Quantum feedback of a double-dot qubit

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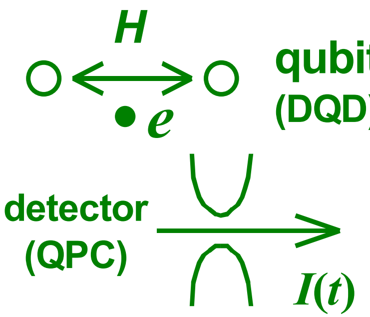
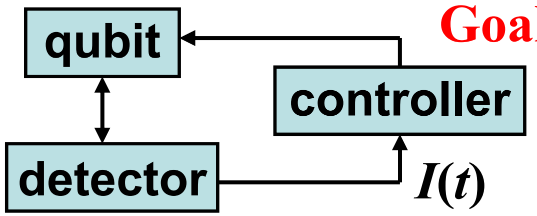
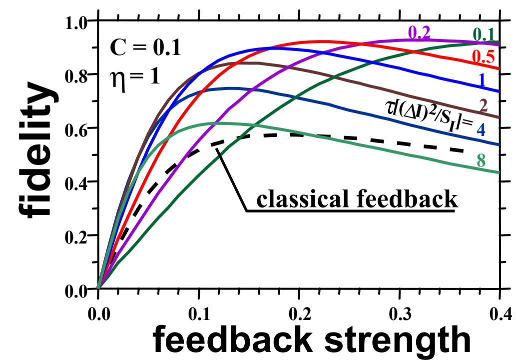




Goal: keep Rabi oscillations forever



Outline:

- Problem to be discussed
- Introduction
  - Quantum feedback in optics
  - Bayesian formalism for quantum measurement
- Bayesian quantum feedback of a qubit
- Simple quantum feedback of a qubit

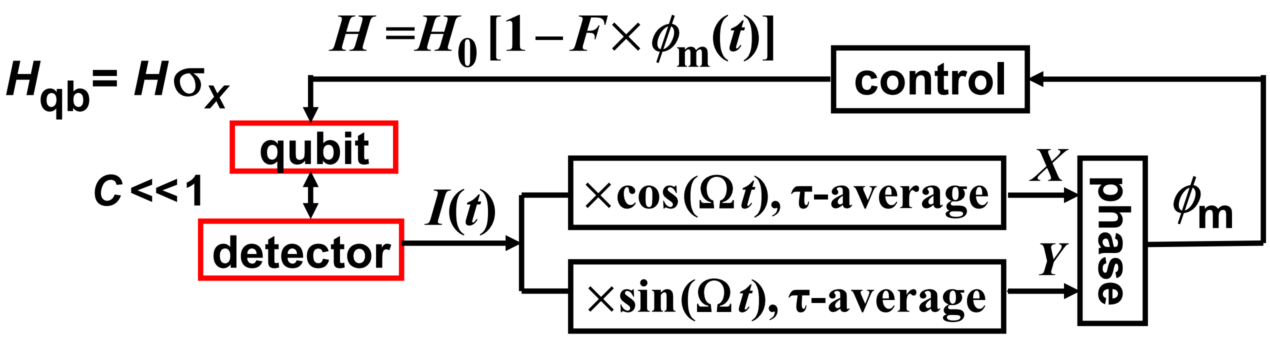
Support:

cond-mat/0404696

Proposal of quantum feedback setup

(problem to be discussed)

(A.K., cond-mat/0404696)



Goal: maintain coherent (Rabi) oscillations in a qubit for arbitrary long time

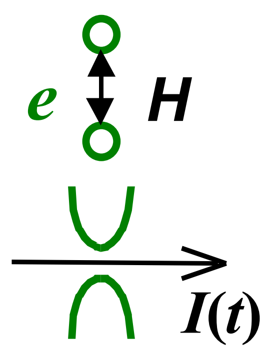
Idea: use quadrature components of the detector current $I(t)$ for monitoring of the oscillation phase (as in a classical feedback!)

Result: works surprisingly well (fidelity up to 95%)

2DEG implementation:

qubit: double quantum dot occupied by one electron

detector: quantum point contact (QPC)



DQD qubit and QPC detector have been demonstrated experimentally:

Buks *et al.*, Nature (1998)
Sprinzak *et al.*, PRL (1999)
Hayashi *et al.*, PRL (2003)

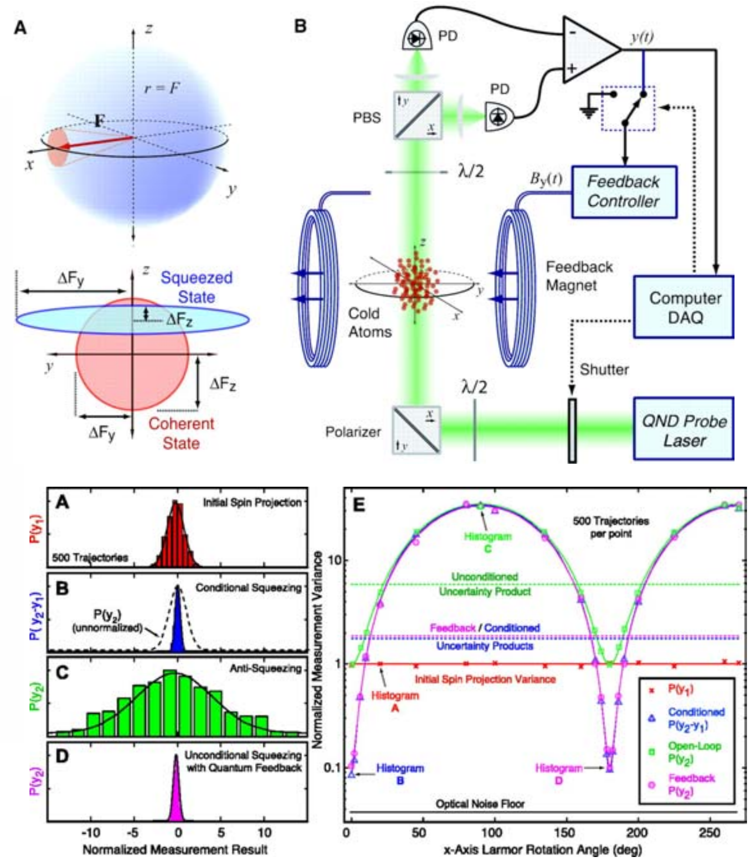
Quantum feedback in optics

Recent experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

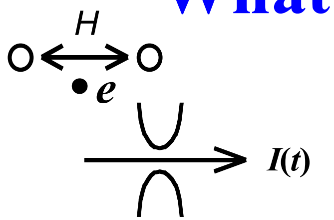
Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

What happens to a qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier (frozen qubit), evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

no measurement result! ensemble averaged

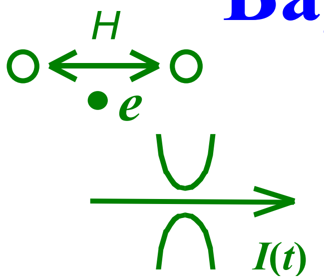
Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems

Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for a single qubit



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1$, $|2\rangle \to I_2$, $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\begin{cases} \dot{\rho}_{11} = -\dot{\rho}_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I / S_I)[I(t) - I_0] \\ \dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I / S_I)[I(t) - I_0] - \gamma\rho_{12} \end{cases}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2 / 4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2 / 4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

$I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I / 2 + \xi(t)$, $S_\xi = S_I$ **Averaging over ξ(t) ⇒ master equation**

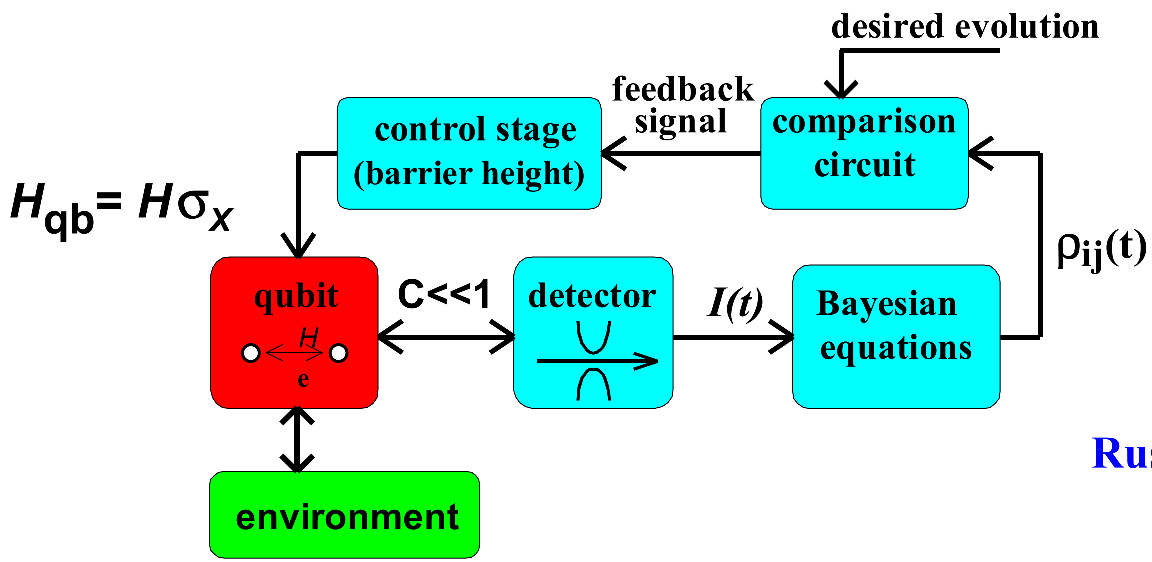
Ideal detector (η=1) does not decohere a single qubit;
then random evolution of qubit *wavefunction* can be monitored

Theoretically, quantum point contact is an ideal detector (η=1),
experimentally, η ~ 0.8 demonstrated (Buks *et al*., 1998)

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Bayesian quantum feedback of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit “fresh”)

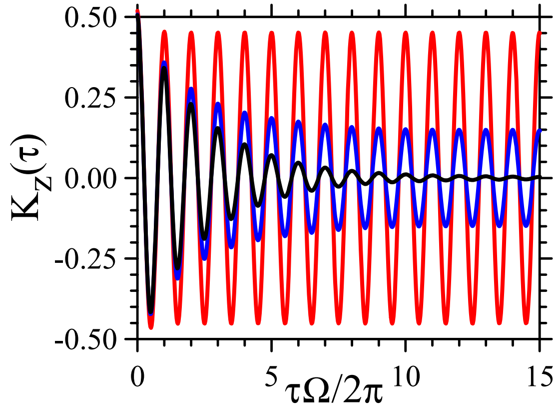
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F \times \Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

(no extra environment)

Qubit correlation function



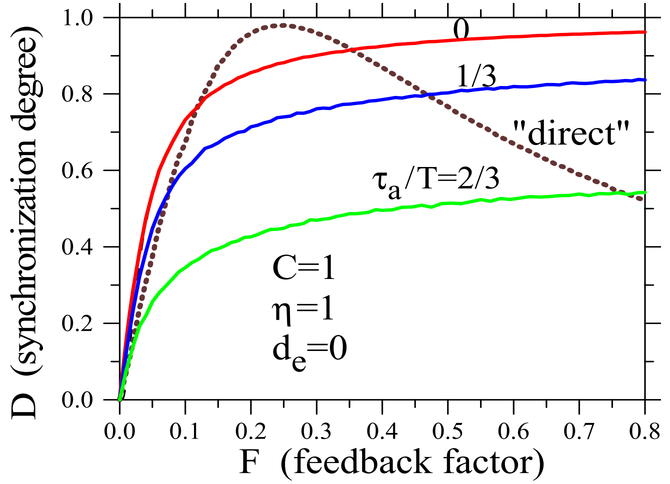
$$K_z(\tau) = \frac{\cos\Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos\Omega t}{2} (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

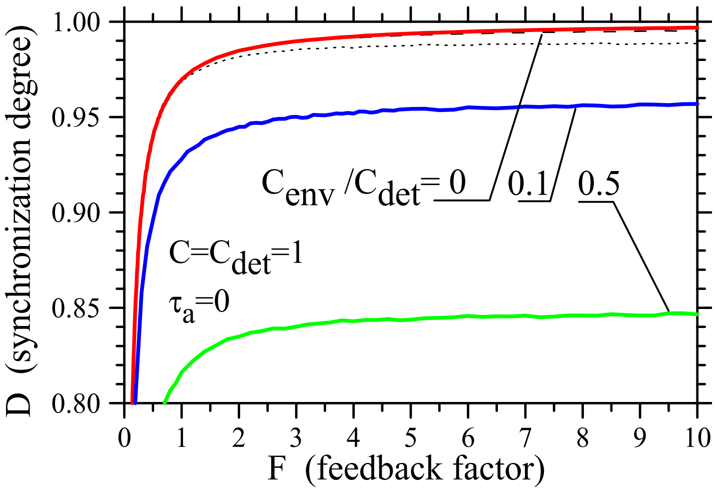
$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

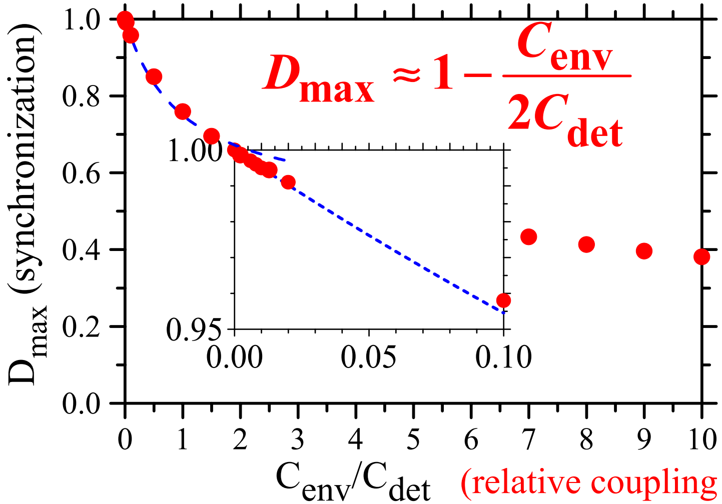
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D = \exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback





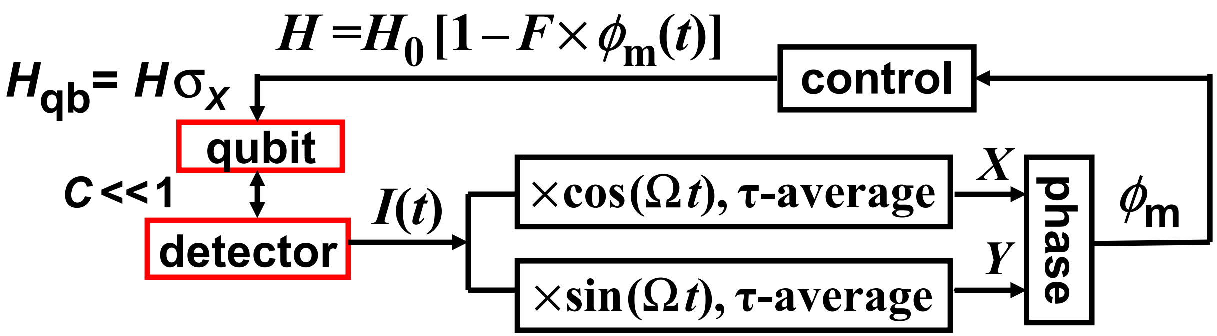
(relative coupling to environment)

Big experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (>>Ω, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

$\rho_{11}-\rho_{22}=\cos(\Omega t)$, $\rho_{12}=i\sin(\Omega t)/2$

Idea: **use two quadrature components of the detector current *I*(*t*) to monitor approximately the phase of qubit oscillations**

(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

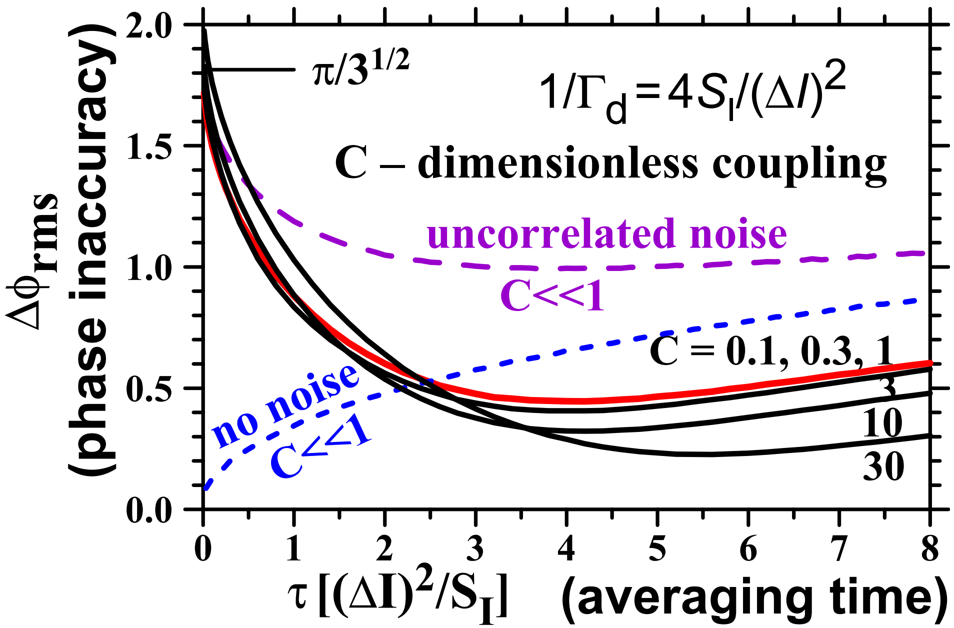
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

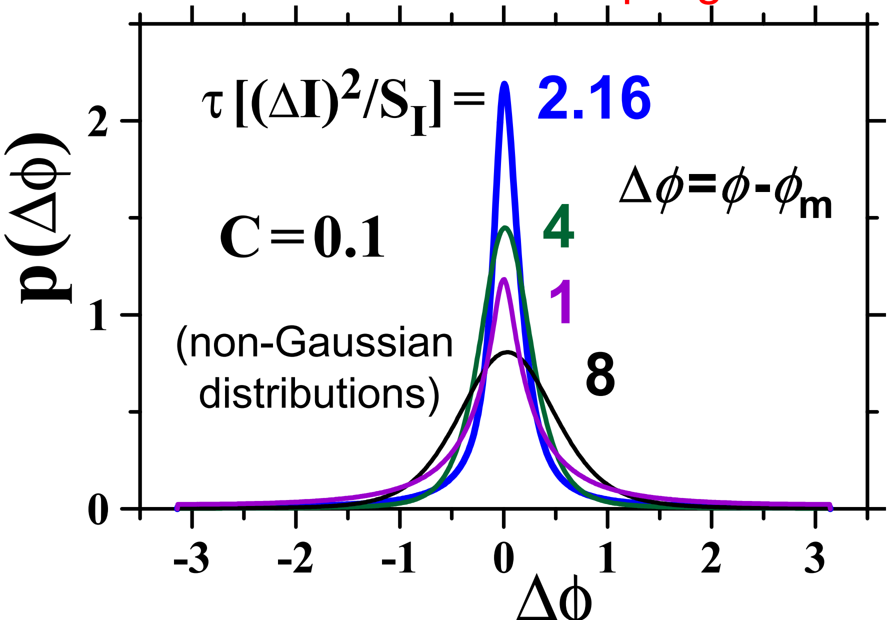
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures
(no feedback yet)

weak coupling C<<1





Best approximation
$\langle X^2+Y^2\rangle = (S_I/\Delta I)^2$
$(2/5)(41^{1/2}-1) \approx 2.16$

Noise improves the monitoring accuracy!
(purely quantum effect, "reality follows observations")

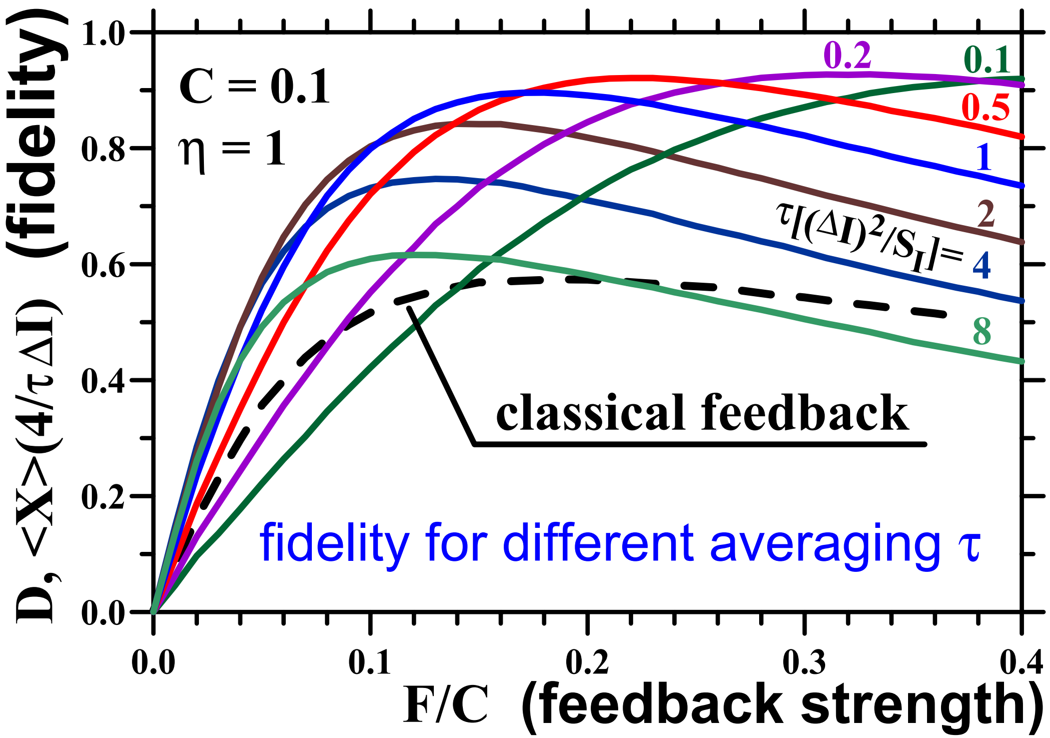
$$d\phi/dt = -[I(t) - I_0]\sin(\Omega t + \phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t) - I_0]\sin(\Omega t + \phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44) \approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

($F_Q \approx 95\%$)

How to verify feedback operation experimentally?

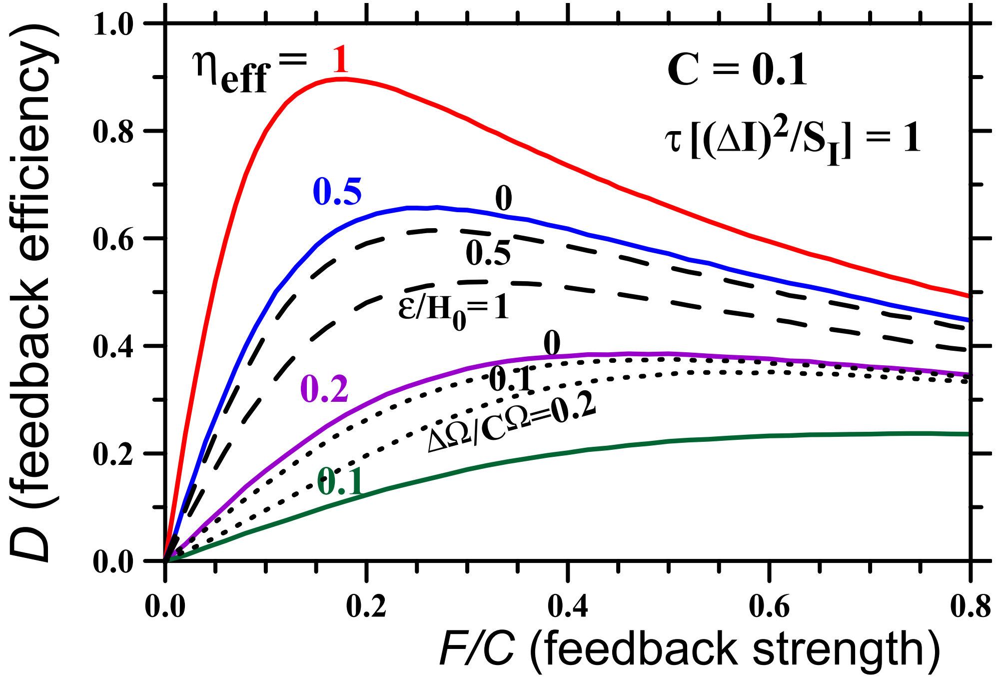
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



Main features:

- Fidelity F_Q up to ~95% achievable (D~90%)
- Natural, practically classical feedback setup
- Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)
- Detector efficiency (ideality) η~0.1 still OK
- Robust to asymmetry ε and frequency shift ΔΩ
- Simple verification: positive in-phase quadrature ⟨X⟩

Simple enough experiment?!

Conclusion

- Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations

- Price for simplicity is a less-then-ideal operation (fidelity is limited by ~95%)

- Feedback operation is much better than expected

- Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)