

Leggett-Garg Inequality with a Kicked Quantum Pump

Andrew N. Jordan,^{1,2} Alexander N. Korotkov,³ and Markus Büttiker²

¹Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA

²Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

³Department of Electrical Engineering, University of California, Riverside, California 92521-0204, USA

(Received 28 October 2005; published 14 July 2006)

A kicked quantum nondemolition measurement is introduced, where a qubit is weakly measured by pumping current. Measurement statistics are derived for weak measurements combined with single-qubit unitary operations. These results are applied to violate a generalization of the Leggett-Garg inequality. The violation is related to the failure of the noninvasive detector assumption, and may be interpreted as either intrinsic detector backaction, or the qubit entangling the microscopic detector excitations. The results are discussed in terms of a quantum point contact kicked by a pulse generator, measuring a double quantum dot.

DOI: [10.1103/PhysRevLett.97.026805](https://doi.org/10.1103/PhysRevLett.97.026805)

PACS numbers: 73.23.-b, 03.65.Ta, 03.67.Lx

An important goal in the research of quantum phenomena in the solid state is to provide realistic tests that demonstrate quantum behavior which no analogous classical system could exhibit. The best known example of such a test is Bell's inequality (BI) [1], but in submicron sized samples the BI serves primarily as a test of entanglement rather than ruling out local hidden variable theories [2]. The seminal work of Leggett and Garg [3] provides another inequality involving only one quantum variable together with a set of projective measurements. This test demonstrates that the predictions of quantum mechanics are incompatible with the philosophical assumptions of macrorealism and a noninvasive detector. An interesting parallel between the two inequalities is that the role of hidden variables in the BI is played by trajectories in the Leggett-Garg inequality (LGI). The belief that the quantum system really takes a definite classical trajectory between two points (chosen from an arbitrary probability distribution) may be disproved with the LGI.

This Letter proposes a generalization of the LGI using quantum nondemolition (QND) measurements weakly measuring the quantum state by pumping current. Weak measurements, in contrast to projective measurements, obtain partial information about the state from an inherently noisy output, so wave function collapse happens continuously. In the solid state, the typically weak coupling between system and detector implies that weak measurements are the norm. A generic problem arising in making a projective measurement out of many weak measurements is that the quantum system has its own Hamiltonian dynamics that effectively rotates the measurement basis, preventing projective measurement. The way around this problem is with QND measurements.

Kicked QND.—The scheme we employ is that of kicked QND measurements, introduced by Braginsky *et al.* and Thorne *et al.* [4] for the harmonic oscillator. In Ref. [5], the idea is introduced for the two-state system by two of the authors by making an analogy to a cat playing with a string

that moves in a circle: rather than chasing the string [6], the cat sits in one spot waiting for the string to come to it, and only then bats at it [7]. The motion in a circle comes from the evolution of a two-state system, where $H = \Delta\sigma_x/2$ is the qubit Hamiltonian, and Δ is the tunnel coupling of the symmetric qubit which defines the Rabi oscillation period, $\tau_q = 2\pi/\Delta$. Although kicked measurement may be realized in a wide variety of systems, we will focus on a quantum point contact (QPC) kicked by a voltage pulse generator. This detector measures σ_z , the position of the electron in a double quantum dot (DD) charge qubit as depicted in Fig. 1. The QPC detector is growing in experimental importance [8–11], and Hayashi *et al.* [8] applied rectangular voltage pulses similar to the ones we consider.

An experimentally appealing variation on the idea of kicked measurement is illustrated in Fig. 1, where a sequence of voltage kicks of duration $\tau_v \ll \tau_q$ is applied to the QPC, alternating in sign every half oscillation period. The parameters of the measurement process with an ideal

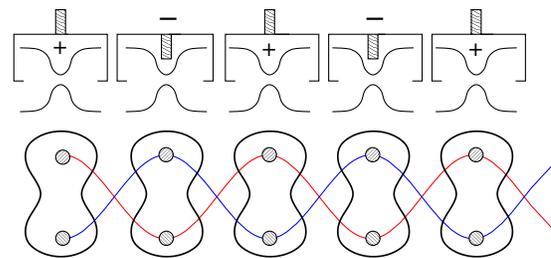


FIG. 1 (color online). Visualization of the kicked QND measurement. A voltage pulse is applied to the QPC, followed by a quiet period of zero voltage bias, lasting for a Rabi oscillation period, followed by another pulse, and so on. The up or down variation is depicted, where the kicks come every half period, and the sign of the voltage pulse alternates with every kick. Readout of the coherent superposition of trajectories (red or blue) occurs by measuring the sign of the current, and corresponds to an elementary quantum pump.

QPC detector [12–15] are specified by the currents, $I_{1,2}$, that correspond to the different positions of the electron in the DD, and the detector shot noise power $S_I = eI(1 - T)$ (where T is the transparency). The typical time needed to distinguish the qubit signal from the background noise is the measurement time $T_M = 4S_I/(I_1 - I_2)^2$. If the qubit starts in state $|1\rangle$ (or $|2\rangle$), so the kicks are in (or out of) phase with the coherent oscillations, then the physical current produced by the QPC is $(I_1 - I_2)(\tau_V/\tau_q) > 0$ [or $(I_2 - I_1)(\tau_V/\tau_q) < 0$]. Thus, by simply determining the *sign* of the current, a measurement of the quantum state can be made. Besides being a phase detector, this apparatus is also an elementary quantum pump [16], where the kicks provide one time-changing parameter of zero average, and the intrinsic quantum dynamics of the qubit provide the other changing parameter of zero average that nevertheless causes a net flow of current [17].

To characterize the result of each measurement kick, dimensionless variables are introduced by defining the current origin at $I_0 = (I_1 + I_2)/2$ and scaling the current per pulse as $I - I_0 = x(I_1 - I_2)/2$, so $I_{1,2}$ are mapped onto $x = \pm 1$ (positive or negative current in the pumping proposal). The weak (static) coupling between QPC and DD implies that $\tau_V \ll T_M$. In these units, we take x to be normally distributed with variance $D = T_M/\tau_V \gg 1$. The typical number of kicks needed to distinguish the two states is D kicks. The measurement result I after N kicks is $I = (1/N) \sum_{n=1}^N x_n$, and we seek the conditional probability distribution $P(I, N|\rho)$ of measuring the result I , starting with a given density operator ρ prepared before the first kick. The probability of measuring the result x_n after one kick is determined by the state of the qubit just before the measurement, and is given by

$$P(x_n) = \rho_{11}^{(n)} P_1(x_n) + \rho_{22}^{(n)} P_2(x_n), \quad (1)$$

where ρ_{ij} are the elements of the density matrix in the z basis, and the notation $P_j(x_n)$ is introduced for the $j = 1, 2$ distributions of the n th kick. These two distributions describe the detector output for the n th kick, if the electron resides only on one of the two dots. The density matrix of the qubit is updated based on information obtained from the measurement that just occurred. This is done with the quantum Bayes rule [13] that defines a nonunitary quantum map [18]:

$$\begin{aligned} \rho_{11}^{(n+1)} &= 1 - \rho_{22}^{(n+1)} = \frac{\rho_{11}^{(n)} P_1(x_n)}{\rho_{11}^{(n)} P_1(x_n) + \rho_{22}^{(n)} P_2(x_n)}, \\ \rho_{12}^{(n+1)} &= [\rho_{21}^{(n+1)}]^* = \rho_{12}^{(n)} \sqrt{\rho_{11}^{(n+1)} \rho_{22}^{(n+1)} / \rho_{11}^{(n)} \rho_{22}^{(n)}}. \end{aligned} \quad (2)$$

The quantum Bayesian formalism provides additional insight into the quantum detection process, and is well suited to analyze kicked QND measurements. Equivalence with the quantum trajectories approach is shown in Ref. [19] [see also Ref. [20]]. The advantage of QND measurement in the quantum Bayesian approach follows from using

Eqs. (2) to calculate the probability distribution $P(I, N|\rho)$ of current I after N kicks, starting with the density matrix ρ :

$$P(I, N|\rho) = \rho_{11} P(I, N|1) + \rho_{22} P(I, N|2), \quad (3)$$

where ρ_{11}, ρ_{22} are the diagonal matrix elements of the original density matrix, and the functions $P(I, N|j)$, $j = 1, 2$ are defined as Gaussian probability distributions of the current, with average $(-1)^{j-1}$, and variance D/N . These two distributions describe the total detector output for N kicks, if the electron resides only on one of the two dots. In Eq. (3), the N weak measurements simply compose to give one N -times stronger measurement. As N is increased, the distributions limit to delta functions giving either $I = 1, -1$ with probability ρ_{11}, ρ_{22} , respectively. A one-sigma confidence is obtained when $N = D$ (see above). The QND measurement output only involves the diagonal density matrix elements, so the current output behaves *exactly* as if it were simply collecting information about a classical bit from a noisy process. In spite of this fact, the Eqs. (2) allow us to deduce the DD electron's density matrix prepared after the N measurements from our knowledge of I :

$$\rho' = \frac{1}{\rho_{11} e^\gamma + \rho_{22} e^{-\gamma}} \begin{pmatrix} \rho_{11} e^\gamma & \rho_{12} \\ \rho_{12}^* & \rho_{22} e^{-\gamma} \end{pmatrix}, \quad (4)$$

where $\gamma = IN/D$ is the rescaled measurement result. The conditional quantum dynamics of Eq. (4) is illustrated in Fig. 2 for all pure states, where (x, y, z) are coordinates on the Bloch sphere. The x and y behavior follows from z , which is in turn conditioned on the detector output I , so the sphere is colored according to the conditional evolution of z . If γ is positive (negative), then states are “attracted” toward the North (South) pole. As γ grows increasingly positive or negative, we become more confident which state the qubit has collapsed to, so the sphere is more and more red ($|1\rangle$) or blue ($|2\rangle$), but notice that this depends on the initial state. The conditional evolution of several representative states is indicated with black arrows.

Generalized LGI.—While the point of the kicked QND proposal was to effectively turn off the qubit unitary evolution while the measurement is taking place, kicked measurement provides a simple way of generating a single-qubit rotation: waiting some fraction r of a Rabi oscillation between kicks defines a phase shift $\phi = 2\pi r$ on the DD qubit. Consider now an experiment, comprised of N_1 kicks, followed by a single-qubit unitary operation $\mathbf{U}(\phi)$, followed by N_2 kicks. The measurement results are defined as $I_1 = (1/N_1) \sum_{n=1}^{N_1} x_n$, $I_2 = (1/N_2) \sum_{n=N_1+1}^{N_1+N_2} x_n$. We seek the normalized probability distribution $P(I_1; I_2)$ of finding current I_1 after N_1 kicks, and I_2 after N_2 subsequent kicks. This distribution may also be interpreted as a “joint counting statistics.” After the first N_1 kicks, the measured current I_1 will occur with a probability density given by (3), and prepares a post-measurement density matrix ρ' (4). The subsequent unitary operation rotates this density matrix, $\rho^{\text{new}} = \mathbf{U}\rho'\mathbf{U}^\dagger$. The following set of

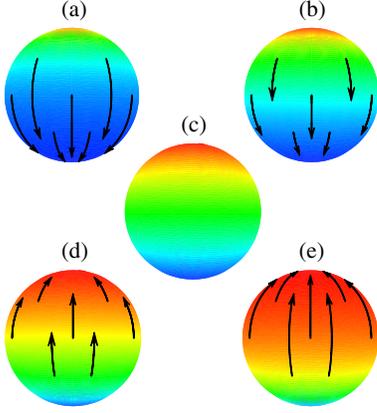


FIG. 2 (color online). The conditional evolution of all pure states under kicked QND measurement is represented on the Bloch sphere. From (a)–(e), the rescaled result of the measurement is $\gamma = (-1, -0.5, 0, 0.5, 1)$, respectively. As the detector obtains more information, we can with greater statistical certainty distinguish the post-measurement quantum state, so the Bloch sphere is more and more red ($|1\rangle$) or blue ($|2\rangle$), depending on the value of γ measured (color is assigned according to $\rho_{11} - \rho_{22}$ of the final state). The conditional evolution of several representative states is also indicated with black arrows. The North, South pole represent the states $|1\rangle, |2\rangle$.

N_2 kicks start with ρ^{new} and continue to measure in the z basis as before. Equation (3) may be applied again to obtain

$$P(I_1; I_2) = [\rho_{11}P(I_1, N_1|1) + \rho_{22}P(I_1, N_1|2)] \\ \times [\rho_{11}^{\text{new}}P(I_2, N_2|1) + \rho_{22}^{\text{new}}P(I_2, N_2|2)], \quad (5)$$

where the new density matrix elements are

$$\rho_{11}^{\text{new}} = [\cos^2(\phi/2)\rho_{11}e^\gamma + \sin^2(\phi/2)\rho_{22}e^{-\gamma} \\ - \sin\phi\text{Im}\rho_{12}]/(\rho_{11}e^\gamma + \rho_{22}e^{-\gamma}), \\ \rho_{12}^{\text{new}} = [\text{Re}\rho_{12} + (i/2)\sin\phi(\rho_{11}e^\gamma - \rho_{22}e^{-\gamma}) \\ + i\cos\phi\text{Im}\rho_{12}]/(\rho_{11}e^\gamma + \rho_{22}e^{-\gamma}). \quad (6)$$

Note that the outcome of the first N_1 kicks, I_1 , appears in the expression involving the second set of kicks, so the distribution does not factorize. It is straightforward to generalize the results (5) and (6) to any number of dislocations in the pulse sequence, each of which has a phase shift.

We now demonstrate how these results may be used to violate a generalized LGI. A generalized LGI has been discussed by Ruskov *et al.* [21] for the current correlations and the spectral noise peak generated by a qubit. Our setup has the advantage of full tunability of phase shifts and measurement strength and thus permits a LGI test over a wide range of parameters. The original proposal [3] derived an inequality involving correlation functions from three experiments, each consisting of two projective measurements done at specified times starting from the same initial condition. The beauty of weak measurements is that

the inequality may be violated with only *one* set of measurements together with statistical averaging. To derive the weak measurement generalization of the LGI, consider three kicks surrounding two phase shifts $\phi_{1,2}$. Define the correlation function $B = S_{12} + S_{23} - S_{13}$, where $S_{nm} = \langle I_n I_m \rangle$; $n, m = 1, 2, 3$. The assumptions of “macrorealism and a noninvasive detector” [3] are introduced with a white, additive, noise model of the detection process (characteristic of QPC electron transport). The measured result I_n can be decomposed into a system signal and detector noise contribution, $I_n = C_n + \xi_n$. The signal C_n describes the DD state at measurement n , while the detector noise term, ξ_n , is white Gaussian noise (discussed previously), of zero average and variance $\langle \xi_n^2 \rangle = D/N_n$. The signal contribution C_n may be endowed with classical hidden variables $\{\lambda\}$, chosen from any probability distribution. The signal can now change arbitrarily between measurements, but only in a bounded way, $-1 \leq C_n(\{\lambda\}) \leq 1$. The noninvasive detector assumption implies that the detector noise does not affect the measured system in the past or the future, so $\langle \xi_n C_m(\{\lambda\}) \rangle = 0$, for any n, m . These assumptions imply that $S_{nm} = \langle C_n(\{\lambda\})C_m(\{\lambda\}) \rangle_r$, where $\langle \dots \rangle_r$ denotes further averaging over the hidden variables $\{\lambda\}$, as well as over realizations or initial conditions. From the bound on each of the signal contributions, it is straightforward to show that $B \leq 1$, concluding the weak measurement generalization of the LGI.

Starting with any DD electron state, we find quantum mechanically from the generalization of (5) and (6) that

$$B = \cos\phi_1 + \cos\phi_2 - \cos\phi_1 \cos\phi_2 \\ + \sin\phi_1 \sin\phi_2 \exp(-N_2/2D), \quad (7)$$

for an arbitrary number of kicks N_1, N_2, N_3 made around the phase shifts. The first three terms in (7) cannot violate the LGI, and it is the last term that is responsible for the violation. In the weak measurement limit, $N_2 \ll D$, the Bell-like parameter takes the form, $B \approx \cos\phi_1 + \cos\phi_2 - \cos(\phi_1 + \phi_2)$, and is maximally violated for $\phi_1 = \phi_2 = \pi/3$ so $B = 3/2$. The physical interpretation for the suppression of the critical term in (7) is the following: if measurement 2 had not been made, the system travels in a coherent superposition of trajectories (red and blue in Fig. 1) between $1 \rightarrow 3$. The intermediate measurement gives the necessary third point, but also yields information (at a rate D^{-1} per kick) about which trajectory the quantum system “really” took [22]. This information manifests itself in making it harder to violate the LGI. In the projective measurement limit, $N_2 \gg D$, we are statistically confident which trajectory the system took, and it becomes impossible to violate the LGI.

An alternative picture may be seen by reconsidering three incident electron groups on the QPC, spaced by a phase shift $\phi_{1,2}$ on the DD qubit. Rather than directly project the QPC electrons after each passes (as is necessary for the quantum Bayesian approach), we use a well-known property of quantum circuits, that the predictions of quan-

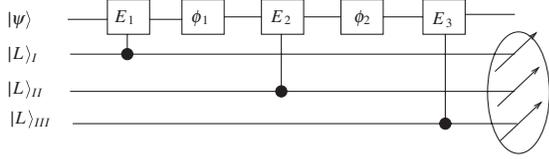


FIG. 3. Quantum circuit analysis of the generalized LGI: three sets of initially unentangled QPC electrons in the left scattering states $|L\rangle_{I,II,III}$ are entangled with the DD electron state $|\psi\rangle$ with operations $E_{1,2,3}$. The DD electron undergoes two different phase shifts $\phi_{1,2}$ between the passing electron bunches. Projective measurements are made on the QPC electrons in the current collector, and are delayed until the end of all unitary operations in the entanglement picture.

tum mechanics are identical if the projective measurements are delayed to the end of all unitary operations. Then the above procedure is identical to the quantum circuit drawn in Fig. 3, where each initial left scattering state $|L\rangle$ encodes many transport electrons. Rather than attribute the correlations (7) to detector backaction, another interpretation is to see the above procedure as the DD qubit effectively creating entanglement between the transport electrons. For simplicity, we consider the $1/2$ transparency point [see Ref. [14] for a more general discussion]. Following the detector treatment in Refs. [12,15], the transmission (reflection) amplitudes $t_{1,2}(r_{2,1}) = \sqrt{1/2} \pm \epsilon$ of the two scattering matrices corresponding to the two positions of the DD electron are expanded in the detector sensitivity ϵ to second order. If the qubit is in state $|1\rangle$, $|2\rangle$ then the outgoing detector scattering states are $|\chi_{1,2}\rangle = |s\rangle(1 - \epsilon^2/2) \pm \epsilon|a\rangle$, where $|s\rangle$, $|a\rangle = (|L\rangle \pm |R\rangle)/\sqrt{2}$ are combinations of the left or right scattering states. Before measurements are made, the state is given by

$$\begin{aligned}
 |\Psi\rangle = & \mathbf{U}(\phi_1)\mathbf{U}(\phi_2)|\psi\rangle|0\rangle(1 - 3\epsilon^2/2) + \epsilon|\psi'\rangle \\
 & \times [\cos\phi_+(|1\rangle + |3\rangle) + \cos\phi_-|2\rangle] + \epsilon|\psi''\rangle \\
 & \times [\sin\phi_+(|3\rangle - |1\rangle) + \sin\phi_-|2\rangle] + \epsilon^2|\psi\rangle \\
 & \times [\cos\phi_-(|1'\rangle + |3'\rangle) + \cos\phi_+|2'\rangle] + \epsilon^2|\psi'''\rangle \\
 & \times [\sin\phi_-(|3'\rangle - |1'\rangle) + \sin\phi_+|2'\rangle], \quad (8)
 \end{aligned}$$

where $|0\rangle = |sss\rangle$, $|1, 2, 3\rangle = |ass, sas, ssa\rangle$, $|1', 2', 3'\rangle = |saa, asa, aas\rangle$, and $|klm\rangle \equiv |k\rangle_I|l\rangle_{II}|m\rangle_{III}$. The DD states are defined as $|\psi','','''\rangle = \alpha|1\rangle + \beta|2\rangle$, $-\alpha|1\rangle + \beta|2\rangle$, $i(\beta|1\rangle - \alpha|2\rangle)$, $i(\beta|1\rangle + \alpha|2\rangle)$, and $\phi_{\pm} = (\phi_1 \pm \phi_2)/2$. The first dominant term is separable and alone can produce no correlations, while the remaining terms are entangled. Using projection operators on the right scattering states in order to calculate current correlators recovers the weak measurement limit of (7).

Conclusions.—We have proposed a kicked qubit readout scheme that is both a quantum nondemolition measurement and a quantum pump. Kicked measurements com-

bined with unitary operations were used to formulate and violate a weak measurement generalization of Leggett and Garg's inequality. The fact that our proposal uses one set of pulses to accomplish both weak measurements and phase shifts provides an important advantage for an experiment aimed at violating the LG inequality.

We thank L. P. Kouwenhoven, E. V. Sukhorukov, R. Ruskov, and B. Trauzettel for helpful discussions. This work was supported by MaNEP and the SNF (A. N. J. and M. B.) and the NSA/ARDA/ARO (A. N. K.).

-
- [1] J. S. Bell, *Physics* (Long Island City, N.Y.) **1**, 195 (1964).
 - [2] P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, *Phys. Rev. Lett.* **92**, 026805 (2004); A. V. Lebedev, G. B. Lesovik, and G. Blatter, *Phys. Rev. B* **71**, 045306 (2005); C. W. J. Beenakker, *cond-mat/0508488*.
 - [3] A. J. Leggett and A. Garg, *Phys. Rev. Lett.* **54**, 857 (1985); A. J. Leggett, *J. Phys. Condens. Matter* **14**, R415 (2002).
 - [4] V. B. Braginsky, Yu. I. Vorontsov, and F. Ya. Khalili, *JETP Lett.* **27**, 276 (1978); K. S. Thorne *et al.*, *Phys. Rev. Lett.* **40**, 667 (1978).
 - [5] A. N. Jordan and M. Büttiker, *Phys. Rev. B* **71**, 125333 (2005).
 - [6] D. V. Averin, *Phys. Rev. Lett.* **88**, 207901 (2002).
 - [7] R. Ruskov, K. Schwab, and A. N. Korotkov, *Phys. Rev. B* **71**, 235407 (2005).
 - [8] T. Hayashi *et al.*, *Phys. Rev. Lett.* **91**, 226804 (2003).
 - [9] J. M. Elzerman *et al.*, *Phys. Rev. B* **67**, R161308 (2003).
 - [10] J. R. Petta *et al.*, *Phys. Rev. Lett.* **93**, 186802 (2004).
 - [11] A. K. Hüttel *et al.*, *Phys. Rev. B* **72**, R081310 (2005).
 - [12] S. A. Gurvitz, *Phys. Rev. B* **56**, 15215 (1997).
 - [13] A. N. Korotkov, *Phys. Rev. B* **60**, 5737 (1999); **63**, 115403 (2001).
 - [14] S. Pilgram and M. Büttiker, *Phys. Rev. Lett.* **89**, 200401 (2002); A. A. Clerk, S. M. Girvin, and A. D. Stone, *Phys. Rev. B* **67**, 165324 (2003).
 - [15] D. V. Averin and E. V. Sukhorukov, *Phys. Rev. Lett.* **95**, 126803 (2005).
 - [16] P. W. Brouwer, *Phys. Rev. B* **58**, R10135 (1998); J. E. Avron *et al.*, *Phys. Rev. B* **62**, R10618 (2000); M. Moskalets and M. Büttiker, *Phys. Rev. B* **70**, 245305 (2004).
 - [17] Eventually random bit-flip errors occur if the kicks are imperfect delta functions, causing the pumped current to vanish asymptotically [5]. A sustainable quantum pump may be built with a simple feedback loop; R. Ruskov and A. N. Korotkov, *Phys. Rev. B* **66**, R041401 (2002).
 - [18] This is equivalent to weak entanglement of the QPC electrons with the DD qubit, with projective measurement on the QPC electrons, executing a POVM on the DD qubit; A. N. Jordan and A. N. Korotkov, *cond-mat/0606676*.
 - [19] H.-S. Goan *et al.*, *Phys. Rev. B* **63**, 125326 (2001).
 - [20] C. M. Caves *et al.*, *Phys. Rev. A* **65**, 022305 (2002).
 - [21] R. Ruskov, A. N. Korotkov, and A. Mizel, *Phys. Rev. Lett.* **96**, 200404 (2006).
 - [22] E. Buks *et al.*, *Nature* (London) **391**, 871 (1998).