, for moderate values of the coupling and nonideality the correlation is still significant.

An experimental demonstration of the correlation and quantitative agreement with the results of the Bayesian formalism would prove the validity of this formalism and therefore confirm its other predictions. In particular, an important prediction for practice is the gradual qubit purification due to continuous measurement which can be useful for a quantum computer.

All quantum algorithms require the supply of "fresh" qubits with well-defined initial states. This supply is not a trivial problem since the qubit left alone for some time deteriorates due to interaction with environment. The usual idea is to use the ground state which should be eventually reached and does not deteriorate. However, to speed up the qubit initialization we need to increase the coupling with environment that should be avoided. The other possible idea is to perform the projective measurement after which the state becomes well defined. However, in a realistic case the coupling with the detector is finite, which makes projective measurements impossible. A natural solution of the problem is to tune the qubit continuously in order to overcome the dephasing due to environment and so keep the qubit "fresh."[19]

To realize such state purification the qubit is continuously measured by a weakly coupled detector and the detector signal is plugged into Eqs. (3) and (4), which allows us to monitor the evolution of the qubit density matrix. It is compared with the desired evolution and the difference is used to generate the feedback signal which controls the qubit parameters H and ε in order to reduce the difference. We have performed a Monte Carlo simulation of the qubit purification by the feedback loop in the regime of well-pronounced quantum oscillations[19] and found strong suppression of the qubit dephasing due to the environment in the case when the dephasing rate γ is comparable or weaker than the "measurement rate" $(\Delta I)^2/4S$. It is interesting to notice that even "naive" feedback [which responds to the difference between desired $\rho_{11}(t)$ and properly normalized $I(t)$] leads to some degree of purification.

In conclusion, we have proposed a Bell-type experiment which can test the predictions of the Bayesian formalism for the evolution of an individual qubit due to continuous quantum measurement. The next (and much more difficult) step is the experimental realization of the qubit purification using the quantum feedback loop.

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