Simple quantum feedback of a solid-state qubit

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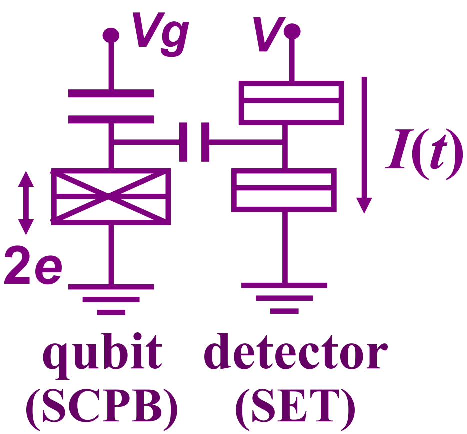
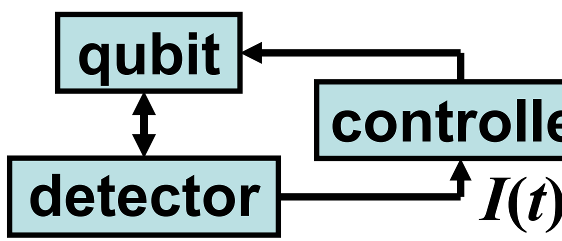
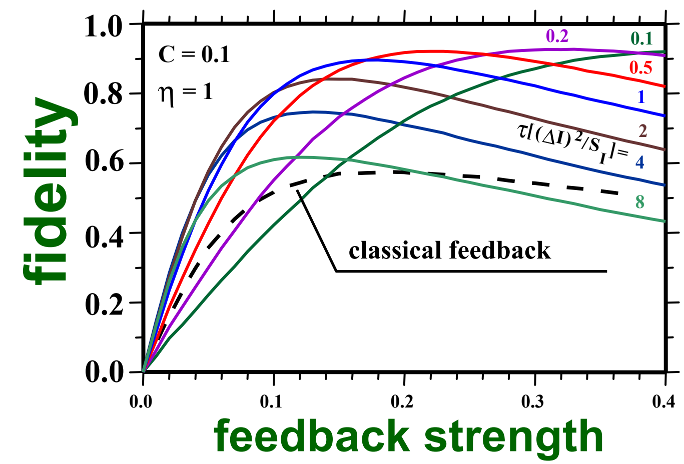


Goal: keep coherent oscillations forever





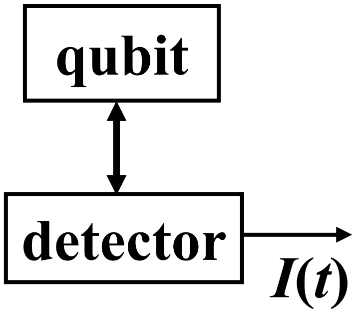
Outline:

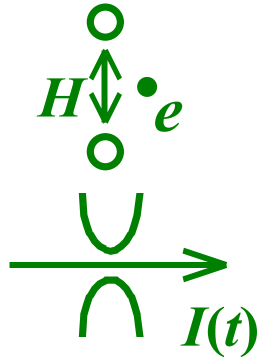
- Introduction (Bayesian formalism for continuous quantum measurement, Bayesian quantum feedback)
- Simple quantum feedback of a qubit

Support:

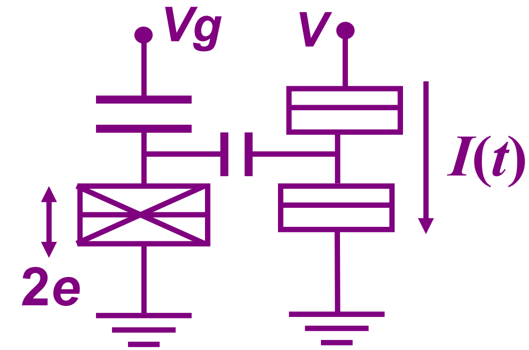
cond-mat/0404696

The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$H = H_{QB} + H_{DET} + H_{INT}$

$H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε – asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}/$Ñ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

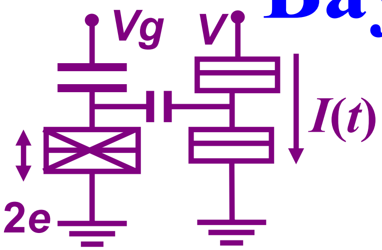
DQD and QPC
(setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

Bayesian formalism for a single qubit



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \rightarrow I_1$, $|2\rangle \rightarrow I_2$ $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \leq 100\%$

Ideal detector (η=1) does not decohere a single qubit;
then random evolution of qubit *wavefunction* can be monitored

For simulations: $I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

Averaging over ξ(t) → conventional master equation

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, **Milburn**, **Wiseman,** Onofrio, Habib, Doherty, etc. (incomplete list)

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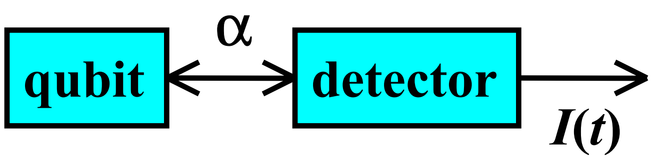
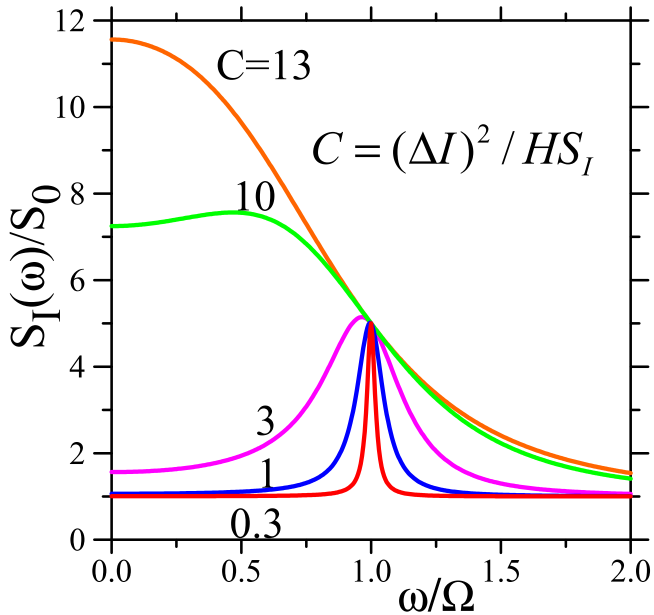
Measured spectrum of qubit coherent oscillations



What is the spectral density $S_I(\omega)$ of detector current?



Assume classical output, eV » $\hbar\Omega$

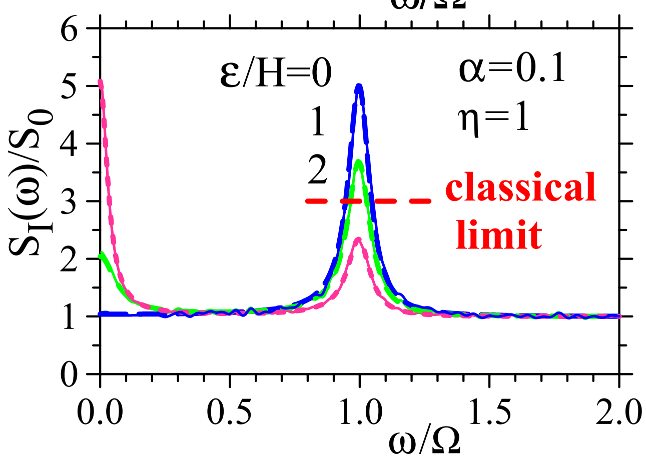
$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003



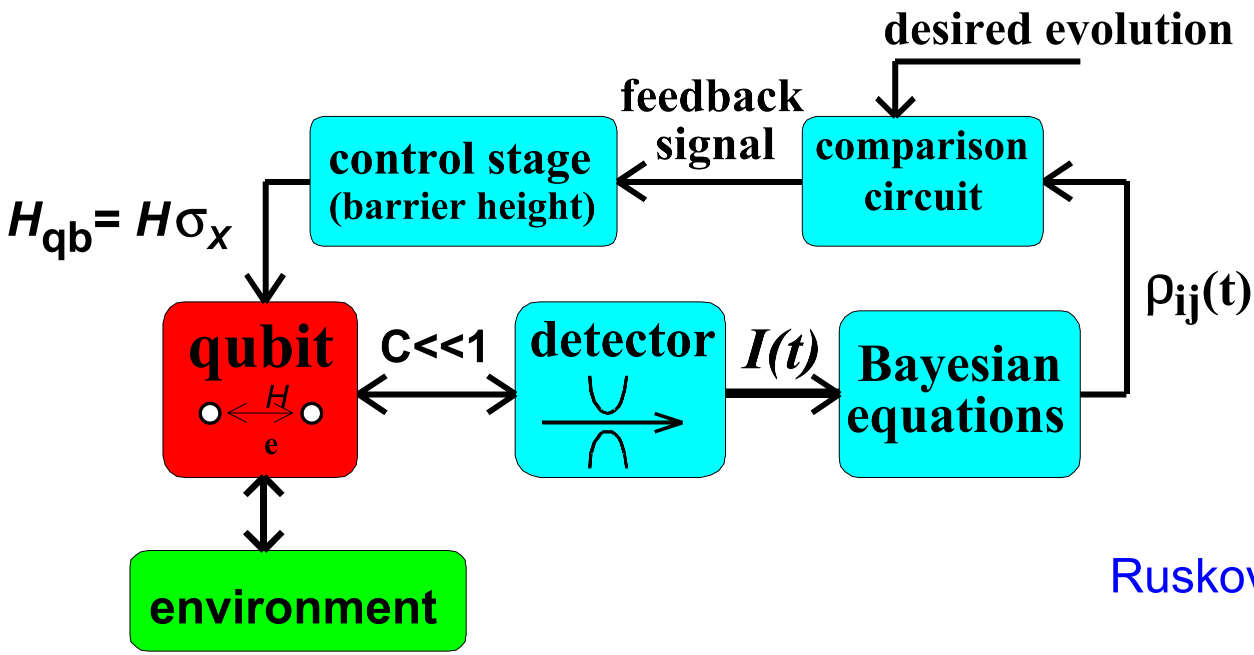
Weak coupling, $\alpha = C/8$ « 1

$$S_I(\omega) = S_0 + \frac{\eta S_0\varepsilon^2/H^2}{1+(\omega\hbar^2\Omega^2/4H^2\Gamma)^2} + \frac{4\eta S_0(1+\varepsilon^2/2H^2)^{-1}}{1+[(\omega-\Omega)\Gamma(1-2H^2/\hbar^2\Omega^2)]^2}$$

Contrary:
Stace-Barrett, 2003 (PRL 2004)

Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov-Korotkov, 2001

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit "fresh")

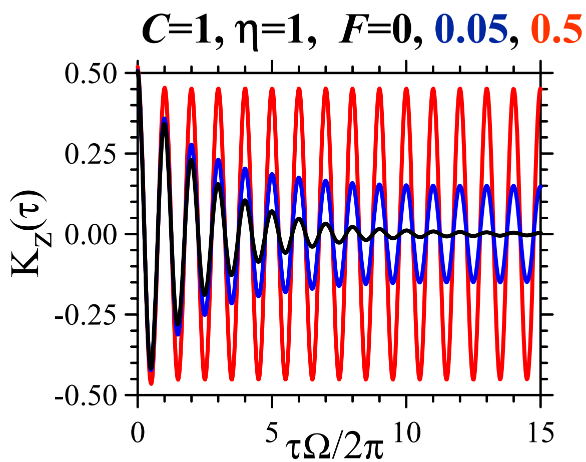
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

(no extra environment)

Qubit correlation function



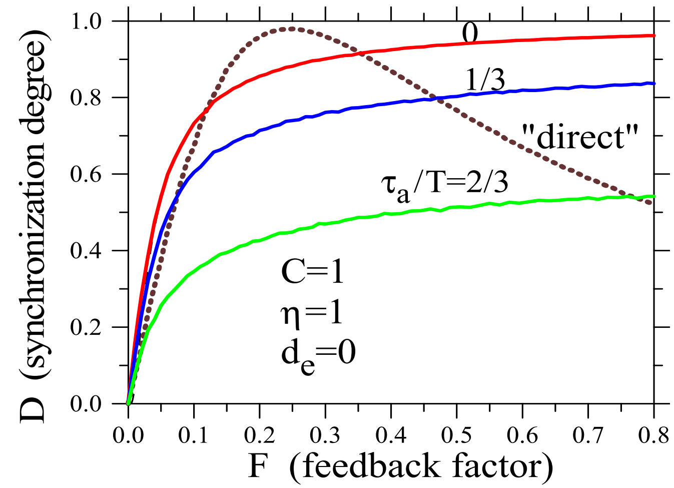
$$K_z(\tau) = \frac{\cos \Omega t}{2} \exp\left[\frac{C}{16F} (e^{-2FH\tau/\hbar} - 1) \right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega t}{2} (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F} (e^{-2FH\tau/\hbar} - 1) \right] + \frac{S_I}{2} \delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

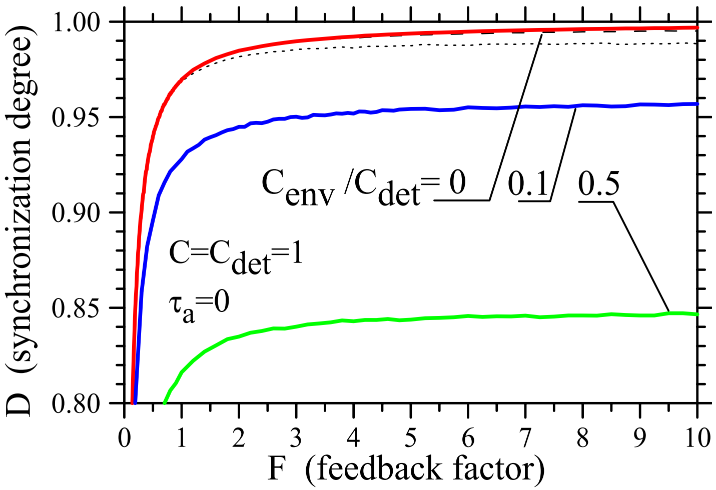
$$D = 2\langle \mathrm{Tr}\rho\rho_{desir} \rangle - 1$$

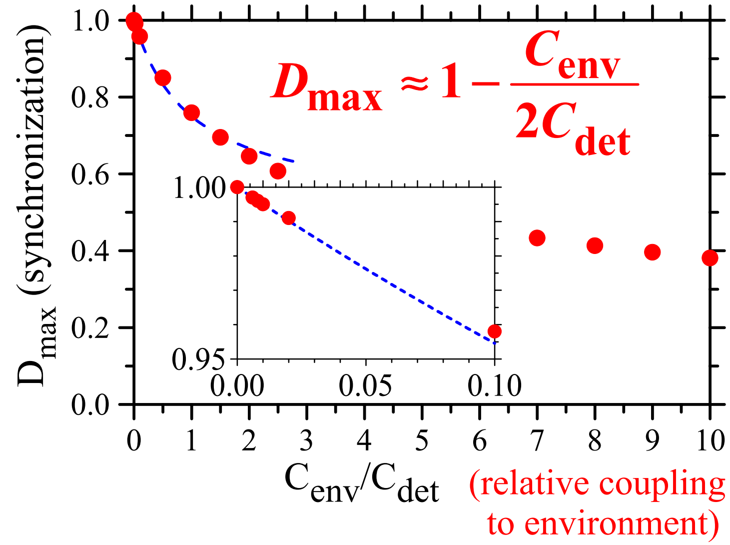
For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%

$$D = \exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Quantum feedback in presence of decoherence by environment





Big experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (>>Ω, GHz-range) of the line delivering noisy signal *I*(*t*) to the "processor"

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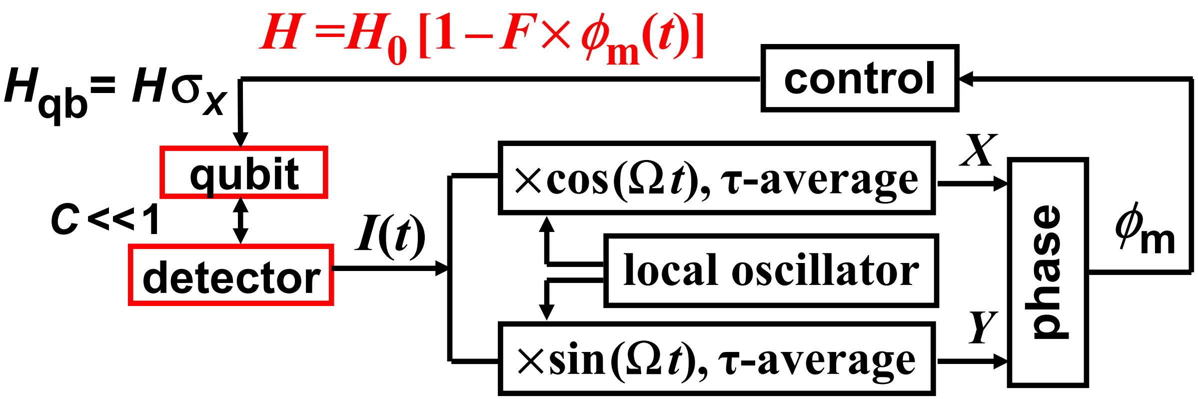
(A.K., cond-mat/0404696)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations

(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0]\cos(\Omega t')\exp[-(t-t')/\tau]\,dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0]\sin(\Omega t')\exp[-(t-t')/\tau]\,dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

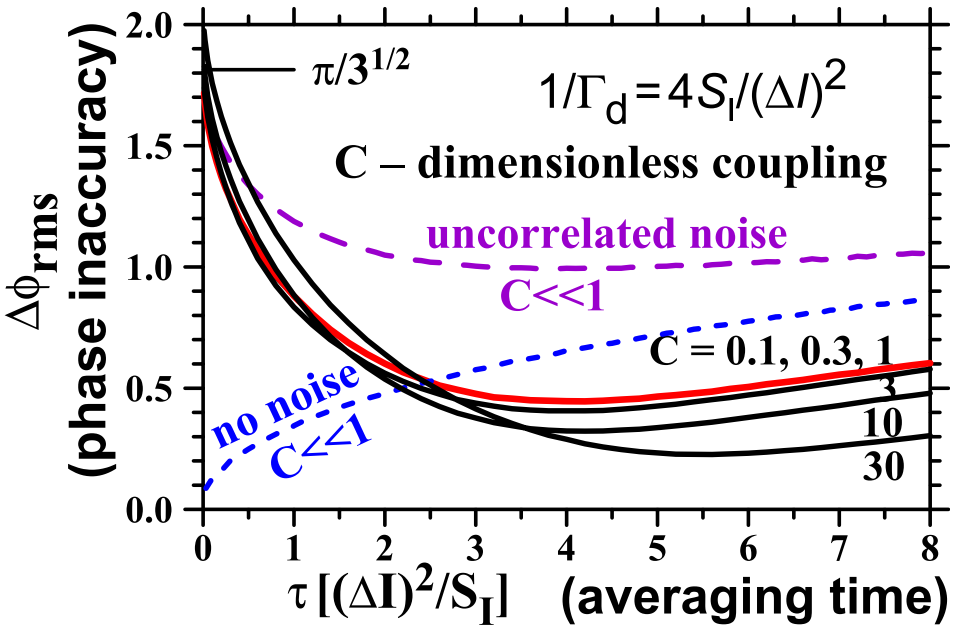
Advantage: simplicity and relatively narrow bandwidth $(1/\tau \sim \Gamma_d \ll \Omega)$

Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

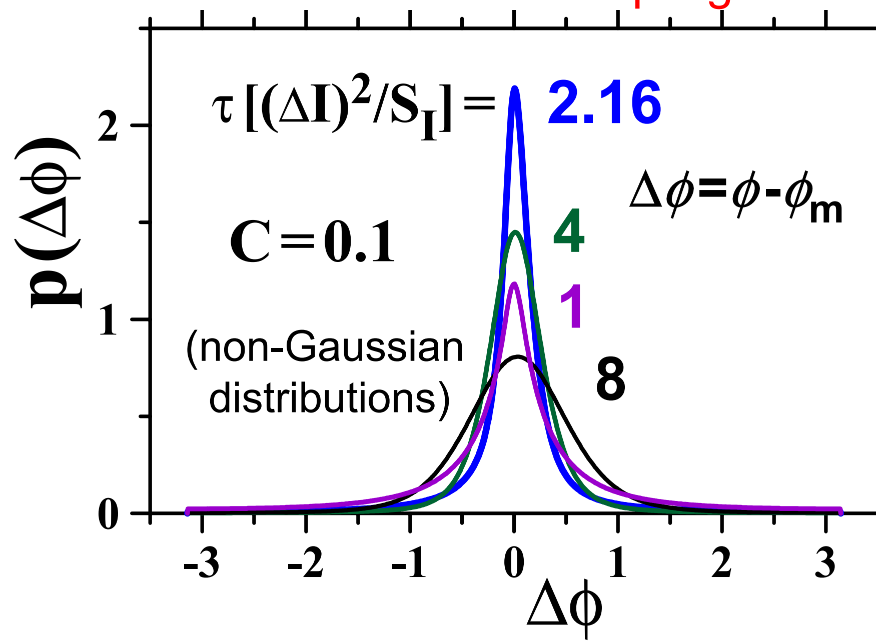
(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures

(no feedback yet)



weak coupling C<<1



Noise improves the monitoring accuracy!

(purely quantum effect, "reality follows observations")

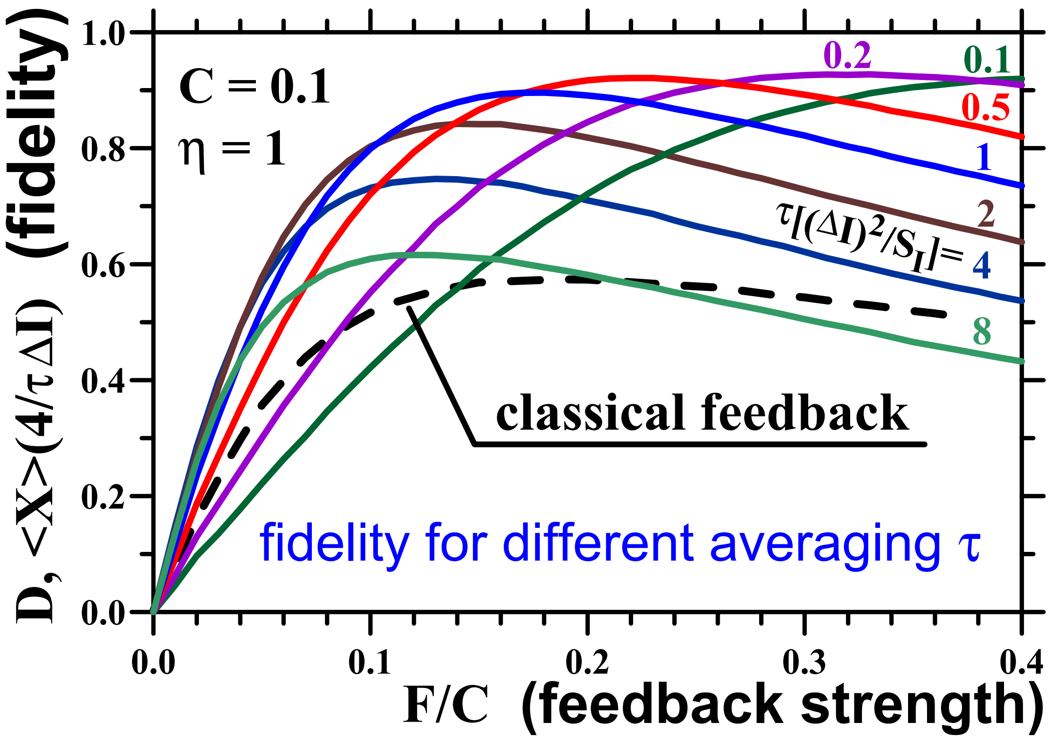
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44)\approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

($F_Q \approx 95\%$)

How to verify feedback operation experimentally?

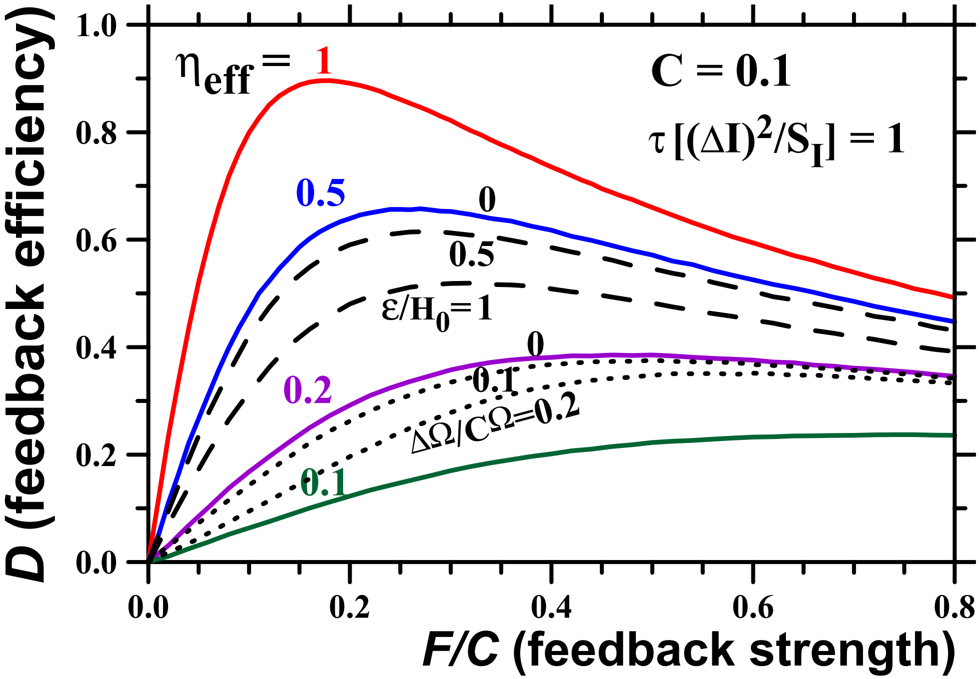
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau \Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



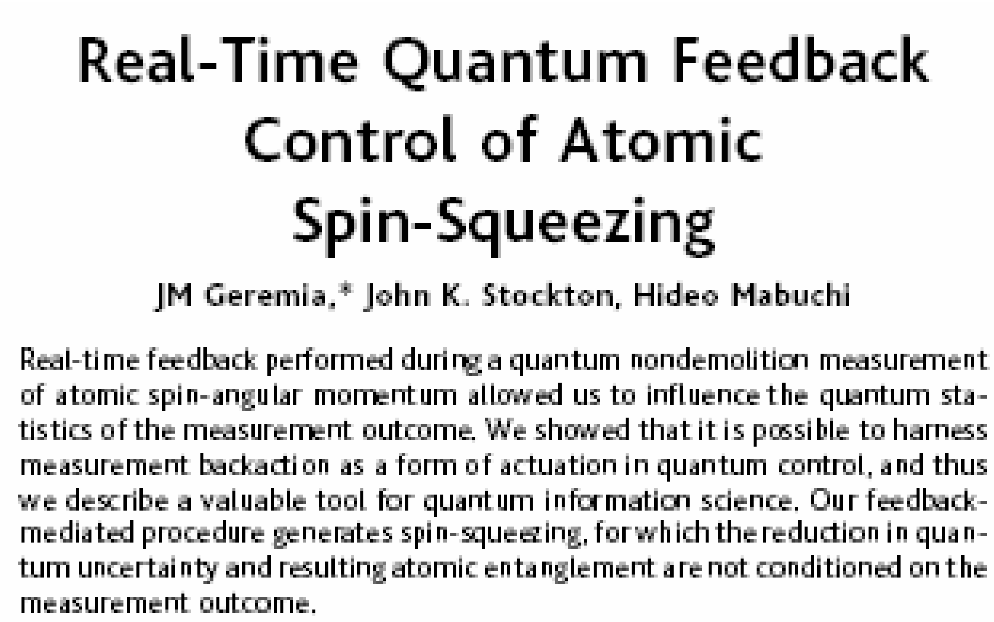
Main features:

- Fidelity F_Q up to ~95% achievable (D~90%)
- Natural, practically classical feedback setup
- Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)
- Detector efficiency (ideality) η~0.1 still OK
- Robust to asymmetry ε and frequency shift ΔΩ
- Simple verification: positive in-phase quadrature ⟨X⟩

Simple enough experiment?!

Quantum feedback in optics

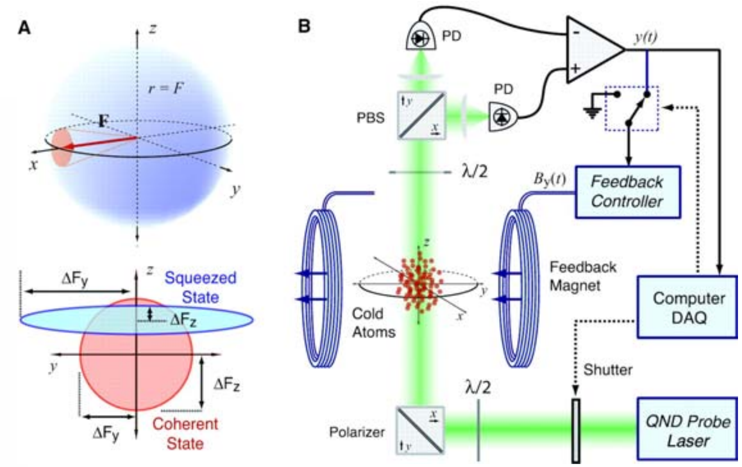
Recent experiment: **Science 304, 270 (2004)**

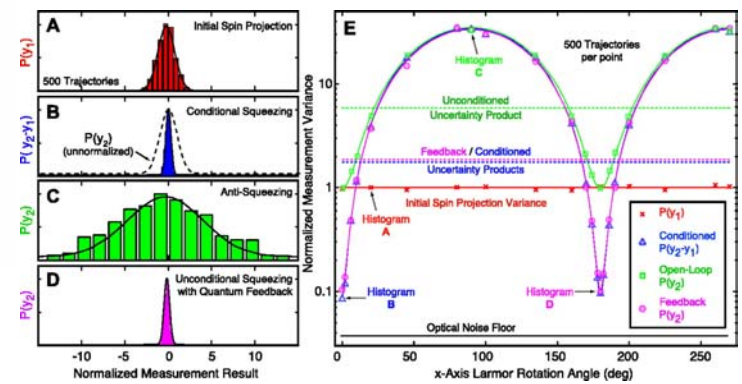


Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.





First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Conclusion

- Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations

- Price for simplicity is a less-then-ideal operation (fidelity is limited by ~95%)

- Feedback operation is much better than expected

- Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)