Quantum nondemolition squeezing of a nanomechanical resonator

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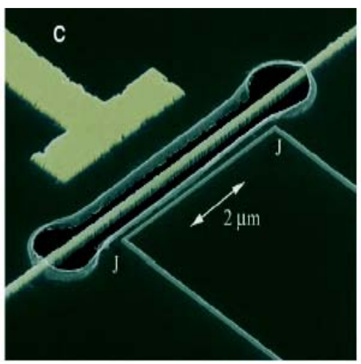
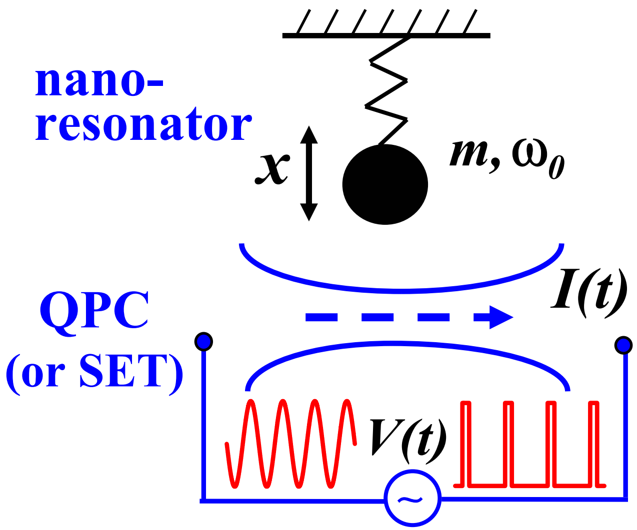
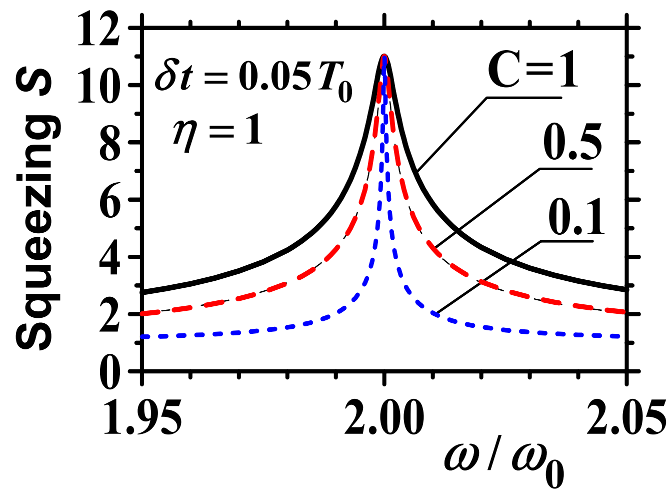




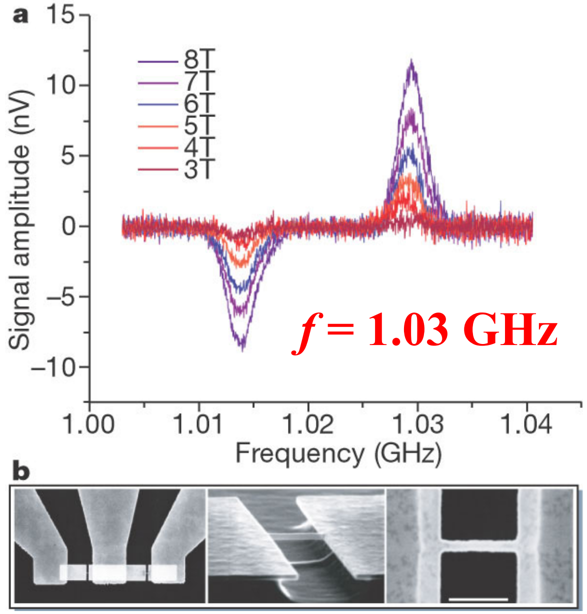


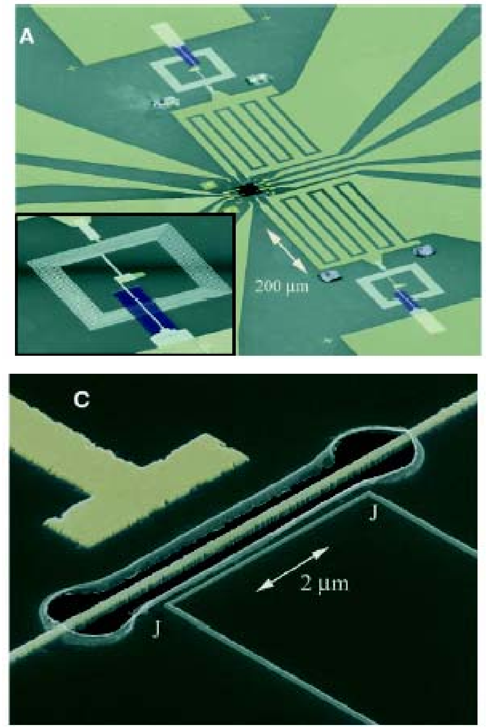
Support:

cond-mat/0411617

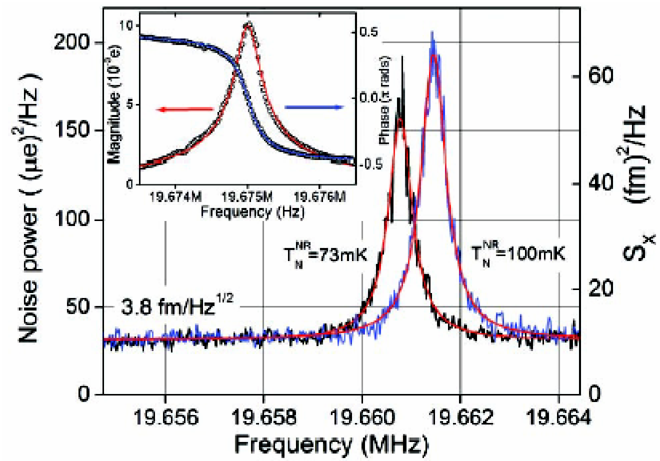
Some experiments on nanoresonators

Ming *et al*. (Roukes' group), Nature-2003

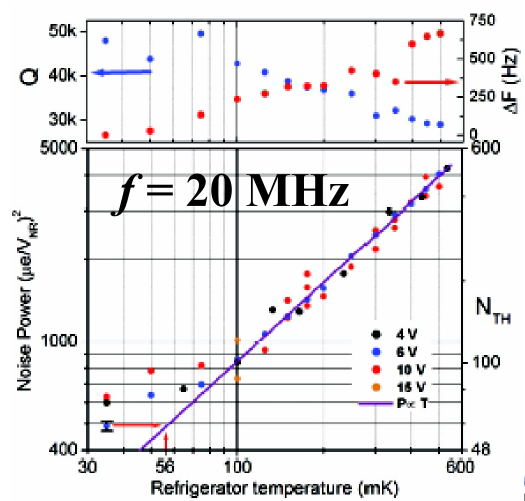




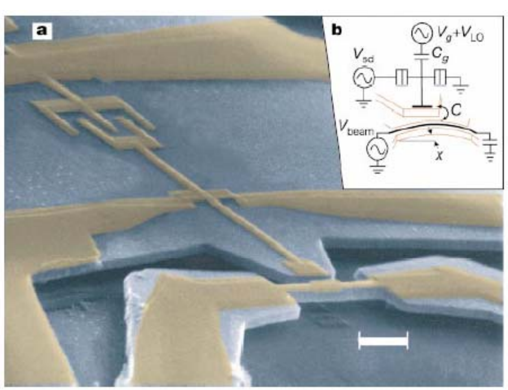
LaHaye, Buu, Camarota, and Schwab, Science-2004

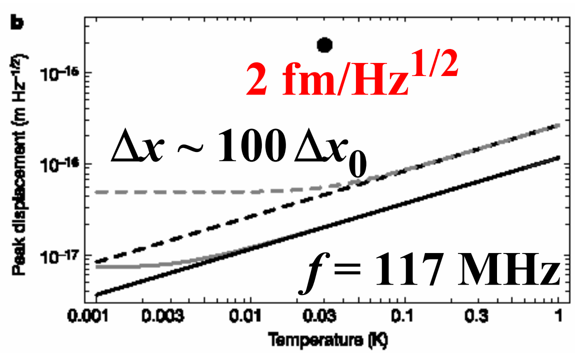


$\Delta x = 5.8\,\Delta x_0$ 3.8 fm/Hz$^{1/2}$

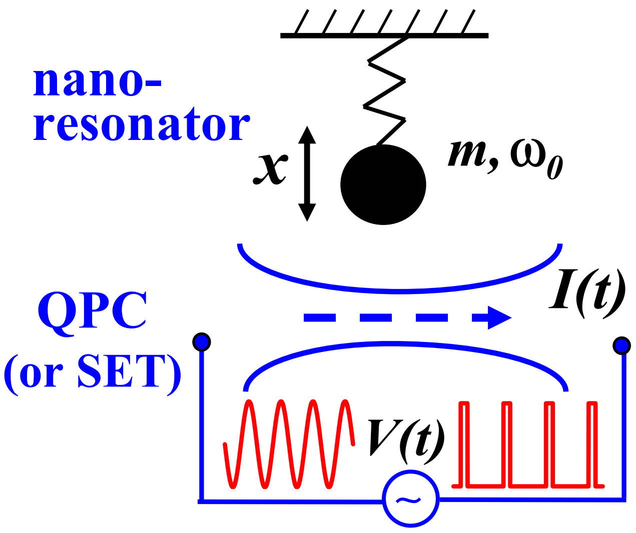


Knobel, Cleland, Nature-2003





QND squeezing of a nanomechanical resonator



Ruskov, Schwab, Korotkov, cond-mat/0406416, cond-mat/0411617

$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2\hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r}(M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r}(\Delta M\,\hat{x}\,a_l^\dagger a_r + H.c.)$$

(Somewhat similar for measurement by SET instead of QPC)

Continuous monitoring heats up nanoresonator (Mozyrsky, Martin, 2002)

Continuous monitoring and quantum feedback can cool nanoresonator down to the ground state (Hopkins, Jacobs, Habib, Schwab, 2003)

New feature: stroboscopic QND measurement using modulation of detector voltage ⇒ **squeezing becomes possible**

Potential application: ultrasensitive force measurements

Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

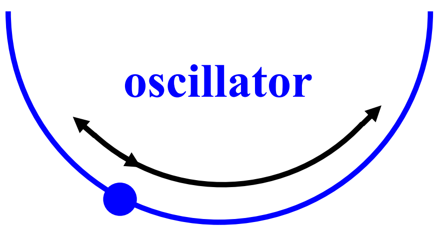
Standard quantum limit

$\Delta p > \hbar / 2\Delta x$

Example: measurement of $x(t_2)$-$x(t_1)$

First measurement: $\Delta p(t_1) > \hbar/2\Delta x(t_1)$, then even for accurate second measurement inaccuracy of position difference is $\Delta x(t_1) + (t_2\text{-}t_1)\hbar/2m\Delta x(t_1) > (t_2\text{-}t_1)\hbar/2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al*., 1978; Thorne *et al*., 1978)

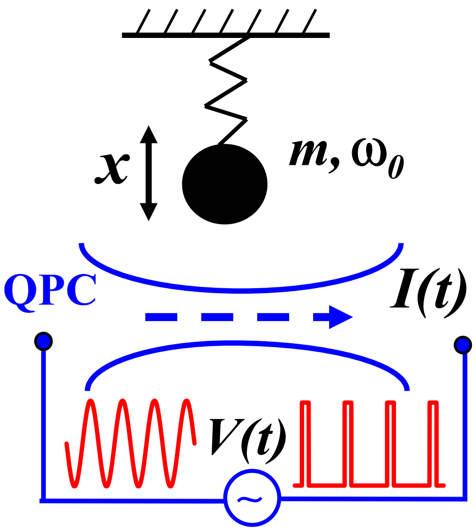


Idea: second measurement exactly one oscillation period later is insensitive to Δp
(or $\Delta t = nT/2$, $T=2\pi/\omega_0$)

Difference in our case:

- **continuous measurement**
- **weak coupling with detector**
- **quantum feedback to suppress "heating"**

Bayesian formalism for continuous measurement of a nanoresonator



$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2 \hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M\, \hat{x}\, a_l^\dagger a_r + H.c.)$$

Current $I_x = 2\pi \left(M + \Delta M\, x\right)^2 \rho_l \rho_r e^2 V/\hbar = I_0 + k\, x$

Detector noise $S_x = S_0 \equiv 2eI_0$

Recipe: quantum Bayes procedure

Nanoresonator evolution (Stratonovich form), same equation as for qubits:

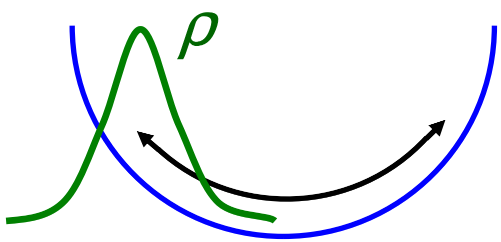
$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0, \rho] + \frac{\rho(x,x')}{S_0}\left\{\underline{\underline{I(t)}}(I_x + I_{x'} - 2\langle I\rangle) - \frac{1}{2}\left(I_x^2 + I_{x'}^2 - 2\langle I^2\rangle\right)\right\}$$

$$\langle I\rangle = \sum I_x \rho(x,x), \quad I(t) = I_x + \xi(t), \quad S_\xi = S_0$$

Ito form (same as in many papers on conditional measurement of oscillators):

$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0, \rho] - \frac{k^2}{4S_0\eta}(x-x')^2 \rho(x,x') + \frac{k}{S_0}(x + x' - 2\langle x\rangle)\,\rho(x,x')\,\xi(t)$$

Evolution of Gaussian states



Assume Gaussian states (following *Doherty-Jacobs* and *Hopkins-Jacobs-Habib-Schwab*), then ρ(*x*,*x*') is described by only 5 magnitudes:

$\langle x\rangle$, $\langle p\rangle$ - average position and momentum (packet center),

D_x, D_p, D_{xp} – variances (packet width)

Assume large *Q*-factor (then no temperature)

Voltage modulation $f(t)V_0$: $\quad k = f(t)k_0, \quad I_x = f(t)(I_{00}+k_0 x), \quad S_I = |f(t)|\,S_0$

Then coupling (measurement strength) is also modulated in time:

$$C = |f(t)|\,C_0, \quad C = \hbar k^2 / S_I m\omega_0^2 = 4/\omega_0\tau_{meas}$$

Packet center evolves randomly and needs feedback (force *F*) to cool down

$$d\langle x\rangle/dt = \langle p\rangle/m + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\,D_x\,\xi(t)$$

$$d\langle p\rangle/dt = -m\omega_0^2\langle x\rangle + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\,D_{xp}\,\xi(t) + F(t)$$

Packet width evolves deterministically and is QND squeezed by periodic *f*(*t*)

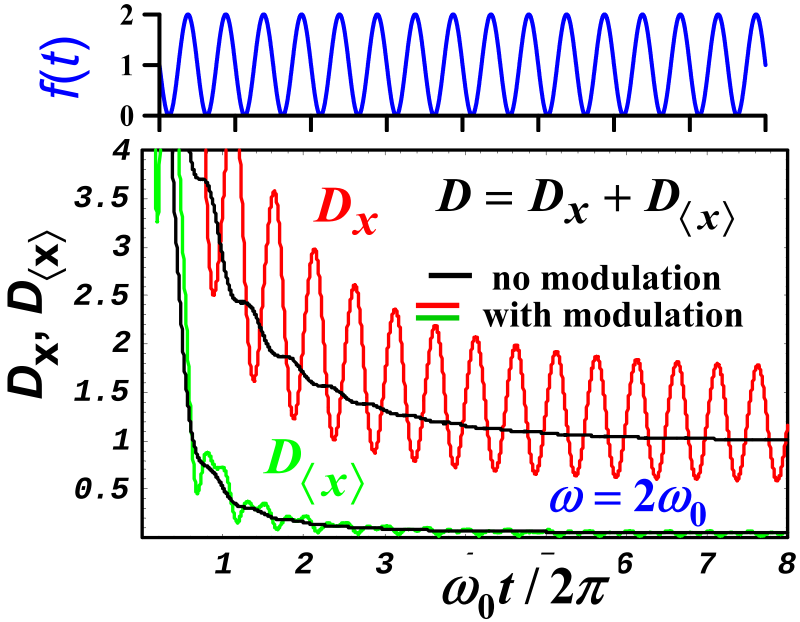
$$d\langle D_x\rangle/dt = (2/m)D_{xp} - (2k_0^2/S_0)\,|f(t)|\,D_x^2$$

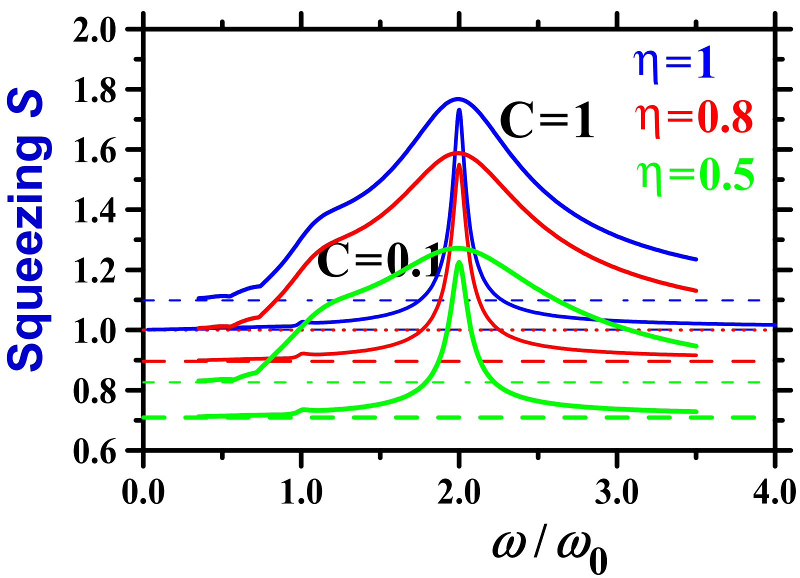
$$d\langle D_p\rangle/dt = -2m\omega_0^2 D_{xp} + (k_0^2\hbar^2/2S_0\eta)\,|f(t)| - (2k_0^2/S_0)\,|f(t)|\,D_{xp}^2$$

$$d\langle D_{xp}\rangle/dt = (1/m)D_p - m\omega_0^2 D_x - (2k_0^2/S_0)\,|f(t)|\,D_x D_{xp}$$

Squeezing by sine-modulation, $V(t)=V_0\sin(\omega t)$

Ruskov-Schwab-Korotkov





Squeezing obviously oscillates in time, maximum squeezing at maximum voltage, momentum squeezing shifted in phase by π/2.

$$S \equiv \max_t (\Delta x_0)^2 / D_x$$

Analytics (weak coupling):

$$S(2\omega_0) = \sqrt{3\eta}, \quad \Delta\omega = 0.36\omega_0 C_0 / \sqrt{\eta}$$

η - detector efficiency, C_0 – coupling

$\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$ – ground state width

$D_x=(\Delta x)^2$, $D_{\langle x\rangle}=\langle\langle x\rangle^2\rangle - \langle\langle x\rangle\rangle^2$

Quantum feedback:

$$F = -m\omega_0\gamma_x\langle x\rangle - \gamma_p\langle p\rangle$$

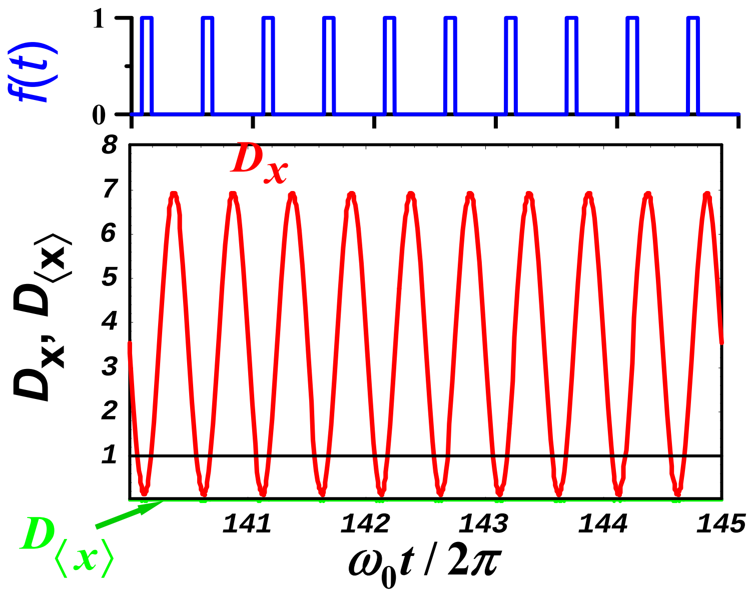
(same as in Hopkins *et al*.; without modulation it cools the state down to the ground state)

Feedback is sufficiently efficient, $D_{\langle x\rangle}$ Ü D_x

Squeezing up to 1.73 at $\omega=2\omega_0$

Squeezing by stroboscopic (pulse) modulation

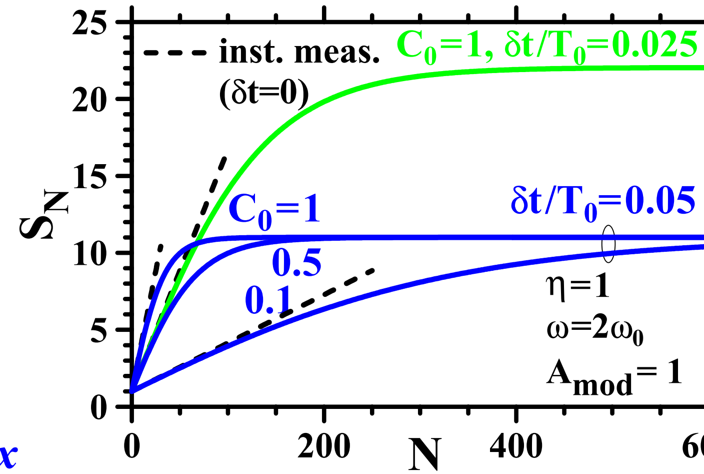
pulse modulation $\omega = 2\omega_0 / n$

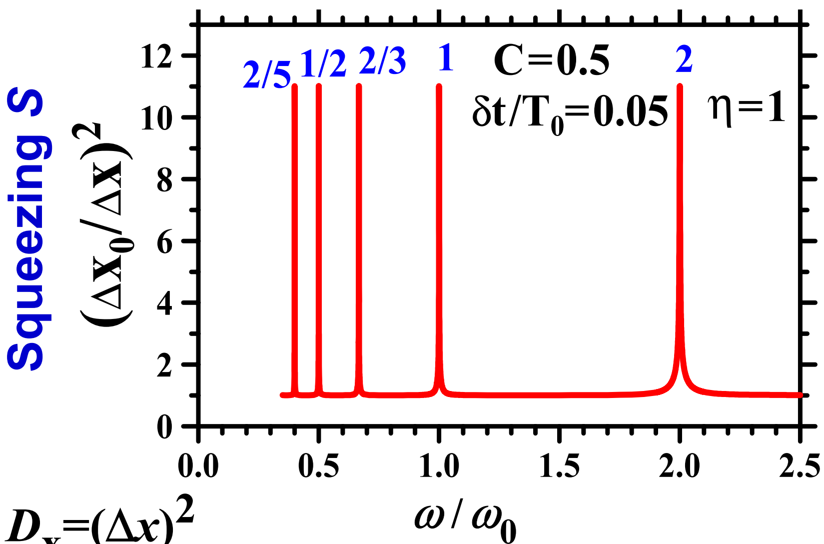


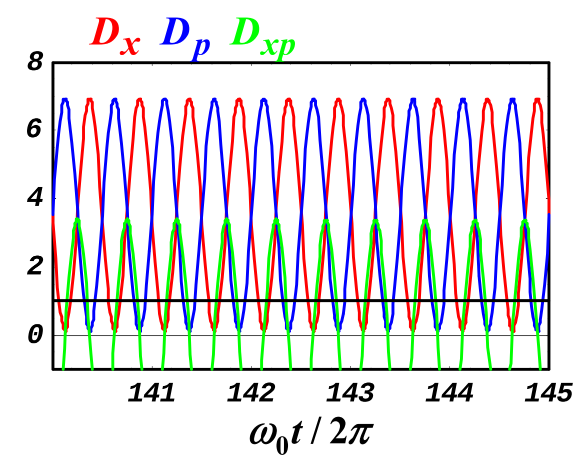
$D_{\langle x \rangle} \ll D_x$

using feedback

Squeezing buildup (in time)







$D_x = (\Delta x)^2$

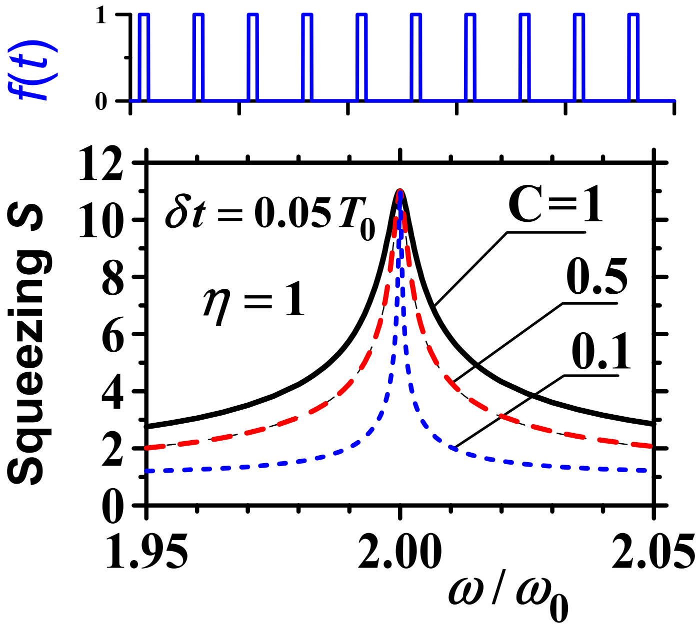
***S* á 1**

Efficient squeezing at $\omega = 2\omega_0/n$

(natural QND condition)

Ruskov-Schwab-Korotkov

Squeezing by stroboscopic modulation



Analytics (weak coupling, short pulses)

Maximum squeezing

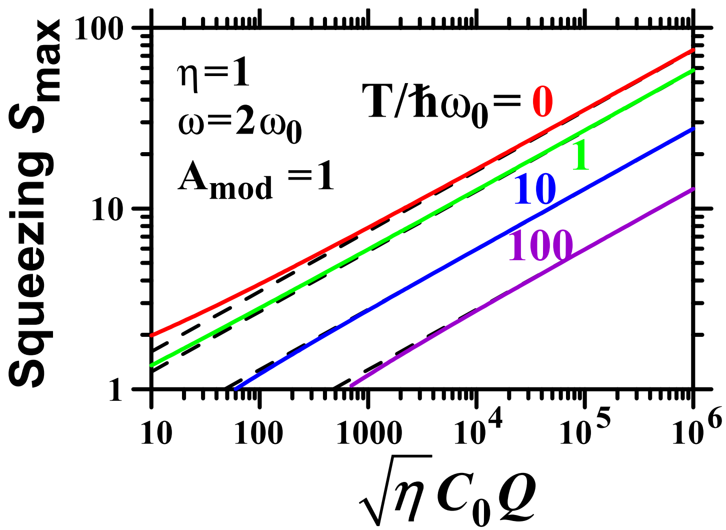
$$S(2\omega_0 / n) = \frac{2\sqrt{3\eta}}{\omega_0 \delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\delta t)^3 \omega_0^4}{\pi n^2 \sqrt{3\eta}}$$

C_0 – dimensionless coupling with detector
δt – pulse duration, $T_0 = 2\pi/\omega_0$
η – quantum efficiency of detector

Squeezing requires $\sim \sqrt{3\eta} / C_0(\omega_0 \delta t)^2$ pulses



Finite *Q*-factor and finite temperature limit

maximum squeezing S_{max}

$$S_{max} = \frac{3}{4} \left[\frac{\sqrt{\eta}\, C_0 Q}{\coth(\hbar\omega_0 / 2T)} \right]^{1/3}$$

(So far in experiment $\eta^{1/2} C_0 Q \sim 0.1$)

Verification of nanoresonator squeezing

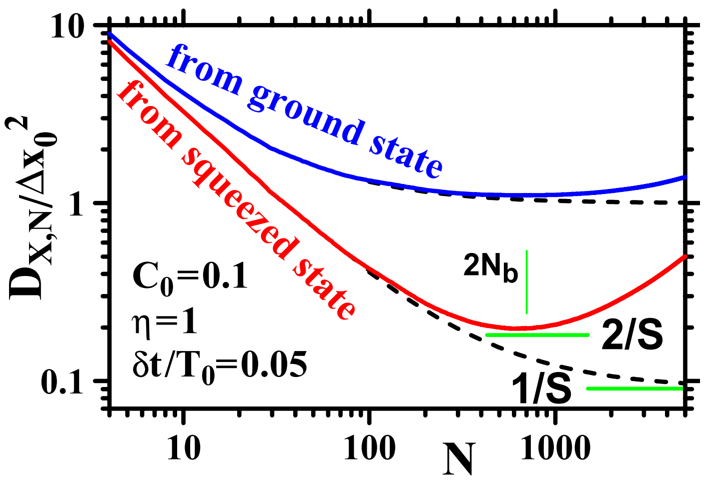
Procedure: 1) prepare squeezed state by stroboscopic measurement,
2) switch off quantum feedback
3) measure in the stroboscopic way $X_N = \frac{1}{N}\sum_{j=1}^{N} x_j$

For instantaneous measurements ($\delta t \to 0$) the variance of X_N is

$$D_{X,N} = \frac{\hbar}{2m\omega_0}\left(\frac{1}{S} + \frac{1}{NC_0\omega_0\delta t}\right) \to \frac{1}{S}(\Delta x_0)^2 \quad \text{at } N \to \infty$$

S – squeezing,
Δx_0 – ground state width

Distinguishable from ground state (S=1) in one run for S à 1
(error probability ~$S^{-1/2}$)



About twice worse for continuous measurements because of extra “heating”

$$\min_N D_{X,N} \sim 2(\Delta x_0)^2 / S$$

Squeezed state is distinguishable in one run (with small error probability), therefore suitable for ultrasensitive force measurement beyond standard quantum limit

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Conclusion

- Modulation of detector voltage with $\omega = 2\omega_0/n$ periodically squeezes x-width of nanoresonator state (“breathing mode”)

- Packet center is randomly “heated” by measurement; quantum feedback can cool it down

- Sine-modulation leads to a small squeezing (<1.73), stroboscopic (pulse) modulation can lead to a strong squeezing (>>1) even for a weak coupling with detector

- Potential application: ultrasensitive force measurement beyond standard quantum limit