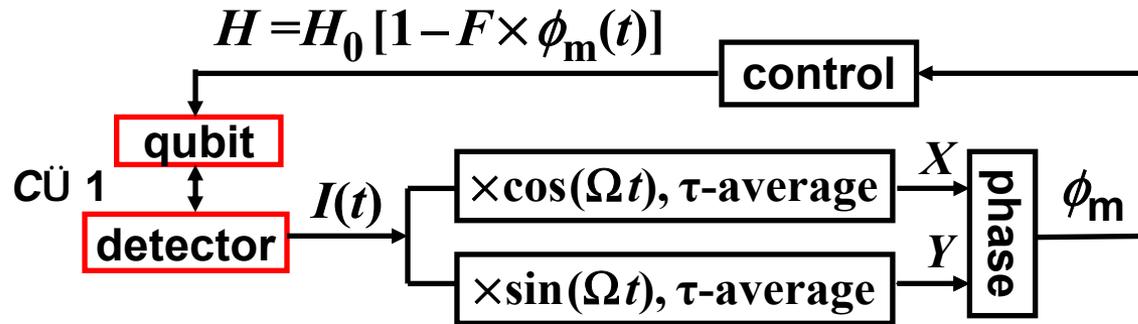


Simple quantum feedback of a solid-state qubit

Alexander Korotkov

University of California, Riverside



Feedback loop maintains Rabi oscillations for infinitely long time

Advantage: simplicity and relatively narrow bandwidth

Anticipated problem: not much information in quadratures

(surprisingly, feedback loop works much better than anticipated!)

Support:



cond-mat/0404696



Simple quantum feedback of a solid-state qubit

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We propose an experiment on quantum feedback control of a solid-state qubit, which is almost within the reach of the present-day technology. Similar to the earlier proposal, the feedback loop is used to maintain the coherent oscillations in a qubit for an arbitrary long time; however, this is done in a significantly simpler way, which requires much smaller bandwidth of the control circuitry.

The main idea is to use the quadrature components of the noisy detector current to monitor approximately the phase of qubit oscillations. The price for simplicity is a less-than-ideal operation: the fidelity is limited by about 95%. The feedback loop operation can be experimentally verified by appearance of a positive in-phase component of the detector current relative to an external oscillating signal used for synchronization.

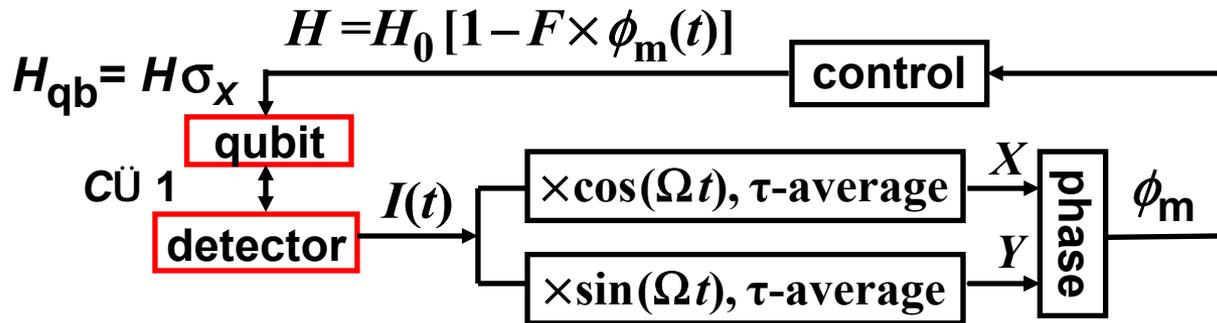
Support:



cond-mat/0404696



Simple quantum feedback of a solid-state qubit



We want to maintain coherent (Rabi) oscillations for arbitrary long time,
 $\rho_{11} - \rho_{22} = \cos(\Omega t)$, $\rho_{12} = i \sin(\Omega t)/2$

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations (a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^t [I(t') - I_0] \cos(\Omega t') \exp[-(t - t')/\tau] dt$$

$$Y(t) = \int_{-\infty}^t [I(t') - I_0] \sin(\Omega t') \exp[-(t - t')/\tau] dt$$

$$\phi_m = -\arctan(Y / X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

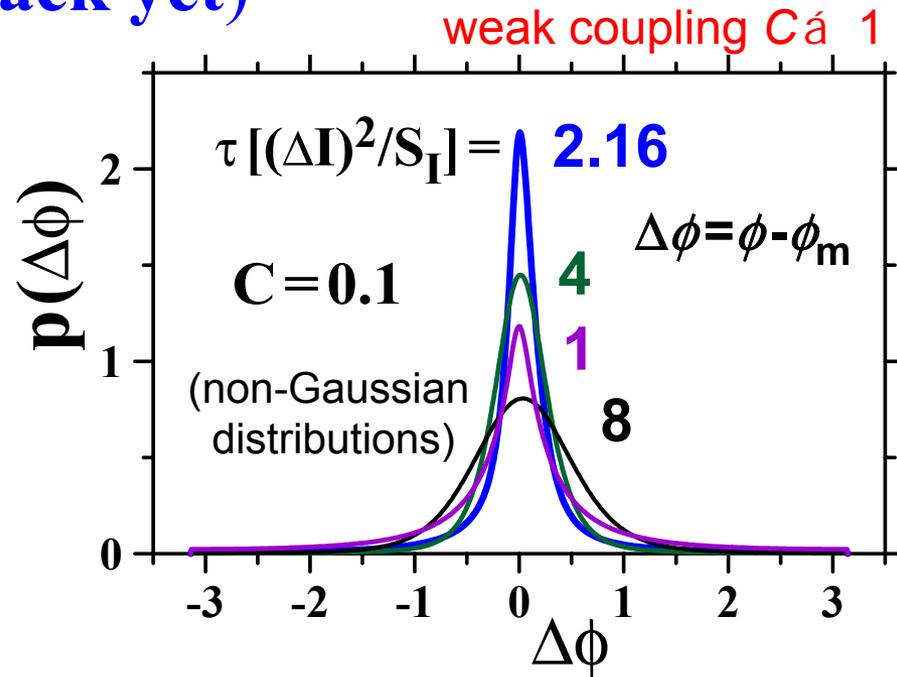
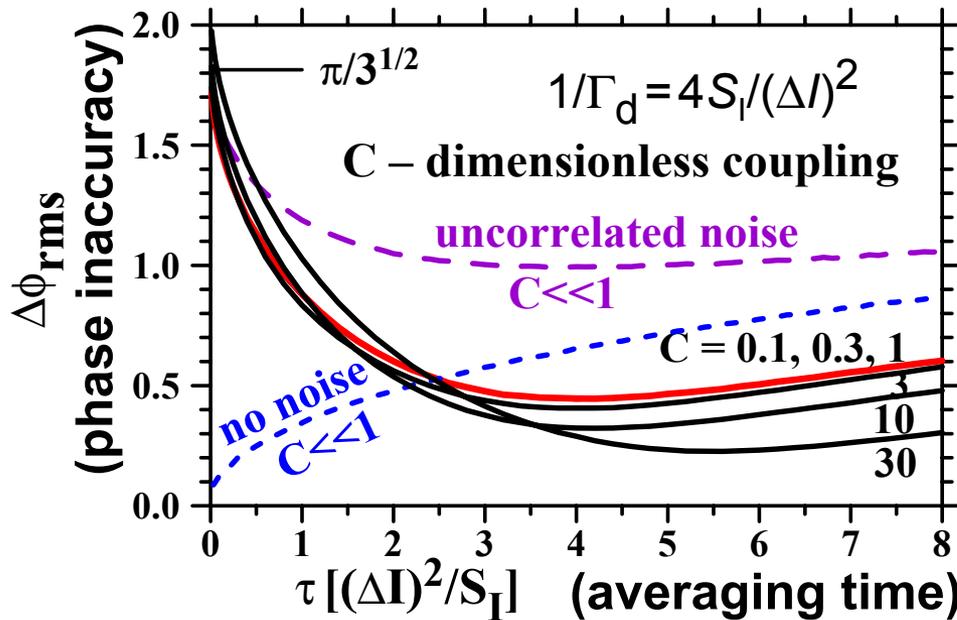
Advantage: simplicity and relatively narrow bandwidth ($1/\tau \sim \Gamma_d \ll \Omega$)

Anticipated problem: without feedback the spectral peak-to-pedestal ratio < 4 , therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)



Accuracy of phase monitoring via quadratures (no feedback yet)



Noise improves the monitoring accuracy!
(purely quantum effect, “reality follows observations”)

Best approximation
 $\langle X^2 + Y^2 \rangle = (S_I/\Delta I)^2$
 $(2/5)(41^{1/2} - 1) \approx 2.16$

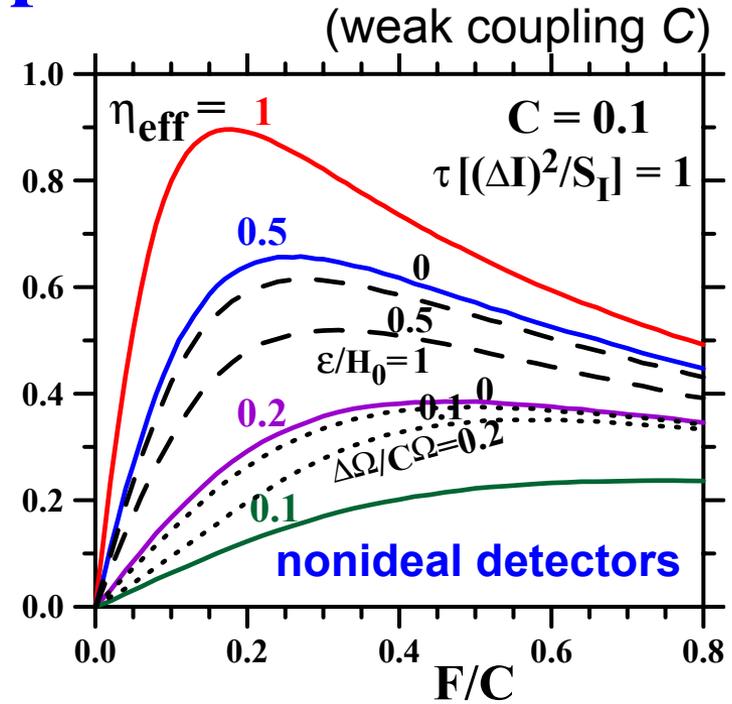
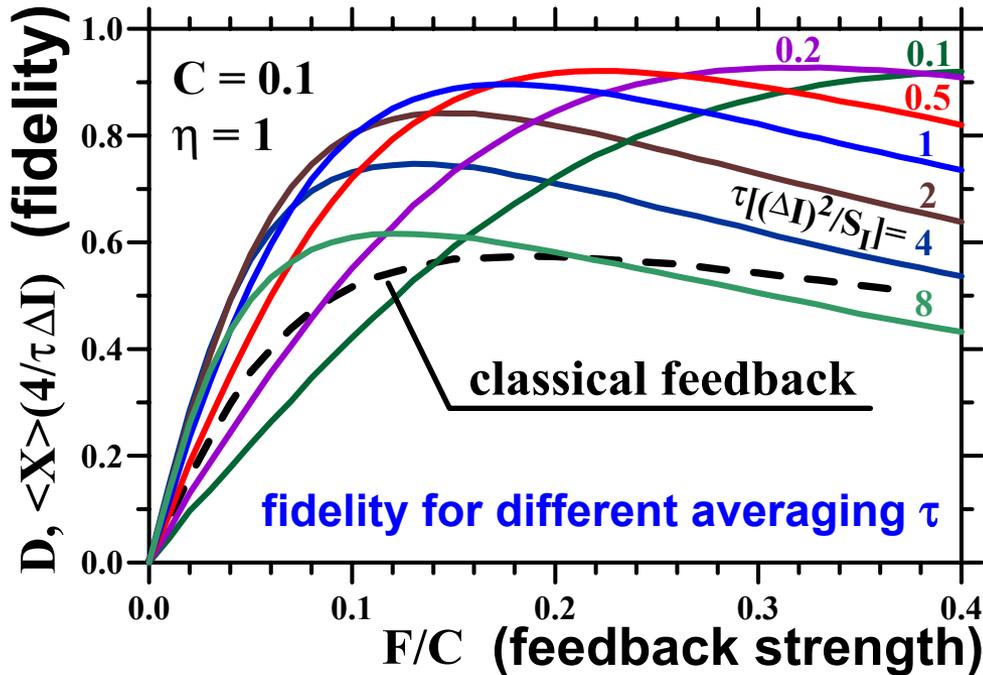
$$d\phi/dt = -[I(t) - I_0] \sin(\Omega t + \phi) (\Delta I / S_I) \quad (\text{actual phase shift, ideal detector})$$

$$d\phi_m/dt = -[I(t) - I_0] \sin(\Omega t + \phi_m) / (X^2 + Y^2)^{1/2} \quad (\text{observed phase shift})$$

Noise enters the actual and observed phase evolution in a similar way



Quantum feedback performance



- Fidelity F up to $\sim 95\%$ achievable ($D \sim 90\%$)
- Natural, practically classical feedback setup
- Averaging $\tau \sim 1/\Gamma \gg 1/\Omega$ (narrow bandwidth!)
- Detector efficiency (ideality) $\eta \leq 0.1$ still OK
- Robust to asymmetry ϵ and frequency shift $\Delta \Omega$
- Very simple verification – just positive in-phase quadrature $\langle X \rangle$

$$D \equiv 2F - 1$$

$$F \equiv \langle \text{Tr } \rho(t) \rho_{\text{des}}(t) \rangle$$

$$D \simeq \langle X \rangle (4 / \tau \Delta I)$$

X – in-phase quadrature of the detector current

Simple experiment?!



Quantum feedback in optics

Recent experiment: Science 304, 270 (2004)

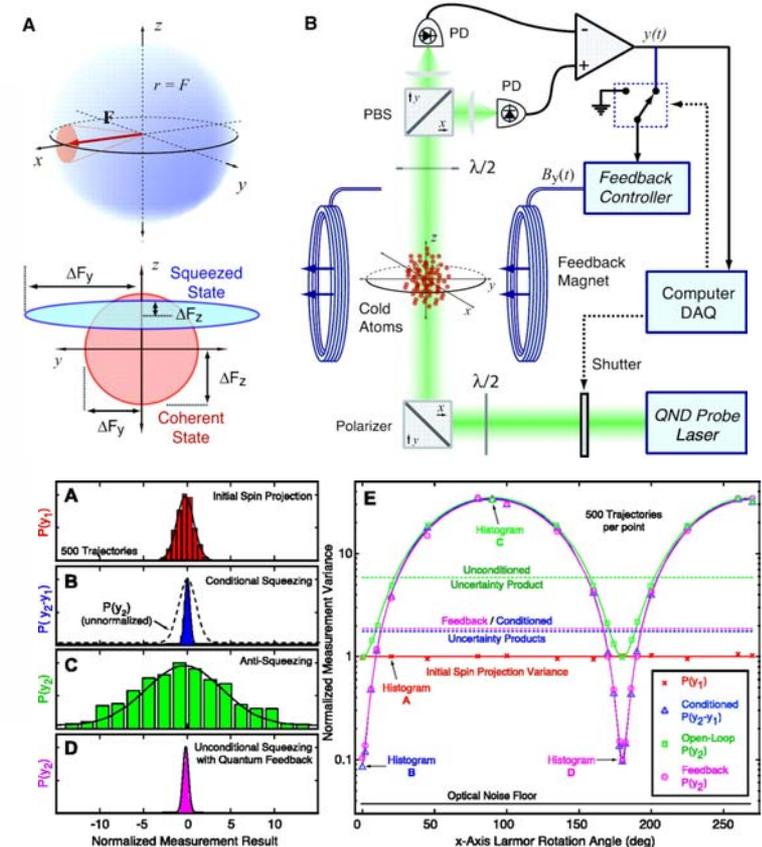
Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.

First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (1993)



**No experimental attempts of quantum feedback in solid-state yet
(even theory is still considered controversial)**

Experiments soon?



Conclusions

(simple quantum feedback of a solid-state qubit)

- **Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations**
- **Much simpler realization than for quantum feedback of Ruskov-Korotkov, Phys. Rev. B 66, 041401(R) (2002)**
- **Price for simplicity is a less-than-ideal operation (fidelity is limited by ~95%)**
- **Feedback performance is much better than expected**
- **Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)**



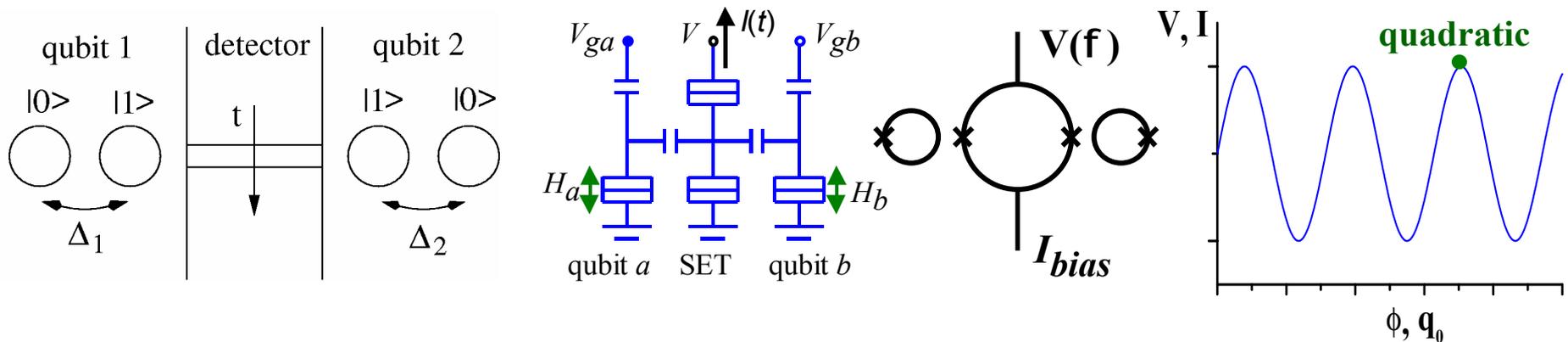
Quadratic quantum measurements

W. Mao,¹ D. Averin,¹ R. Ruskov,² and A. Korotkov²

¹*Stony Brook University* and ²*UC Riverside*

Phys. Rev. Lett. **93**, 056803 (2004)

Studied setup: two qubits and detector



Qubits can be made 100% entangled by measurement

This is done in an easier way than using a linear detector,
as in Ruskov-Korotkov, PRB 67, 241305(R) (2003)

Support:



ARDA



Quadratic quantum measurements

W. Mao,¹ D. Averin,¹ R. Ruskov,² and A. Korotkov²

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Phys. Rev. Lett. 93, 056803 (2004)

We develop a theory of quadratic quantum measurements by a mesoscopic detector. It is shown that the quadratic measurements should have non-trivial quantum information properties, providing, for instance, a simple way of entangling two non-interacting qubits. We also calculate output spectrum of a detector with both linear and quadratic response, continuously monitoring two qubits.

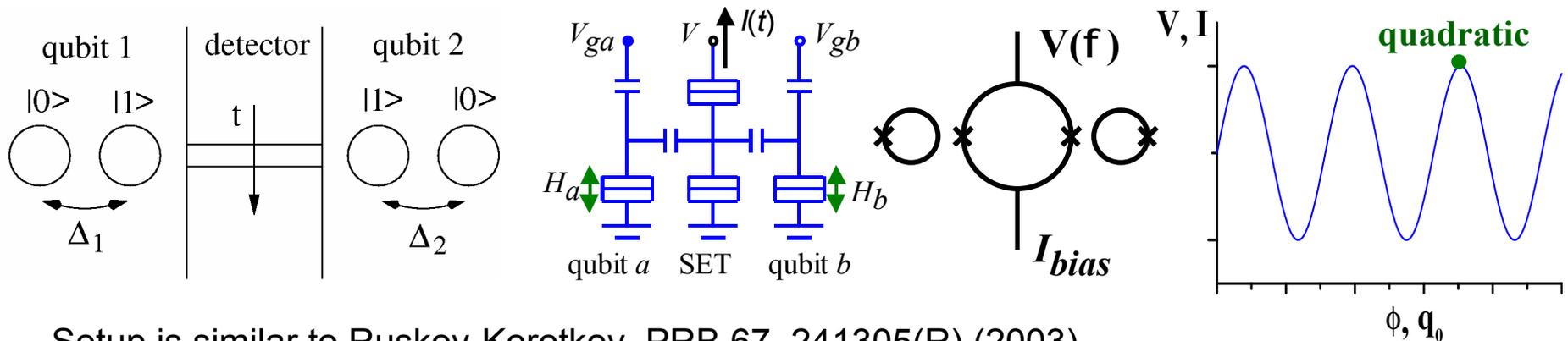
Support:



ARDA

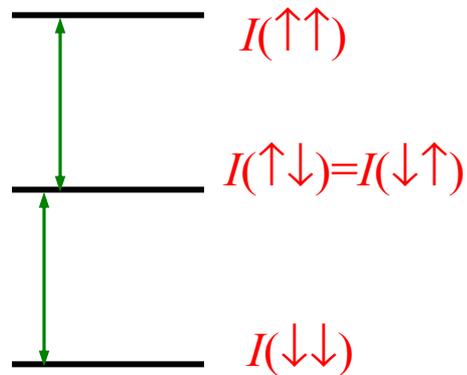


Studied setup: two qubits and detector

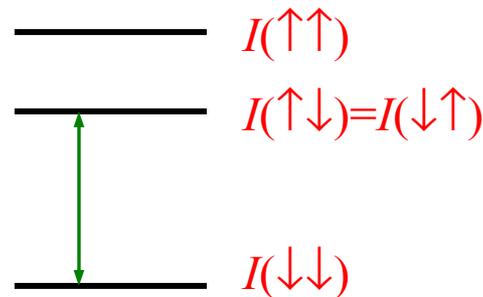


Setup is similar to Ruskov-Korotkov, PRB 67, 241305(R) (2003), but a nonlinear (instead of a linear) detector is considered

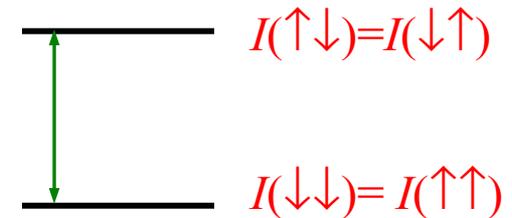
Linear detector



Nonlinear detector



Quadratic detector



Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)



Bayesian formalism for a nonlinear detector

$$H = H_{QBs} + H_{DET} + \sum_{j=1,2} [t(\{\sigma_z^j\})\xi + t^\dagger(\{\sigma_z^j\})\xi^\dagger]$$

$$t(x) = t_0 + \delta_1 \sigma_z^1 + \delta_2 \sigma_z^2 + \lambda \sigma_z^1 \sigma_z^2 \quad \delta_j = 0 \Rightarrow \text{quadratic detector}$$

Assumed: 1) weak tunneling in the detector, 2) large detector voltage (fast detector dynamics, and 3) weak response.

The model describes an ideal detector (no extra noises).

Recipe: Coupled detector-qubits evolution and frequent collapses of the number n of electrons passed through the detector

Two-qubit evolution (Ito form):

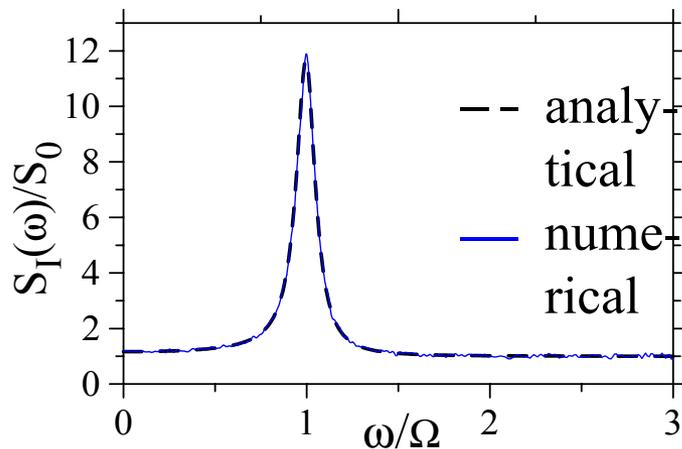
$$\frac{d}{dt} \rho_{kl} = -i[H_{QBs}, \rho]_{kl} + [I(t) - \langle I \rangle] \left[\frac{1}{S_0} (I_k + I_l - 2\langle I \rangle) - i\varphi_{kl} \right] \rho_{kl} - \gamma_{kl} \rho_{kl}$$

$$\gamma_{kl} = (1/2)(\Gamma_+ + \Gamma_-) [(|t_k| - |t_l|)^2 + \varphi_{kl}^2 |t_0|^2], \quad \varphi_{kl} = \arg(t_k t_l^*)$$

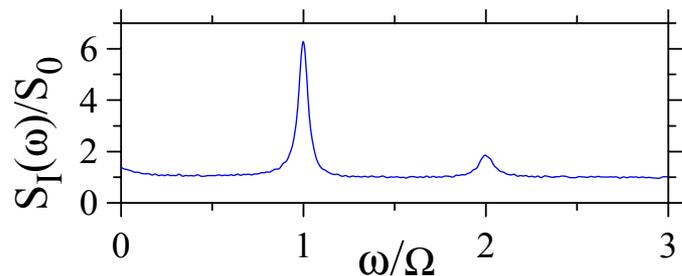
$$\langle I \rangle = \sum_j \rho_{jj} I_j, \quad I_k = (\Gamma_+ - \Gamma_-) |t_k|^2, \quad S_0 = 2(\Gamma_+ + \Gamma_-) |t_0|^2$$



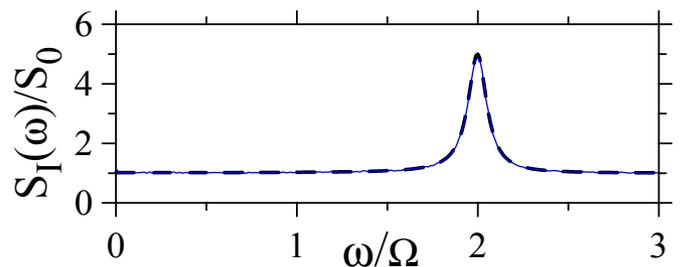
Linear detector



Nonlinear detector



Quadratic detector



1= $\uparrow\uparrow$, 2= $\uparrow\downarrow$, 3= $\downarrow\uparrow$, 4= $\downarrow\downarrow$

Two-qubit detection

(oscillatory subspace)

$$S_I(\omega) = S_0 + \frac{8}{3} \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

$$\Gamma = \eta^{-1} (\Delta I)^2 / 4S_0, \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω , peak/noise = $(32/3)\eta$
(Ω is the Rabi frequency) (Ruskov-Korotkov, 2002)

Extra spectral peaks at 2Ω and 0

(analytical formula for weak coupling case)

$$S_I(\omega) = S_0 + \frac{4\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2 \omega^2}$$

$$(\Delta I = I_{23} - I_{14}, I_1 = I_4, I_2 = I_3)$$

Peak only at 2Ω , peak/noise = 4η

Mao, Averin, Ruskov, Korotkov, 2004

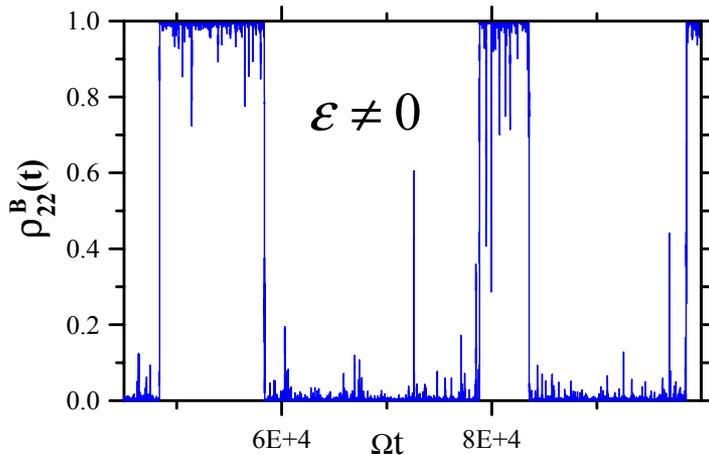
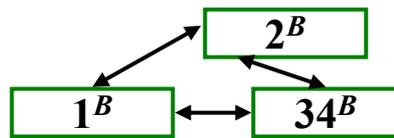
University of California, Riverside



Two-qubit quadratic detection: scenarios and switching

Three scenarios:
(distinguishable by average current)

- 1) collapse into $|\uparrow\downarrow - \downarrow\uparrow\rangle = |1^B\rangle$, current $I_{\mathcal{A}E}$, flat spectrum
- 2) collapse into $|\uparrow\uparrow - \downarrow\downarrow\rangle = |2^B\rangle$, current $I_{\mathcal{A}\mathcal{E}\mathcal{E}}$ flat spectrum
- 3) collapse into remaining subspace $|34^B\rangle$, current $(I_{\mathcal{A}E} + I_{\mathcal{A}\mathcal{E}\mathcal{E}})/2$, spectral peak at 2Ω , peak/pedestal = 4η .



3) Slightly asymmetric qubits, $\varepsilon \neq 0$

$$\Gamma_{2B \rightarrow 34B} = 2\varepsilon^2 \Gamma / \Omega^2$$

Switching between states due to imperfections

- 1) Slightly different Rabi frequencies, $\Delta\Omega = \Omega_1 - \Omega_2$
 $\Gamma_{1B \rightarrow 2B} = \Gamma_{2B \rightarrow 1B} = (\Delta\Omega)^2 / 2\Gamma$, $\Gamma = \eta^{-1} (\Delta I)^2 / 4S_0$

$$S_I(\omega) = S_0 + \frac{(\Delta I)^2 \Gamma}{(\Delta\Omega)^2} \frac{1}{1 + [\omega\Gamma / (\Delta\Omega)^2]^2}$$

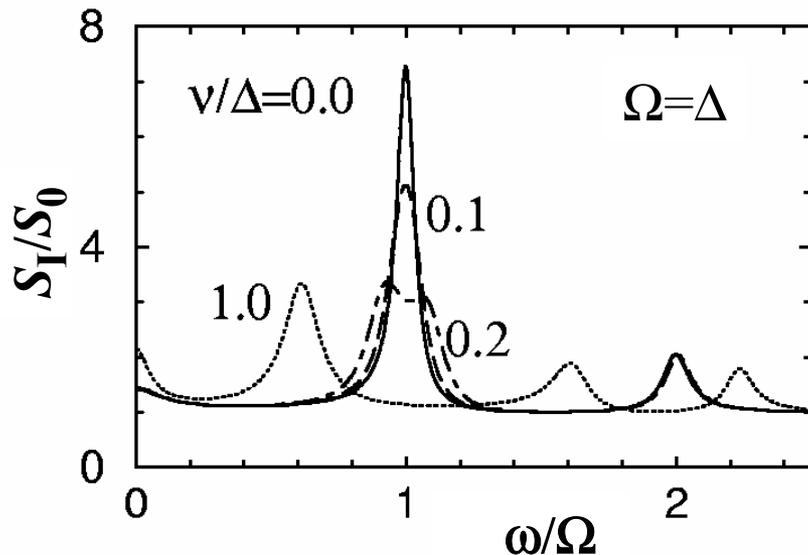
- 2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B \rightarrow 34B} = [(I_1 - I_4) / \Delta I]^2 \Gamma / 2$$

$$S_I(\omega) = S_0 + \frac{2}{3} \frac{4\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2 \omega^2} + \frac{8(\Delta I)^4}{27\Gamma(I_1 - I_4)^2} \frac{1}{1 + [4\omega(\Delta I)^2 / 3\Gamma(I_1 - I_4)^2]^2}$$



Effect of qubit-qubit interaction



$$H_{QBs} = -\sum_j (\varepsilon_j \sigma_z^j + \Delta_j \sigma_x^j) / 2 + \frac{v}{2} \sigma_z^1 \sigma_z^2$$

v - interaction between two qubits

First spectral peak splits (first order in v),
second peak shifts (second order in v)

$$\omega_{1-} = [\Delta^2 + (v/2)]^{1/2} - v/2$$

$$\omega_{1+} = [\Delta^2 + (v/2)]^{1/2} + v/2$$

$$\omega_2 = 2[\Delta^2 + (v/2)]^{1/2} = \omega_{1-} + \omega_{1+}$$

Conclusions (quadratic quantum measurements)

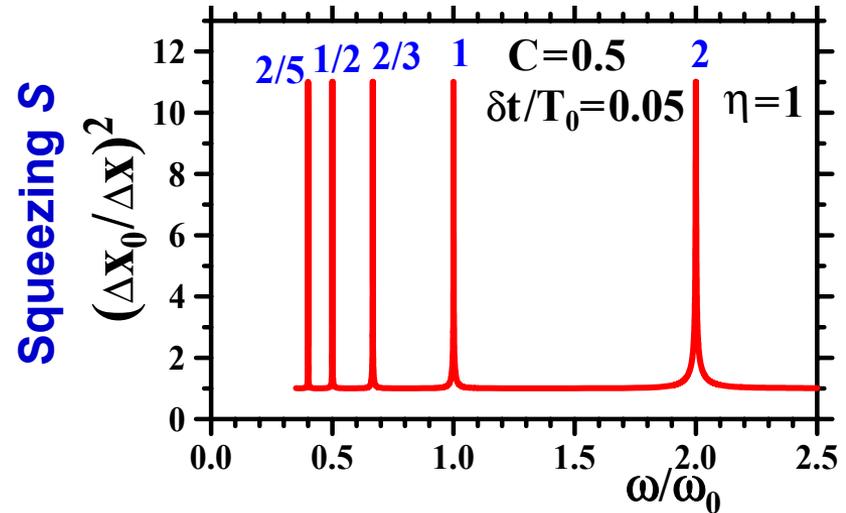
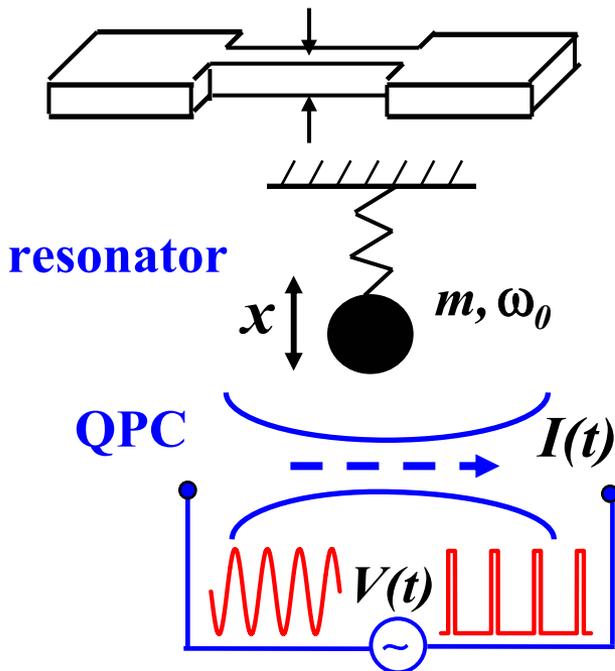
- Conditional (Bayesian) formalism for a nonlinear detector is developed
- Detector nonlinearity leads to the second peak in the spectrum (at 2Ω),
in purely quadratic case there is no peak at Ω
(very similar to classical nonlinear and quadratic detectors)
- Qubits become entangled (with some probability) due to measurement,
detection of entanglement is easier than for a linear detector (current
instead of spectrum)



Quantum nondemolition (QND) squeezing of a nanoresonator

R. Ruskov,¹ K. Schwab,² and A. Korotkov¹

¹UC, Riverside and ²LPS



Application: ultrasensitive force detection; sensitivity beyond standard quantum limit

cond-mat/0406416

Support:



Quantum nondemolition (QND) squeezing of a nanoresonator

R. Ruskov,¹ K. Schwab,² and A. Korotkov¹

¹UC, Riverside and ²LPS

We show that the nanoresonator position can be squeezed significantly below the ground state level by measuring the nanoresonator with a quantum point contact or a single-electron transistor and applying a periodic voltage across the detector. The mechanism of squeezing is basically a generalization of quantum nondemolition measurement of an oscillator to the case of continuous measurement by a weakly coupled detector. The quantum feedback is necessary to prevent the “heating” due to measurement back-action. We also discuss a procedure of experimental verification of the squeezed state.

Support:



ARDA

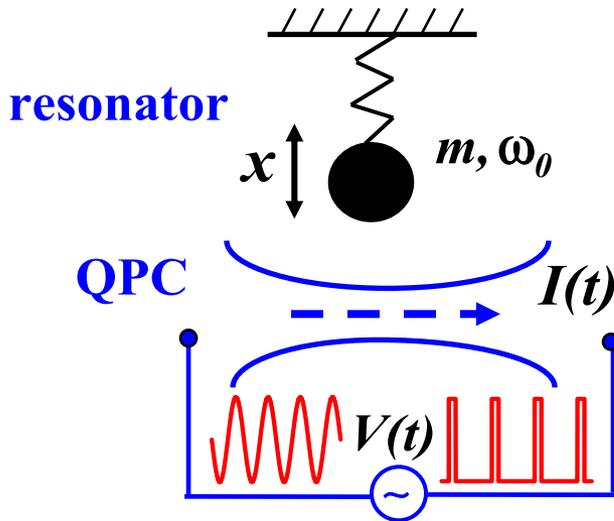


cond-mat/0406416



QND squeezing of a nanoresonator

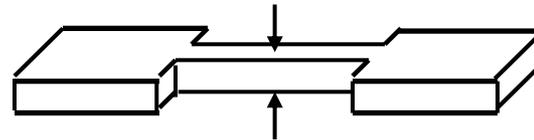
Ruskov, Schwab, Korotkov, cond-mat/0406416



$$\hat{H}_0 = \hat{p}^2 / 2m + m\omega_0^2 \hat{x}^2 / 2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \hat{x} a_l^\dagger a_r + H.c.)$$



$\omega_0 \sim 1$ GHz, $T \sim 50$ mK,
quantum behavior $T < \hbar\omega_0$
or $T\tau_{obs}/Q < \hbar/2$

Model similar to Hopkins, Jacobs, Habib, Schwab, PRB 2003
(continuous monitoring and quantum feedback to cool down)

New feature: Braginsky's stroboscopic QND measurement using
modulation of detector voltage \Rightarrow **squeezing becomes possible**

Potential application: ultrasensitive force measurements

Other most important papers:

Doherty, Jacobs, PRA 1999 (formalism for Gaussian states)

Mozyrsky, Martin, PRL 2002 (ensemble-averaged evolution)



Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

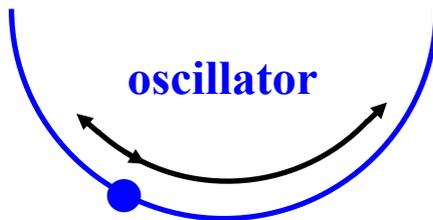
Standard quantum limit

Example: measurement of $x(t_2) - x(t_1)$



First measurement: $\Delta p(t_1) > \hbar / 2\Delta x(t_1)$, then even for accurate second measurement
inaccuracy of position difference is $\Delta x(t_1) + (t_2 - t_1)\hbar / 2m\Delta x(t_1) > (t_2 - t_1)\hbar / 2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al.*, 1978; Thorne *et al.*, 1978)



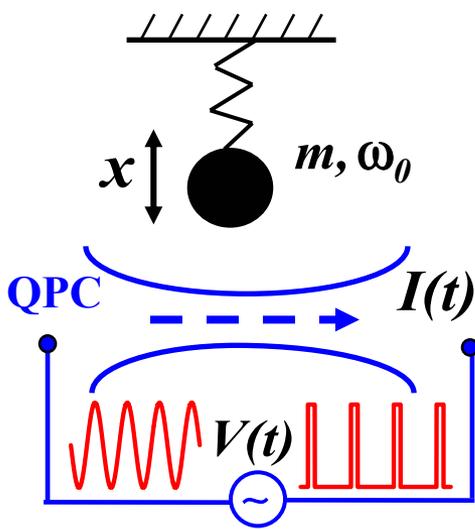
Idea: second measurement exactly one oscillation
period later is insensitive to Δp
(or $\Delta t = nT/2$, $T = 2\pi/\omega_0$)

Difference in our case:

- continuous measurement
- weak coupling with detector
- quantum feedback to suppress “heating”



Bayesian formalism for continuous measurement of a nanoresonator



$$\hat{H}_0 = \hat{p}^2 / 2m + m\omega_0^2 \hat{x}^2 / 2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \hat{x} a_l^\dagger a_r + H.c.)$$

Current $I_x = 2\pi (M + \Delta M x)^2 \rho_l \rho_r e^2 V / \hbar = I_0 + k x$

Detector noise $S_x = S_0 \equiv 2eI_0$

Recipe: quantum Bayes procedure

Nanoresonator evolution (Stratonovich form), same Eqn as for qubits:

$$\frac{d\rho(x, x')}{dt} = \frac{-i}{\hbar} [\hat{H}_0, \rho] + \frac{\rho(x, x')}{S_0} \left\{ I(t)(I_x + I_{x'} - 2\langle I \rangle) - \frac{1}{2} (I_x^2 + I_{x'}^2 - 2\langle I^2 \rangle) \right\}$$

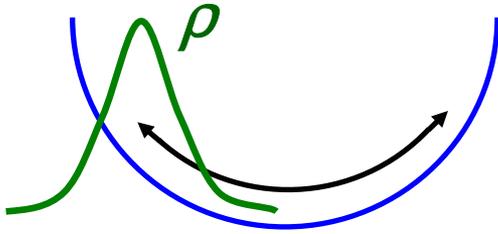
$$\langle I \rangle = \sum I_x \rho(x, x), \quad I(t) = I_x + \xi(t), \quad S_\xi = S_0$$

Ito form (same as in many papers on conditional measurement of oscillators):

$$\frac{d\rho(x, x')}{dt} = \frac{-i}{\hbar} [\hat{H}_0, \rho] - \frac{k^2}{4S_0\eta} (x - x')^2 \rho(x, x') + \frac{k}{S_0} (x + x' - 2\langle x \rangle) \rho(x, x') \xi(t)$$



Evolution of Gaussian states



Assume Gaussian states (following Doherty-Jacobs and Hopkins-Jacobs-Habib-Schwab),

then $\rho(x, x')$ is described by only 5 magnitudes:

$\langle x \rangle, \langle p \rangle$ - average position and momentum (packet center),

D_x, D_p, D_{xp} - variances (packet width)

Assume large Q-factor (then no temperature)

Voltage modulation $f(t)V_0$: $k = f(t)k_0, \quad I_x = f(t)(I_{00} + k_0x), \quad S_I = |f(t)|S_0$

Then coupling (measurement strength) is also modulated in time:

$$C = |f(t)|C_0, \quad C = \hbar k^2 / S_I m \omega_0^2 = 4 / \omega_0 \tau_{meas}$$

Packet center evolves randomly and needs feedback (force F) to cool down

$$d\langle x \rangle / dt = \langle p \rangle / m + (2k_0 / S_0) \text{sgn}[f(t)] D_x \xi(t)$$

$$d\langle p \rangle / dt = -m\omega_0^2 \langle x \rangle + (2k_0 / S_0) \text{sgn}[f(t)] D_{xp} \xi(t) + F(t)$$

Packet width evolves deterministically and is QND squeezed by periodic $f(t)$

$$d\langle D_x \rangle / dt = (2/m) D_{xp} - (2k_0^2 / S_0) |f(t)| D_x^2$$

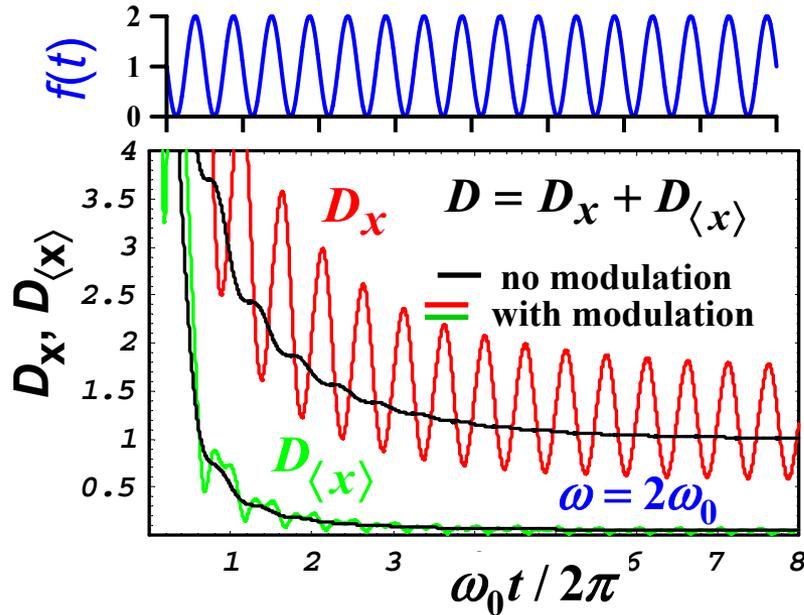
$$d\langle D_p \rangle / dt = -2m\omega_0^2 D_{xp} + (k_0^2 \hbar^2 / 2S_0 \eta) |f(t)| - (2k_0^2 / S_0) |f(t)| D_{xp}^2$$

$$d\langle D_{xp} \rangle / dt = (1/m) D_p - m\omega_0^2 D_x - (2k_0^2 / S_0) |f(t)| D_x D_{xp}$$



Squeezing by sine-modulation, $V(t)=V_0 \sin(\omega t)$

Ruskov-Schwab-Korotkov



Squeezing obviously oscillates in time, maximum squeezing at maximum voltage, momentum squeezing shifted in phase by $\pi/2$.

$$S \equiv \max_t (\Delta x_0)^2 / D_x$$

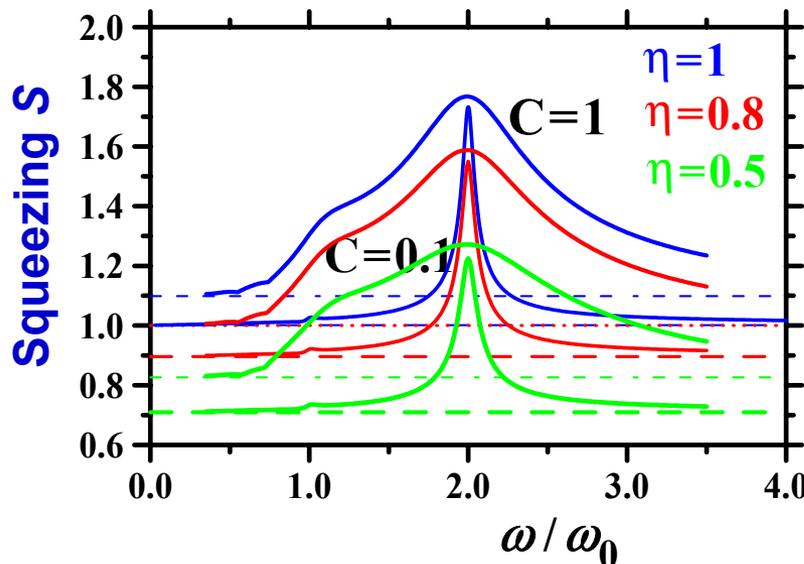
Analytics (weak coupling):

$$S(2\omega_0) = \sqrt{3\eta}, \quad \Delta\omega = 0.36\omega_0 C_0 / \sqrt{\eta}$$

η - detector efficiency, C_0 - coupling

$\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$ - ground state width

$$D_x = (\Delta x)^2, \quad D_{\langle x \rangle} = \langle \langle x \rangle^2 \rangle - \langle \langle x \rangle \rangle^2$$



Quantum feedback:

$$F = -m\omega_0 \gamma_x \langle x \rangle - \gamma_p \langle p \rangle$$

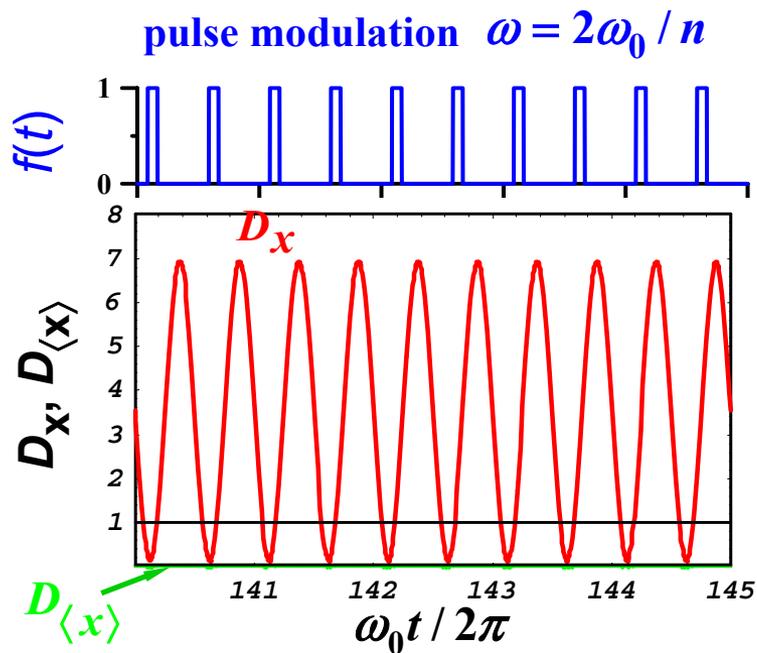
(same as in Hopkins *et al.*; without modulation it cools the state down to the ground state)

Feedback is sufficiently efficient, $D_{\langle x \rangle} \dot{U} D_x$

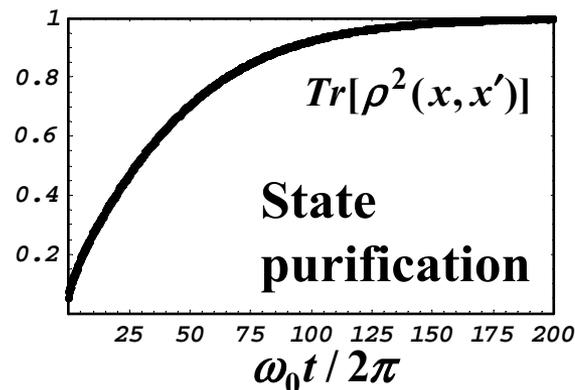
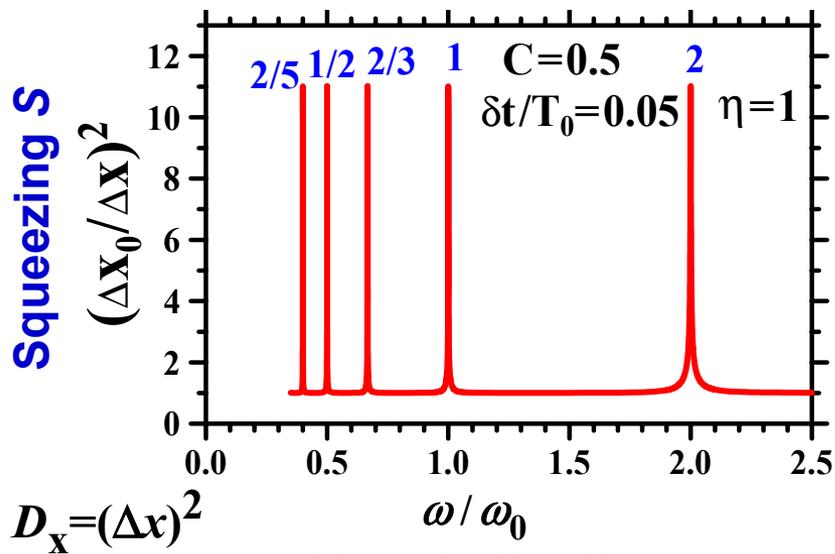
Squeezing up to 1.73 at $\omega=2\omega_0$



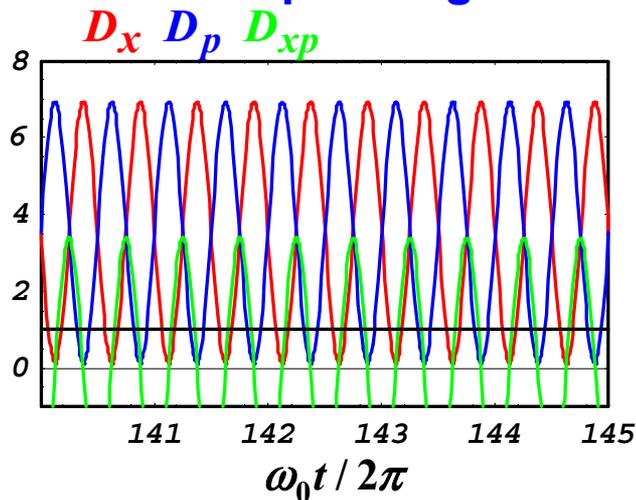
Squeezing by stroboscopic (pulse) modulation



$D_{\langle x \rangle} \ll D_x$
using feedback



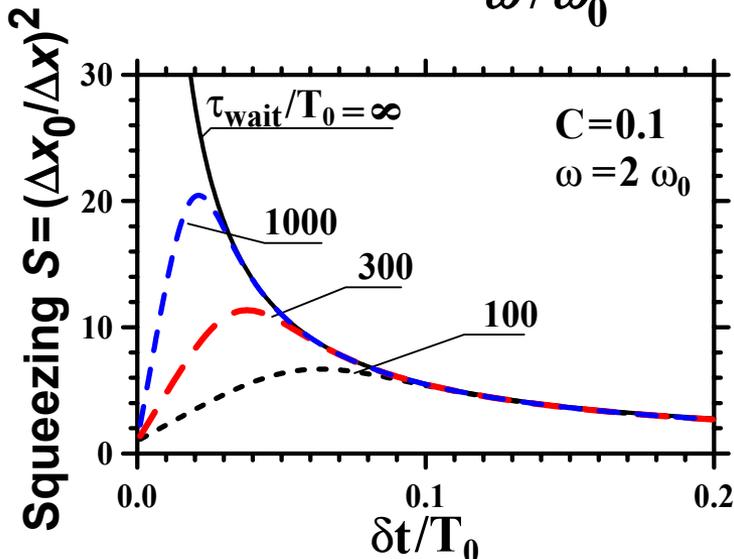
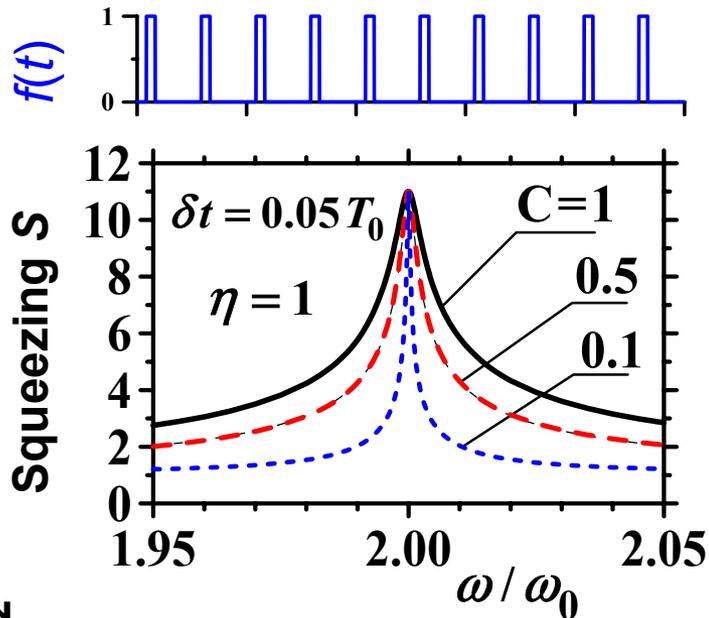
Momentum squeezing as well



Sá 1
Efficient squeezing at $\omega = 2\omega_0 / n$
(natural QND condition)



Squeezing by stroboscopic modulation



Analytics (weak coupling, short pulses)

Maximum squeezing

$$S(2\omega_0 / n) = \frac{2\sqrt{3\eta}}{\omega_0 \Delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\Delta t)^3 \omega_0^4}{\pi n^2 \sqrt{3\eta}}$$

C_0 – dimensionless coupling with detector

Δt – pulse duration, $T_0 = 2\pi/\omega_0$

η – quantum efficiency of detector

(long formula for the line shape)

Finite Q-factor limits the time we can afford to wait before squeezing develops, $\tau_{\text{wait}} / T_0 \sim Q/\pi$

Squeezing saturates as $\sim \exp(-n/n_0)$ after

$n_0 = \sqrt{3\eta} / C_0(\omega_0 \Delta t)^2$ measurements

Therefore, squeezing cannot exceed

$$S \approx \sqrt{C_0 Q} 4\sqrt{\eta}$$



Observability of nanoresonator squeezing

- Procedure:** 1) prepare squeezed state by stroboscopic measurement,
2) switch off quantum feedback
3) measure in the stroboscopic way $X_N = \frac{1}{N} \sum_{j=1}^N x_j$

For **instantaneous measurements** ($\delta t \rightarrow 0$) the variance of X_N is

$$D_{X,N} = \frac{\hbar}{2m\omega_0} \left(\frac{1}{S} + \frac{1}{NC_0\omega_0\delta t} \right) \rightarrow \frac{1}{S} (\Delta x_0)^2 \quad \text{at } N \rightarrow \infty$$

S – squeezing,
 Δx_0 – ground state width

Then distinguishable from ground state (S=1)
in one run for $S \gg 1$ (error probability $\sim S^{-1/2}$)

Not as easy for **continuous measurements** because of extra “heating”.

$D_{X,N}$ has a minimum at some N and then increases.

However, numerically it seems $\min_N D_{X,N} \sim 2(\Delta x_0)^2 / S$ (only twice worse)

Example: $\min_N D_{X,N} / (\Delta x_0)^2 = 0.078$ for $C_0=0.1$, $\eta=1$, $\delta t/T_0=0.02$, $1/S=0.036$

Squeezed state is distinguishable in one run (with small error probability), therefore suitable for ultrasensitive force measurement beyond standard quantum limit



Conclusions

(QND squeezing of a nanoresonator)

- **Periodic modulation of the detector voltage modulates measurement strength and periodically squeezes the width of the nanoresonator state (“breathing mode”)**
- **Packet center oscillates and is randomly “heated” by measurement; quantum feedback can cool it down (keep it near zero in both position and momentum)**
- **Sine-modulation leads to a small squeezing (<1.73), stroboscopic (pulse) modulation can lead to a strong squeezing ($\gg 1$) even for a weak coupling with detector**
- **Still to be done: correct account of Q -factor and temperature**
- **Potential application: ultrasensitive force measurement beyond standard quantum limit**

