Quantum feedback control for quantum computing

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Outline:

- Quantum computing
- Paradox of quantum collapse
- Continuous quantum measurement
- Quantum feedback control

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Rusko Ruskov (postdoc),
Qin Zhang (grad. student)

Support:

Quantum computing (1)

- Exponential speedup for certain problems
  (difficult to solve, easy to check)
- Not a general-purpose computing

Most important mathematical problem (for practice):
factorization of a number into primes

Classical algorithm: $\sim N^{1/2}$ operations (best: $\sim N^{1/3}$ operations)

Example: $N \sim 2^{1024} \Rightarrow \sim 10^{150}$ operations (10 GHz x 10^{130} yrs)

Age of Universe: 10^{10} yrs

Quantum algorithm: $\sim \log^2(N)$ operations
(P. Shor – A. Kitaev)

Example: $N \sim 2^{1024} \Rightarrow \sim 10^{6}$ operations (1 MHz x 1 sec)

Quantum computing (2)

Main idea: massive parallelization of quantum computation

qubit = quantum bit

1000 bits $\Rightarrow 2^{1000}$ different numbers (states), only one at a time

1000 qubits $\Rightarrow 2^{1000}$ different states **simultaneously**
(2^{1000} computers in parallel)

Why? - Qubits can be in superposition of quantum states
(weird feature of quantum world)

Quantum computer can break codes of any imaginable length

But! Quantum cryptography is unbreakable (different subject)

Main problem in building a quantum computer: decoherence (just technical, but severe problem), at present up to 7 qubits

Collapse paradox in quantum physics, defeat of realism

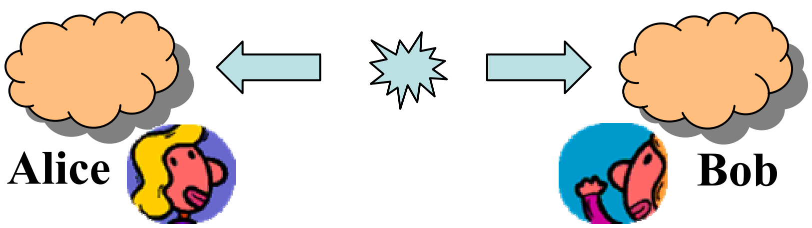
Niels Bohr: "If you are not confused by quantum physics then you haven't really understood it"

Richard Feynman: "I think I can safely say that nobody understands quantum mechanics"

Collapse paradox

(main paradox of QM):

Act of observation changes a quantum system



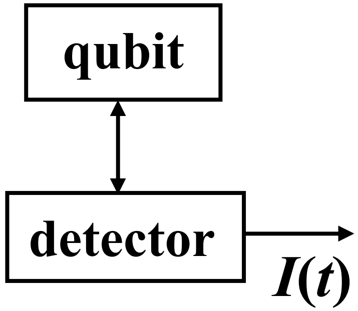
Two parts of a system still feel each other over distance (without any communication)

1935: Einstein, Podolsky, Rosen (EPR)
quantum theory cannot be correct

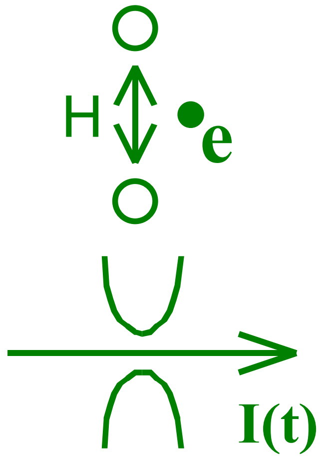
1964: J. Bell (theory)
it can be checked, who was wrong: Bohr or Einstein

1982: A. Aspect et al. (experiment)
quantum mechanics is right (Einstein was wrong)

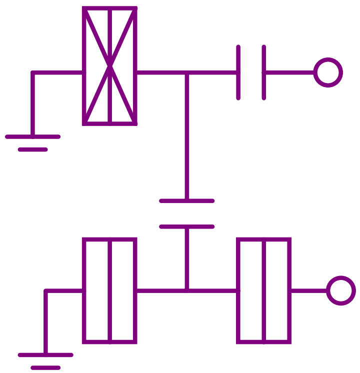
In some sense causality is violated (cause and effect reversed in time) "spooky action at distance"



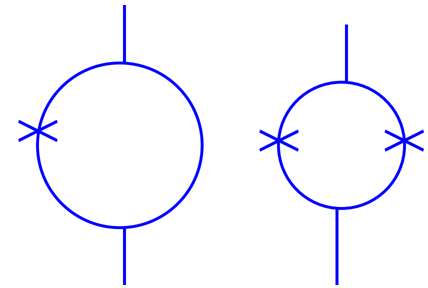
Continuous measurement of solid-state qubits



Double-quantum-qot and quantum point contact (QPC)



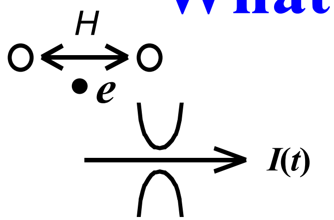
Cooper-pair box and single-electron transistor (SET)



Two SQUIDs

Continuous measurement $\Rightarrow$ continuous collapse

What happens to a qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier (frozen qubit), evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

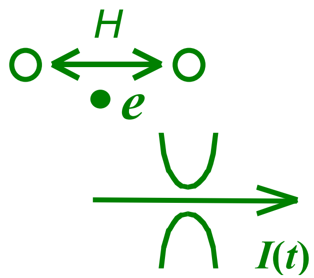
no measurement result! ensemble averaged

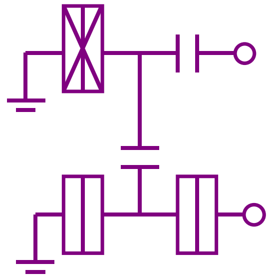
Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
"Orthodox" (1920s)	yes	no
"Conventional" (1980s)	no	yes
Bayesian (A.K., 1998)	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems
Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

|1ò Æ I_1, |2ò Æ I_2

$\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\begin{cases} d\rho_{11}/dt = -d\rho_{22}/dt = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[I(t) - I_0] \\ d\rho_{12}/dt = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[I(t) - I_0] - \gamma\rho_{12} \end{cases}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

For simulations: $I(t) - I_0 \to (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

Averaging over $\xi(t) \Rightarrow$ master equation

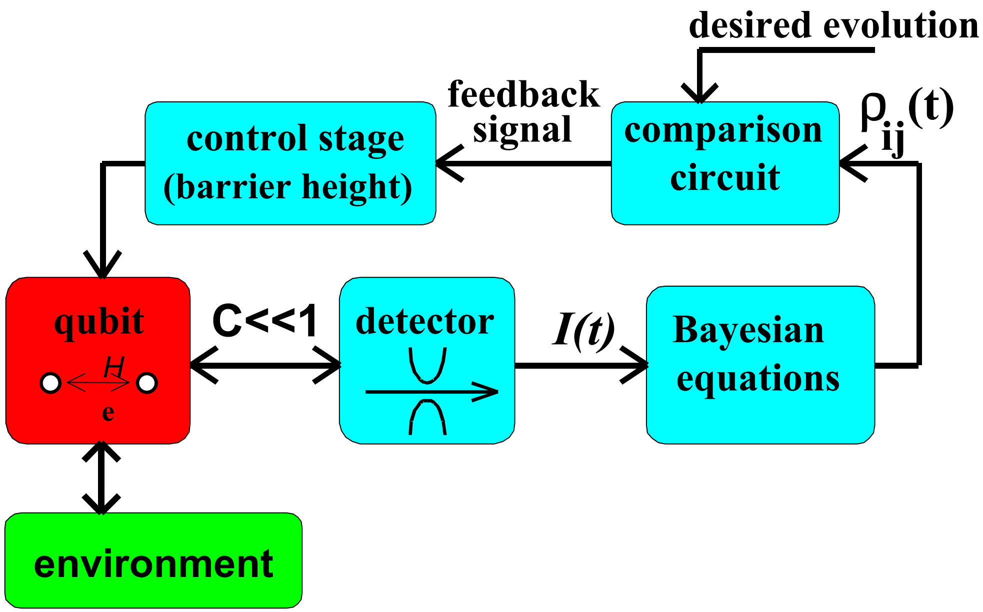
Ideal detector (η=1) does not decohere a single qubit (pure state remains pure), then random evolution of the qubit *wavefunction* can be monitored

Similar formalisms developed earlier. **Key words**: **Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.**

Names: **E.B. Davies, K. Kraus, A.S. Holevo, C.W. Gardiner, H.J. Carmichael, C.M. Caves, M.B. Plenio, P.L. Knight, M.B. Mensky, D.F. Walls, N. Gisin, I.C. Percival, G.J. Milburn, H.M. Wiseman, R. Onofrio, S. Habib, A. Doherty, etc. (very incomplete list)**

Quantum feedback control of a solid-state qubit

Ruskov & A.K., 2002



- similar to classical feedback
- control of a quantum object
- price for measurement is a gradual collapse
- thought to be impossible

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit "fresh")

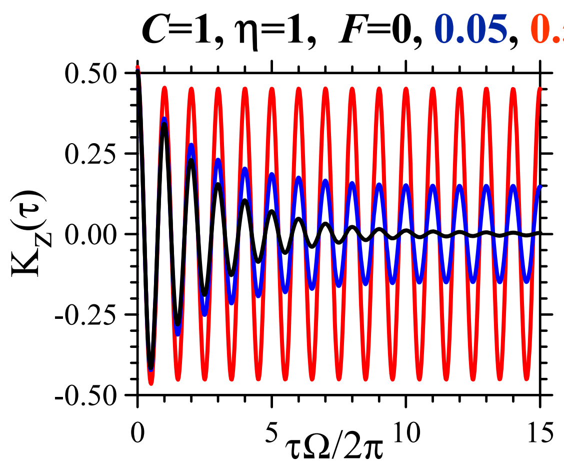
Use: qubit initialization is a quantum computer

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

Quantum feedback in quantum optics is discussed since 1993 (Wiseman-Milburn), recently first successful experiments in Mabuchi's group (2004).

Performance of quantum feedback

Qubit correlation function



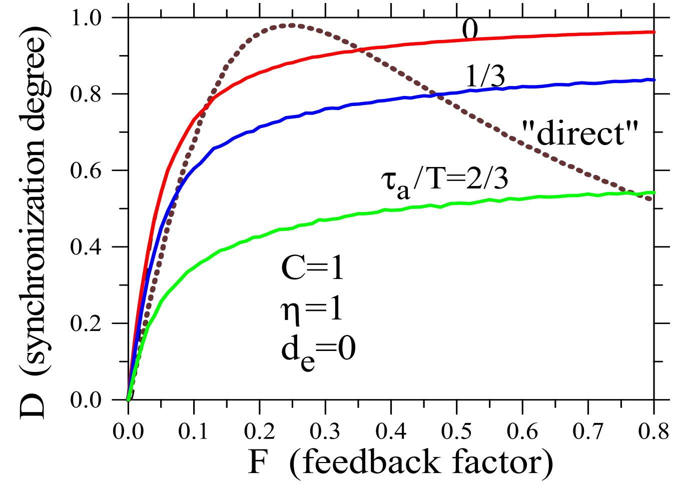
$$K_z(\tau) = \frac{\cos \Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega t}{2} (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

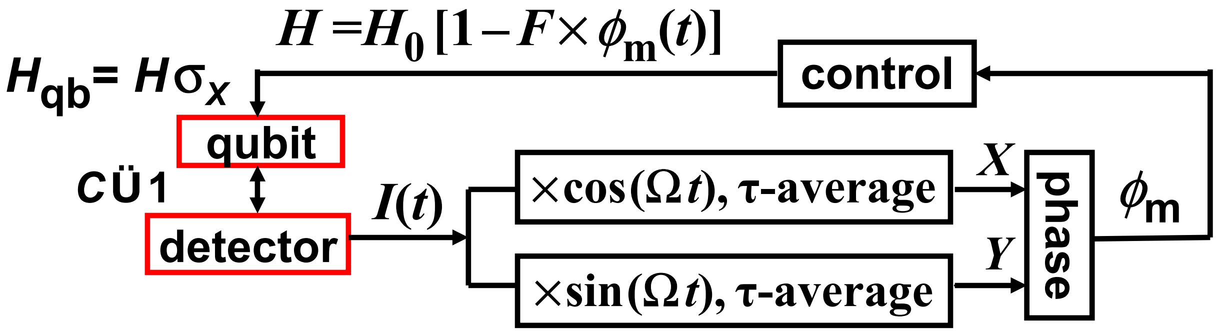
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D = \exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)



We want to maintain coherent (Rabi) oscillations for arbitrary long time,
$\rho_{11}-\rho_{22}=\cos(\Omega t)$, $\rho_{12}=i\sin(\Omega t)/2$

Idea: **use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

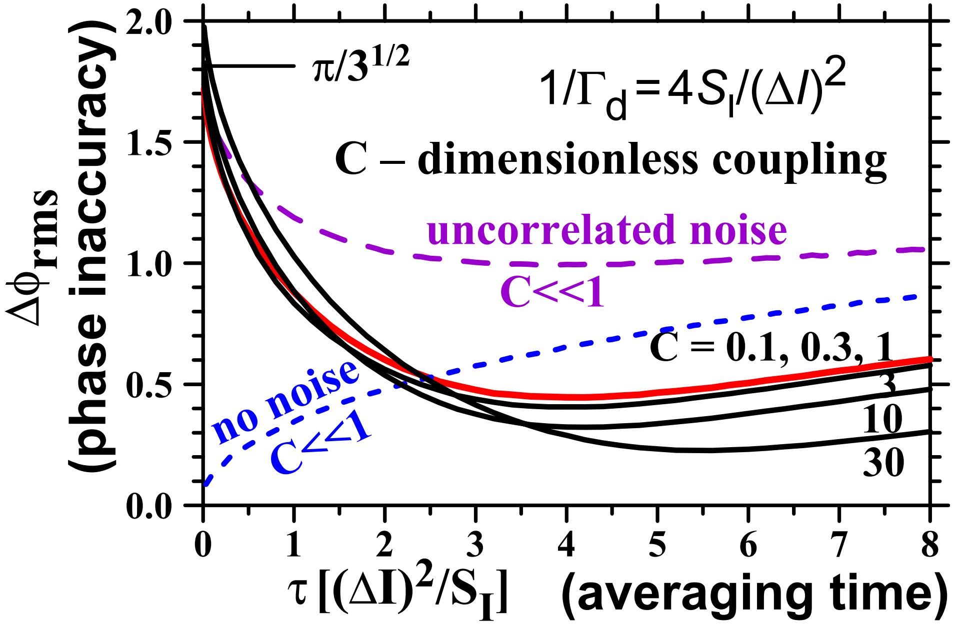
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

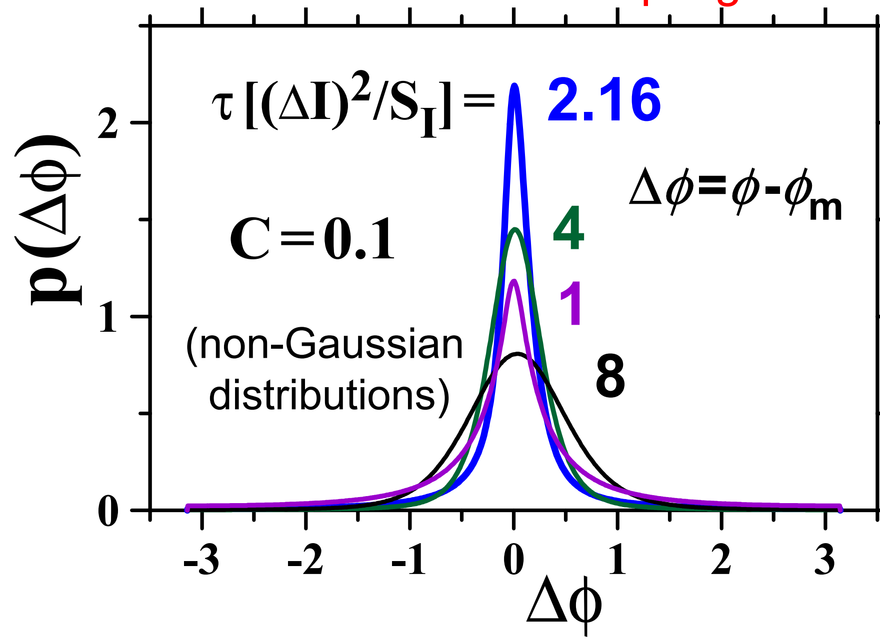
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures
(no feedback yet)

weak coupling C á 1





Best approximation
$\langle X^2+Y^2\rangle=(S_I/\Delta I)^2$
$(2/5)(41^{1/2}-1)\approx 2.16$

Noise improves the monitoring accuracy!
(purely quantum effect, "reality follows observations")

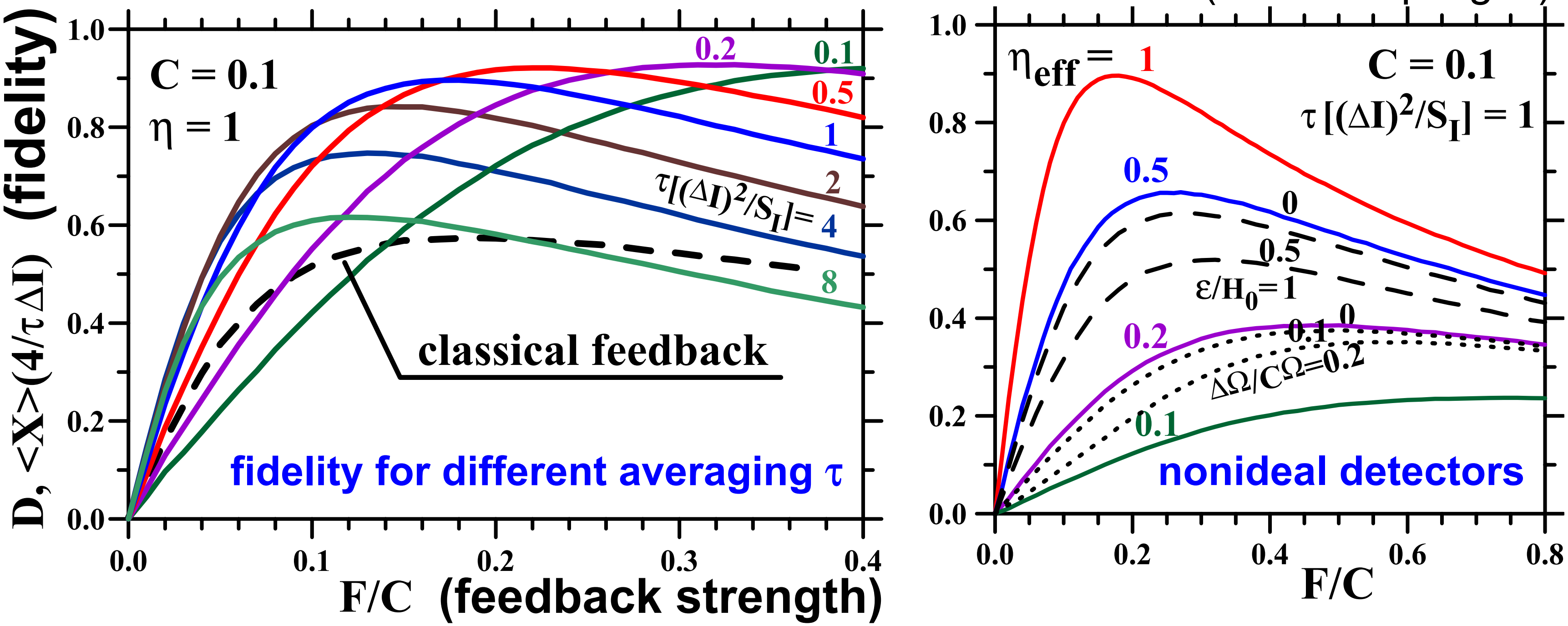
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Simple quantum feedback

(weak coupling *C*)



- **Fidelity *F* up to ~95% achievable (*D*~90%)**
- **Natural, practically classical feedback setup**
- **Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)**
- **Detector efficiency (ideality) η≤0.1 still OK**
- **Robust to asymmetry ε and frequency shift ΔΩ**
- **Very simple verification – just positive in-phase quadrature ⟨*X*⟩**

$D \equiv 2F - 1$

$F \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$

$D \simeq \langle X \rangle (4/\tau \Delta I)$

X – in-phase quadrature of the detector current

Doable experiment?!

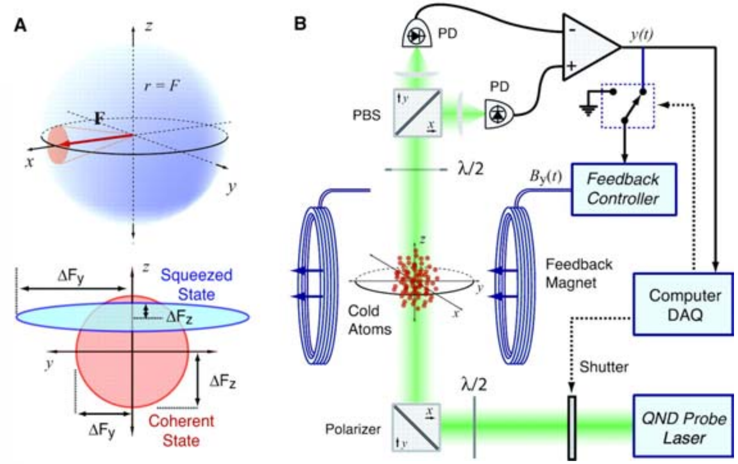
Quantum feedback in optics

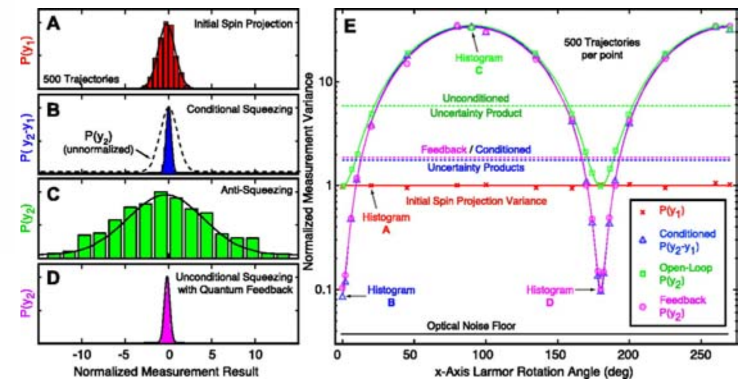
Recent experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.





First detailed theory:

H.M. Wiseman and G. J. Milburn, Phys. Rev. Lett. 70, 548 (**1993**)

No experimental attempts of quantum feedback in solid-state yet (even theory is still considered controversial)

Experiments soon?

Summary

- Weird features of quantum world are the basis of quantum computing
- Extremely weird nature of quantum collapse can be put to use in a quantum computer
- Quantum feedback is possible in spite of collapse, feedback of a qubit is useful in a quantum computer
- Very difficult experiments; however, very important for both practice and philosophy